

Deep Learning

First classifier: **Nearest Neighbor**

```
def train(images, labels):  
    # Machine learning!  
    return model
```



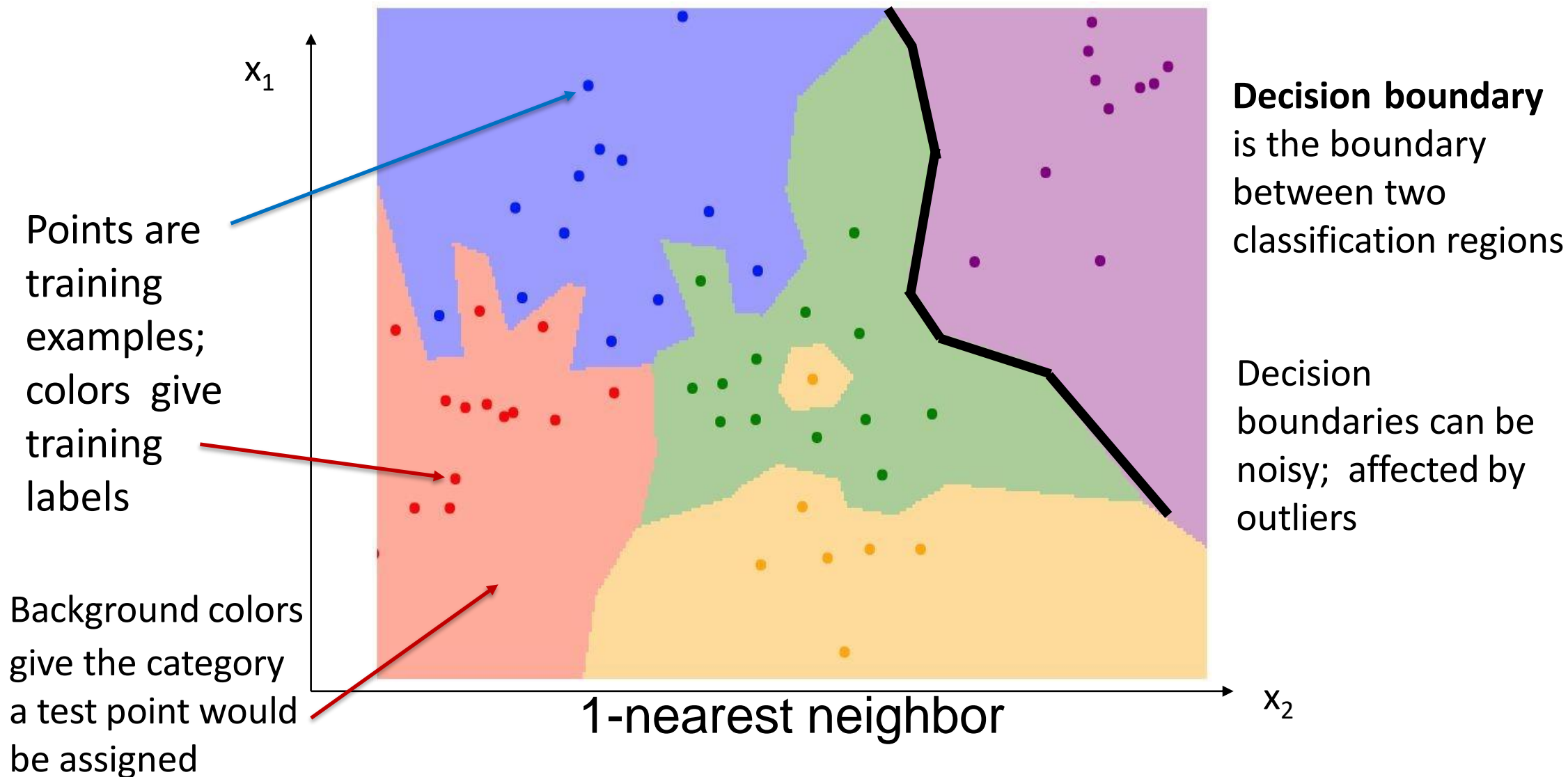
Memorize all
data and labels

```
def predict(model, test_images):  
    # Use model to predict labels  
    return test_labels
```



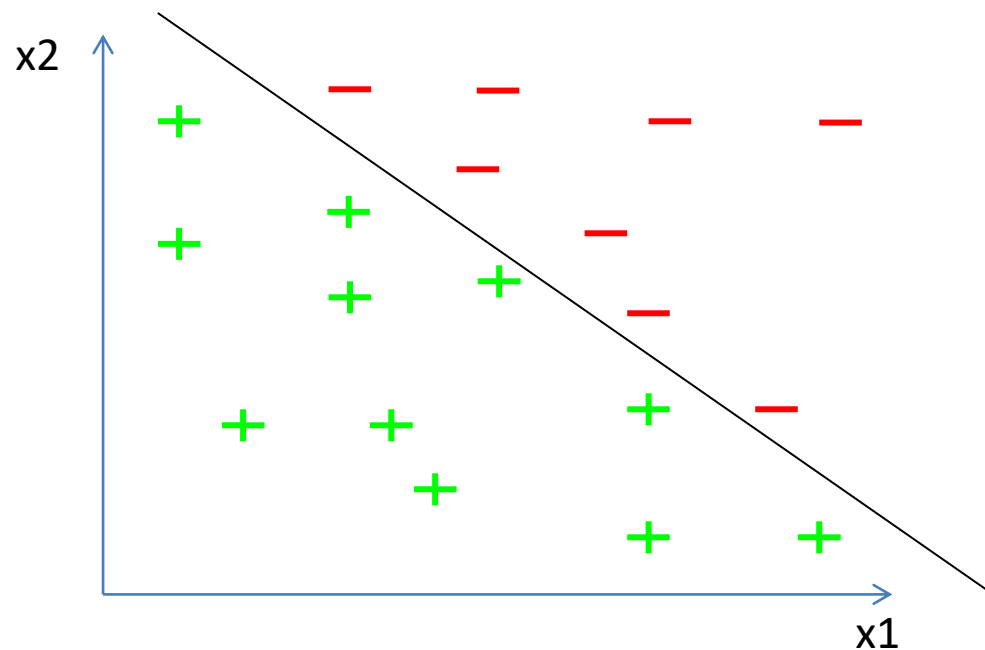
Predict the label
of the most similar
training image

Example



Linear classifiers : Motivation

- kNN produce decision boundaries by calculating them during prediction.
- Can we define a (simple) function during training to define decision boundaries directly?



Parametric Approach: Linear Classifier

$$f(x, W) = Wx$$

Image



$$f(x, W)$$



10 numbers defining class scores



$$W$$

parameters
or weights

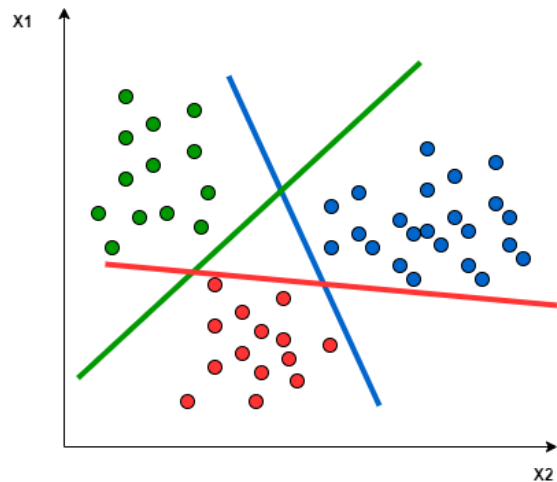
Array of **32x32x3** numbers
(3072 numbers total)

Parametric Approach: Linear Classifier

$$\mathbf{w}_1 \cdot \mathbf{x} = w_{1,1} * x_1 + \dots + w_{1,3072} * x_{3072}$$

Shape: (10,3072)

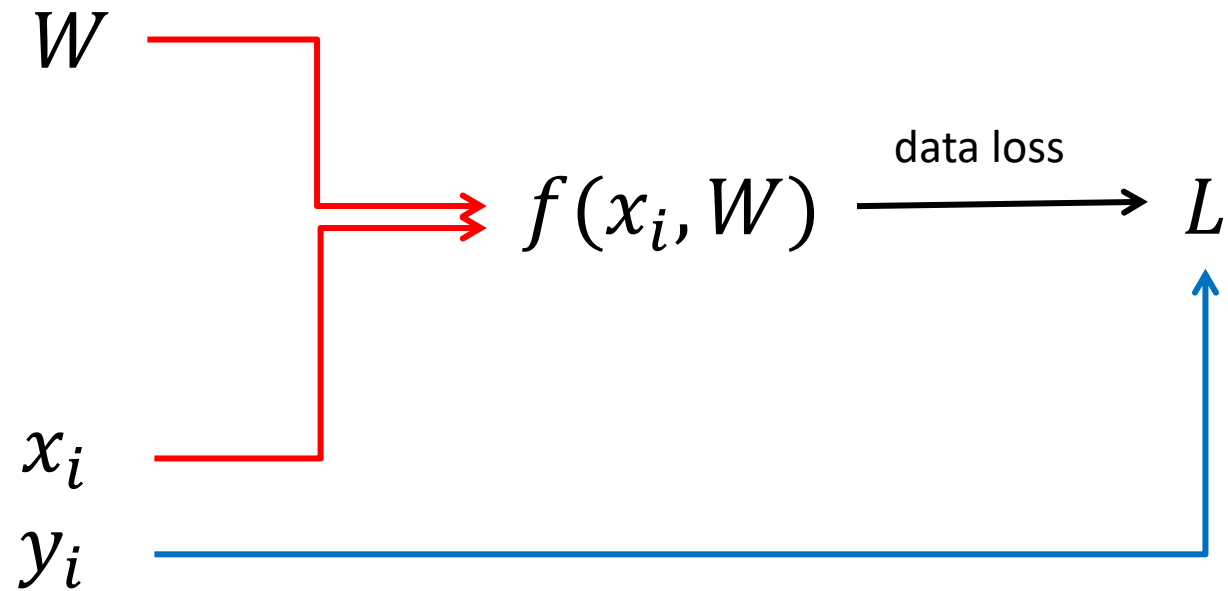
$$f(\mathbf{x}, \mathbf{W}) = \mathbf{W}\mathbf{x} + \mathbf{b}$$



Shape: (10,1)

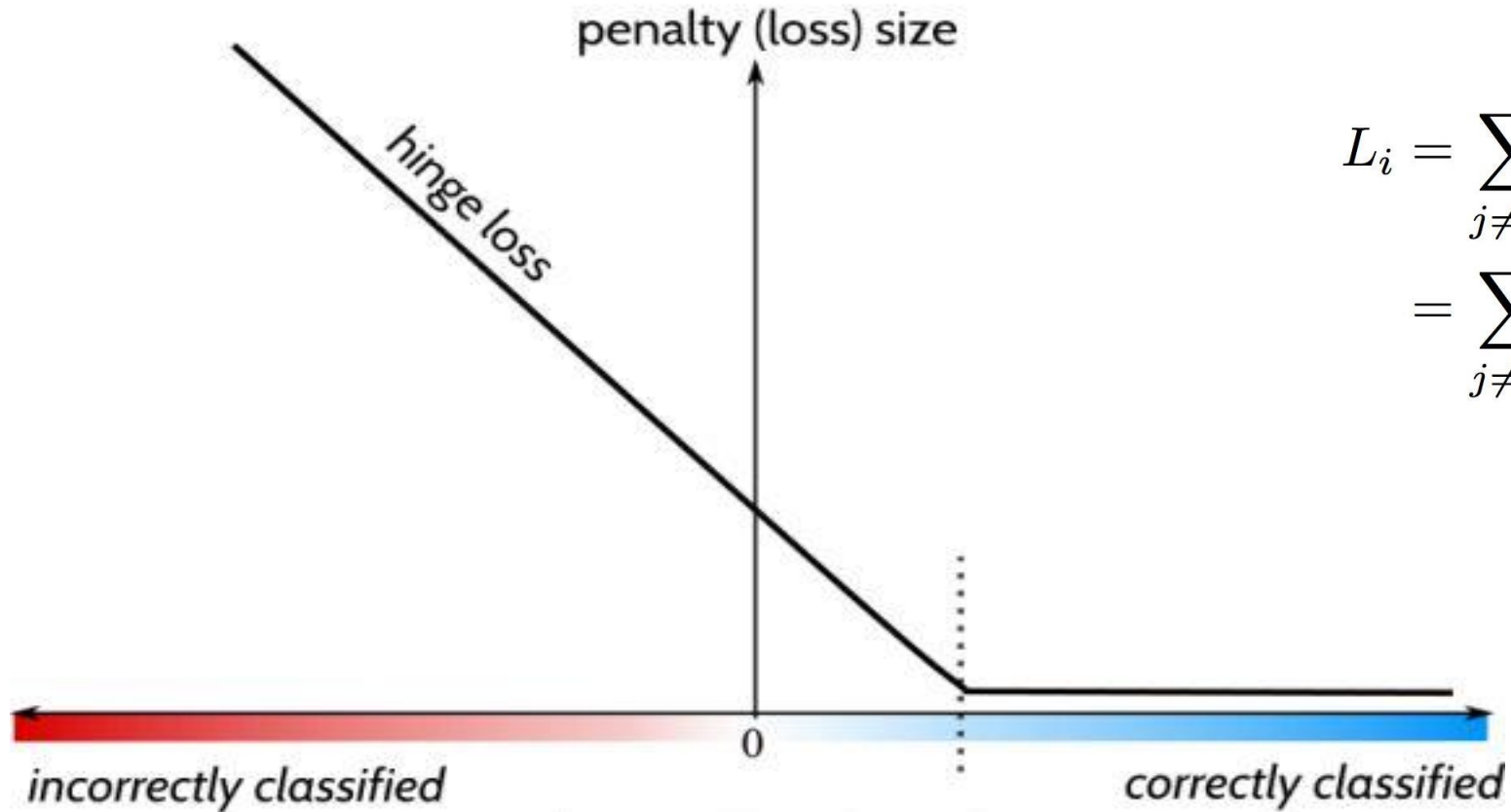
Shape: (3072,1)

Loss Function



L: Metric to assess what loss of data classification our model incurs

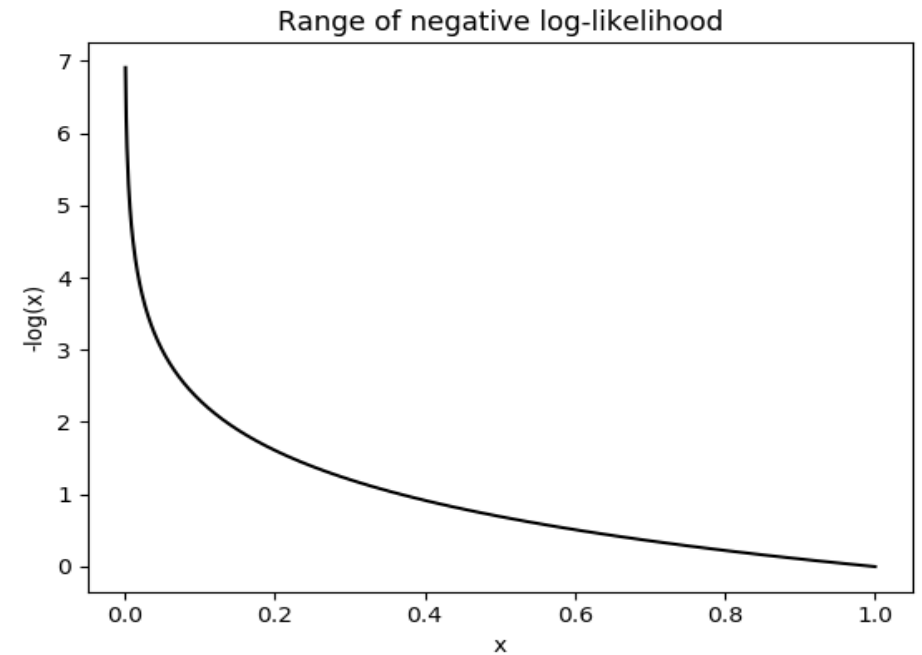
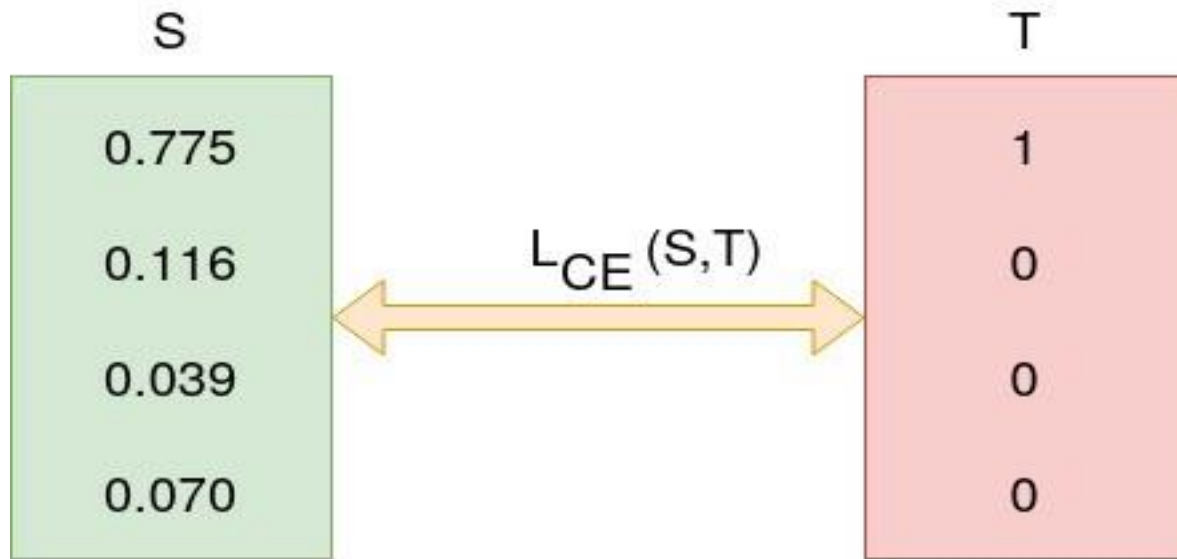
Hinge loss



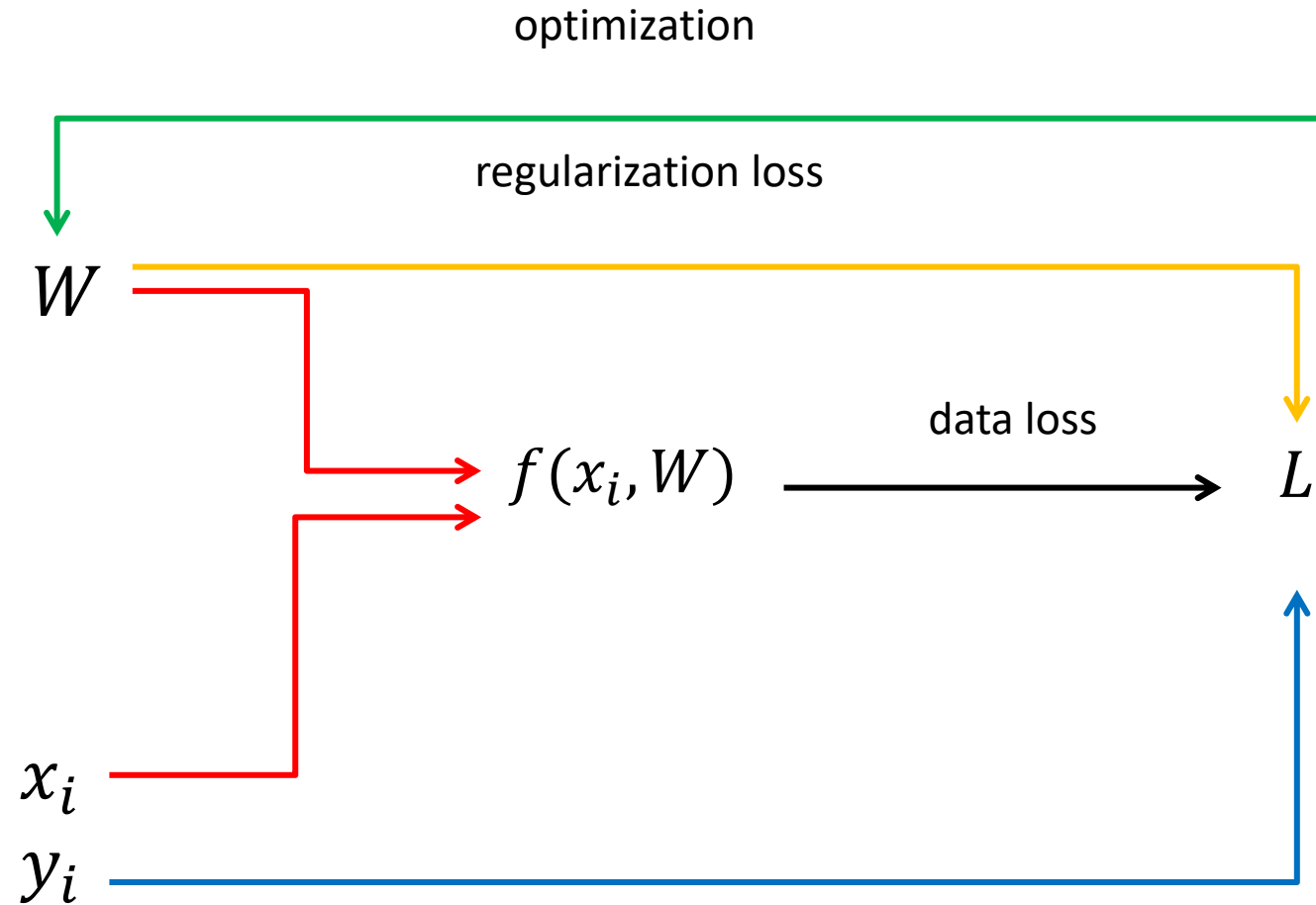
$$L_i = \sum_{j \neq y_i} \begin{cases} 0 & \text{if } s_{y_i} \geq s_j + 1 \\ s_j - s_{y_i} + 1 & \text{otherwise} \end{cases}$$
$$= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Cross-Entropy Loss

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$



Linear Classifier



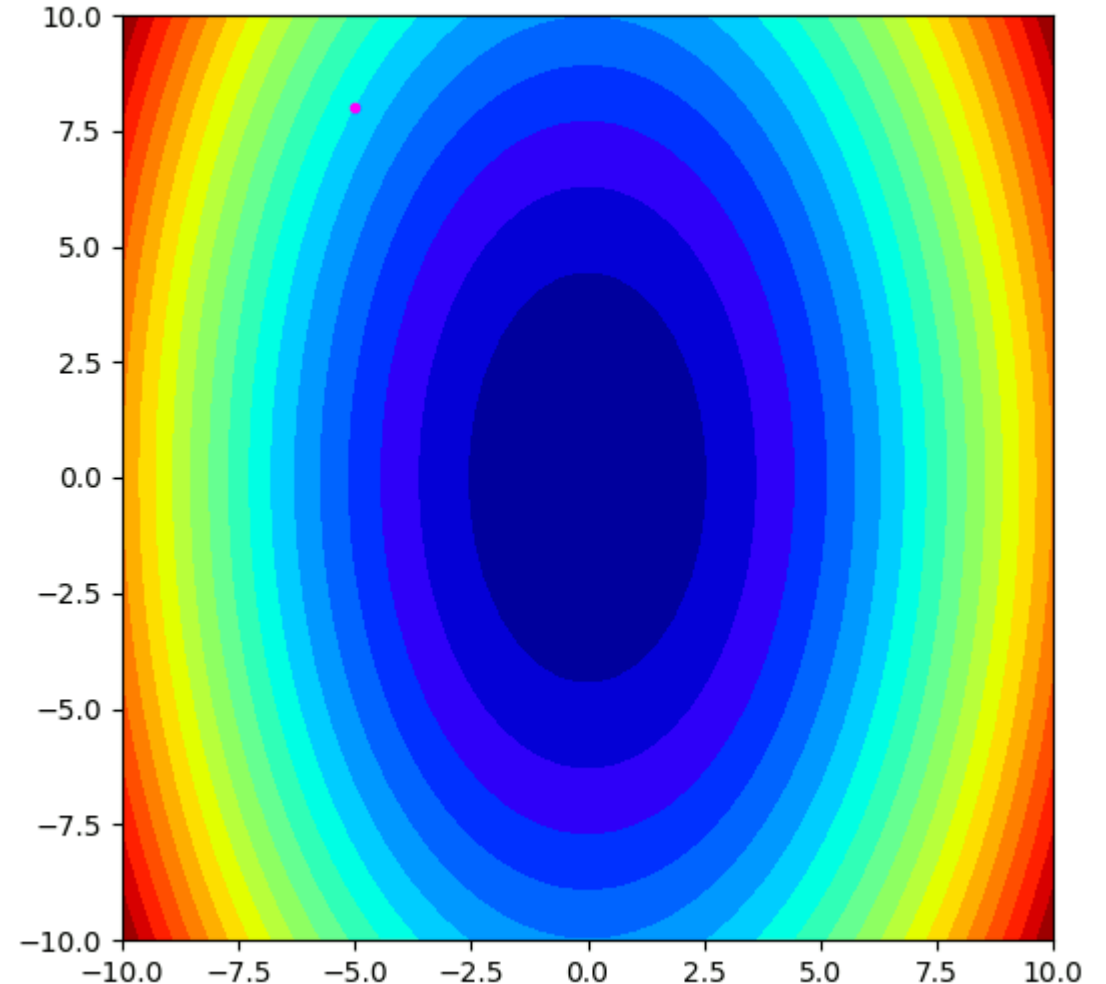
Gradient Descent

Iteratively step in the direction of the negative gradient
(direction of local steepest descent)

```
# Vanilla gradient descent
w = initialize_weights()
for t in range(num_steps):
    dw = compute_gradient(loss_fn, data, w)
    w -= learning_rate * dw
```

Hyperparameters:

- Weight initialization method
- Number of steps
- Learning rate

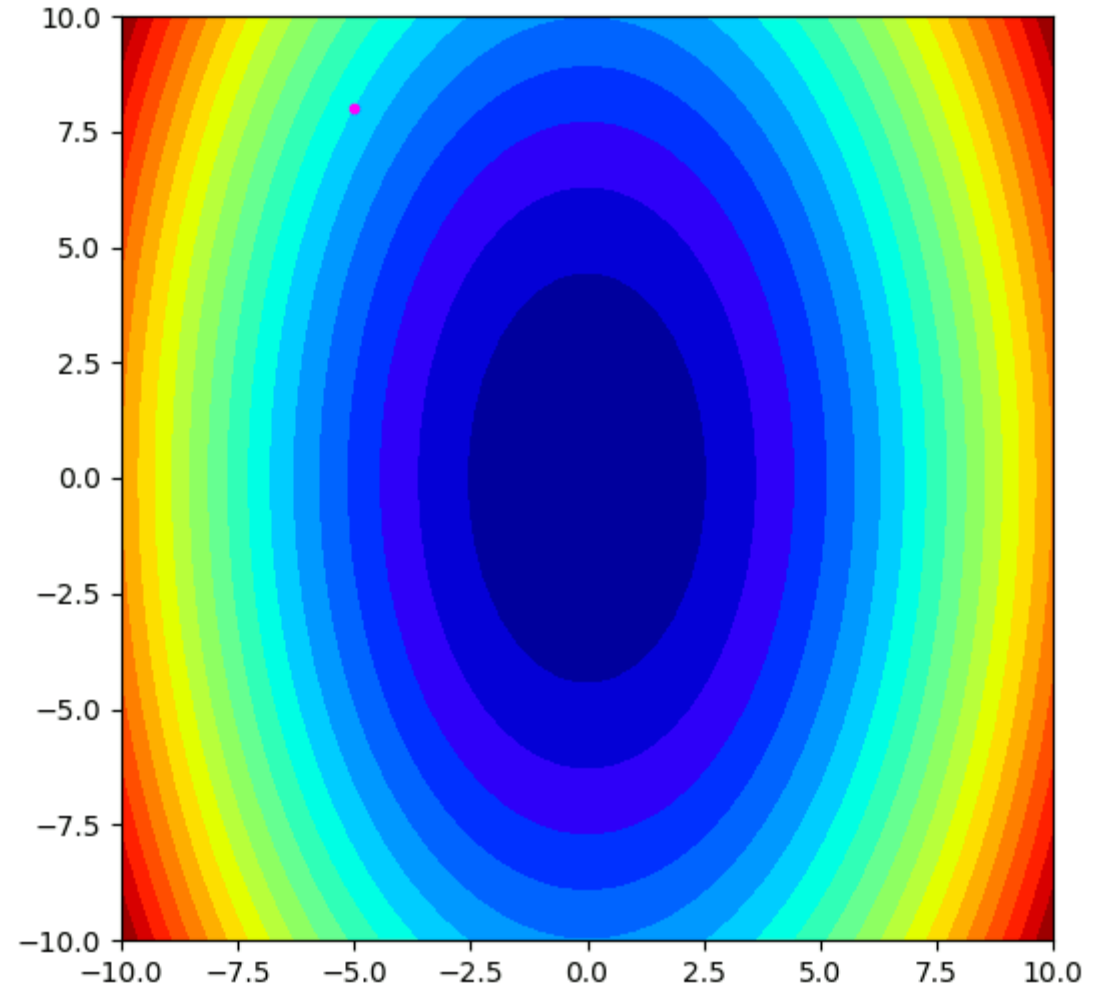


Problems with SGD

Gradients are calculated from minibatches \rightarrow they can be **noisy**

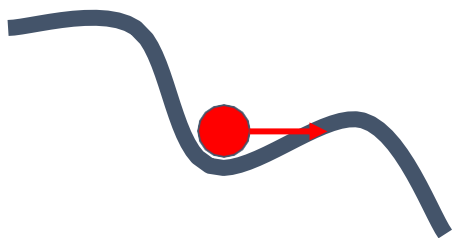
$$L(W) = \frac{1}{N} \sum_{i=1}^N L_i(x_i, y_i, W)$$

$$\nabla_W L(W) = \frac{1}{N} \sum_{i=1}^N \nabla_W L_i(x_i, y_i, W)$$

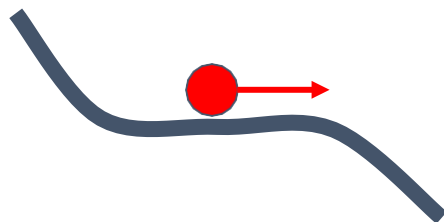


SGD + Momentum

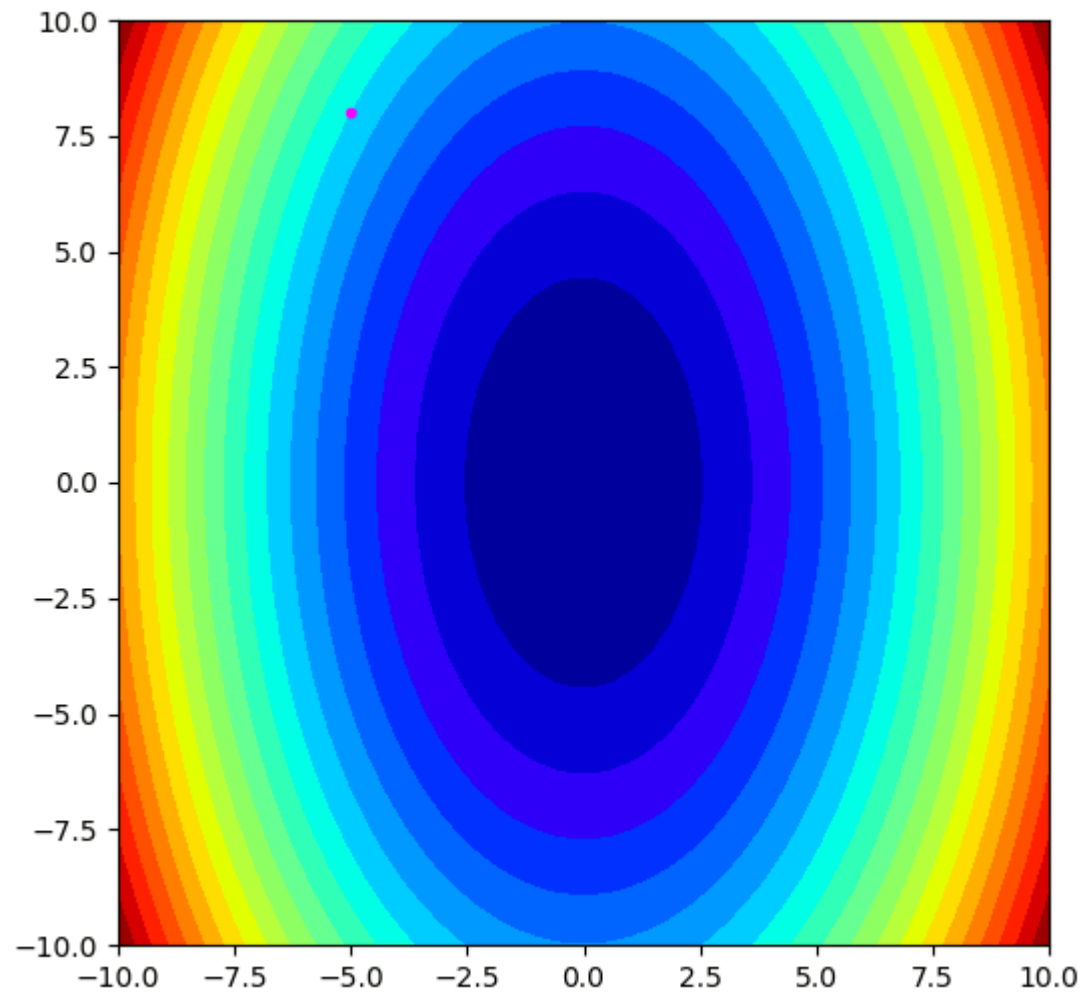
Local Minima



Saddle points

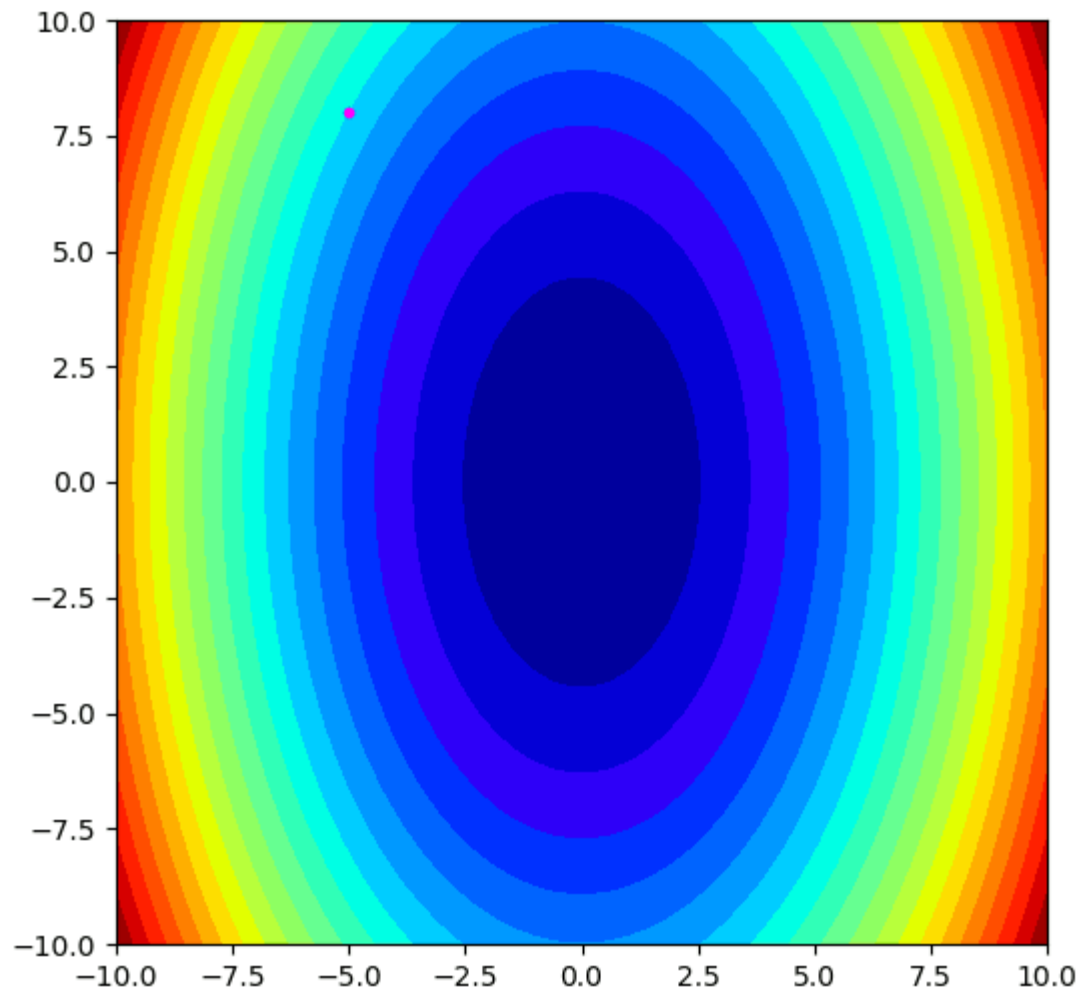


Gradient Noise



— SGD — SGD+Momentum

Adam



- SGD
- SGD+Momentum
- RMSProp
- Adam

Summary

1. Use **Linear Models** for image classification problems
2. Use **Loss Functions** to express preferences over different selection of weights
3. Use **Stochastic Gradient Descent** to minimize our loss functions and train the model

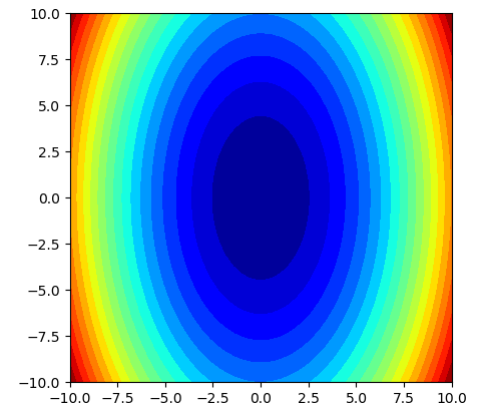
$$s = f(x; W) = Wx$$

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right) \quad \text{Softmax} \quad \text{SVM}$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

$$L = \frac{1}{N} \sum_{i=1}^N L_i + R(W)$$

```
v = 0
for t in range(num_steps):
    dw = compute_gradient(w)
    v = rho * v + dw
    w -= learning_rate * v
```



Cifar10 Linear Classifier

airplane



automobile



bird



cat



deer



dog



frog



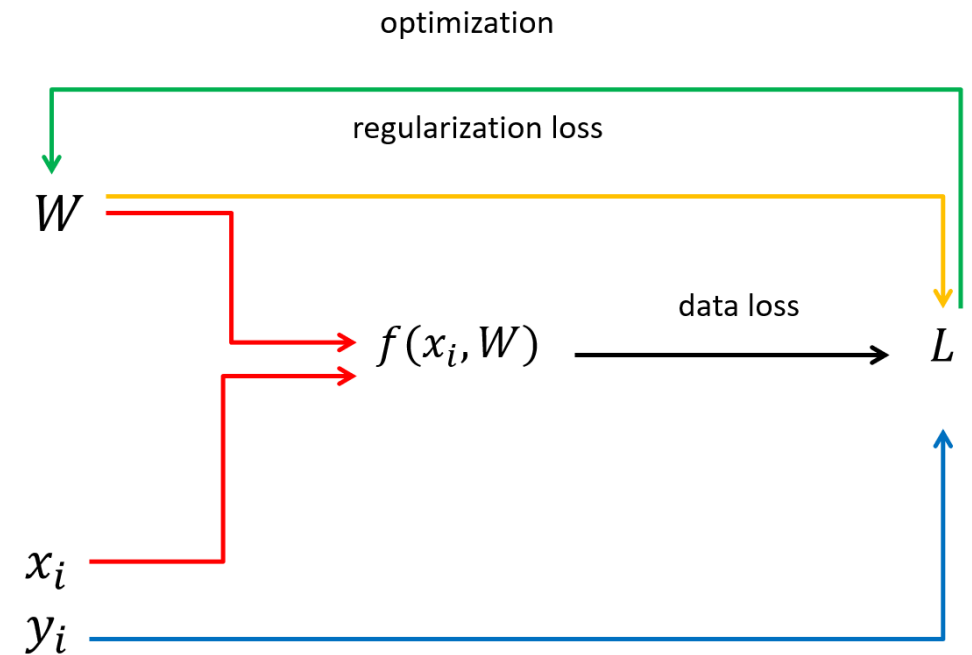
horse



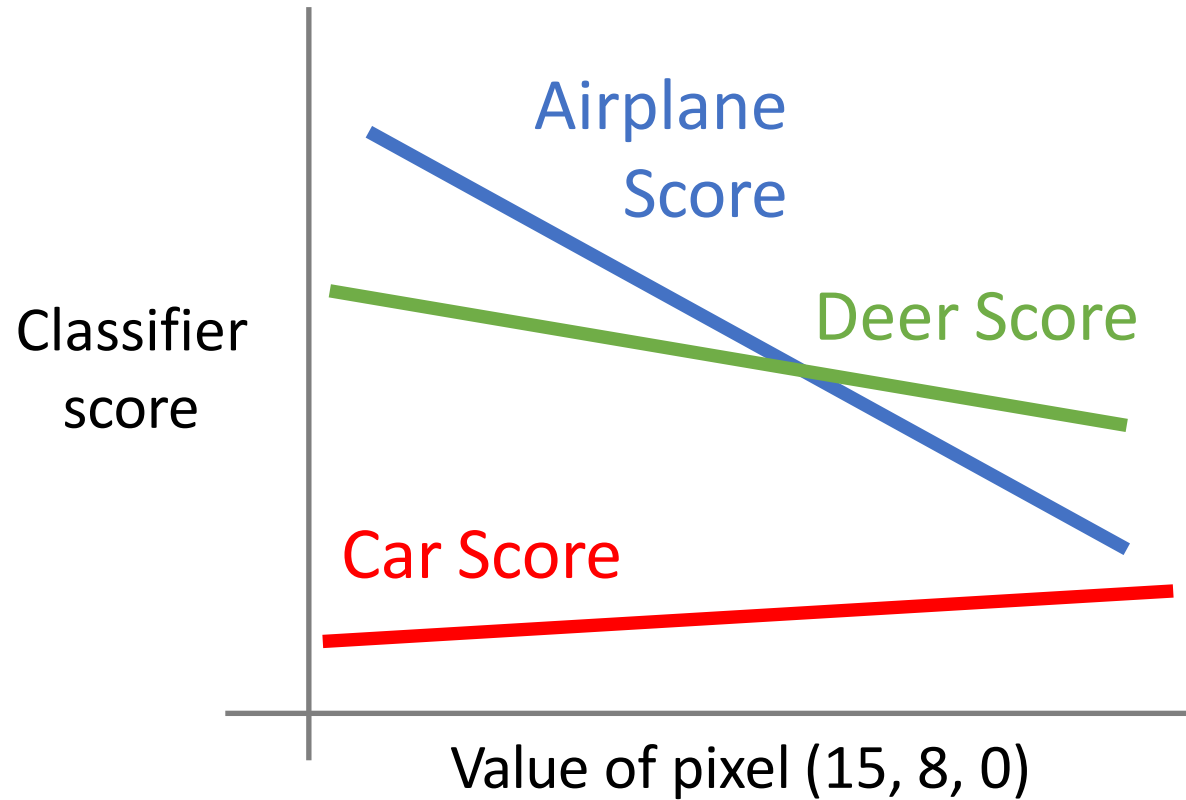
ship



truck



Geometric Interpretation: Linear Classifier

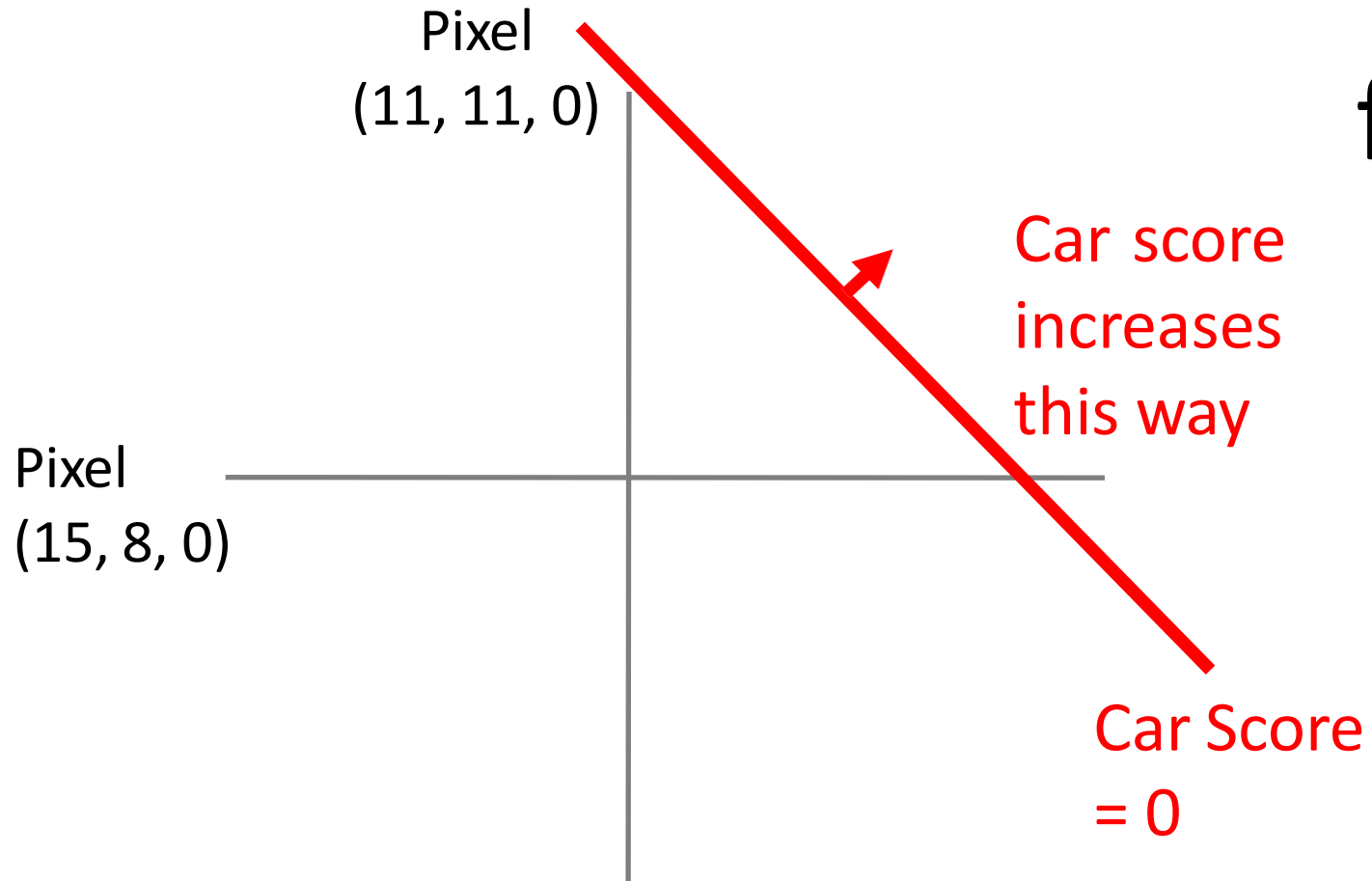


$$f(x, W) = Wx + b$$

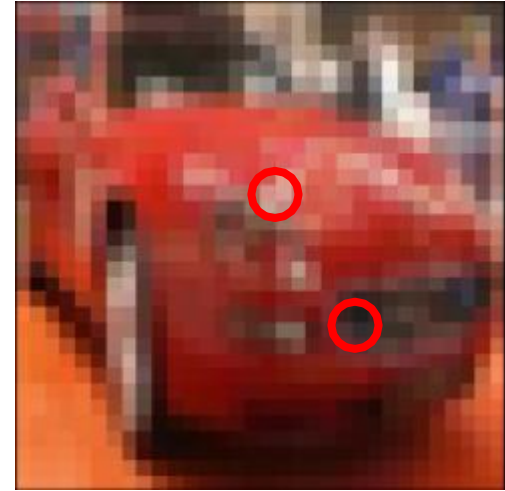


Array of **32x32x3** numbers
(3072 numbers total)

Geometric Interpretation: Linear Classifier

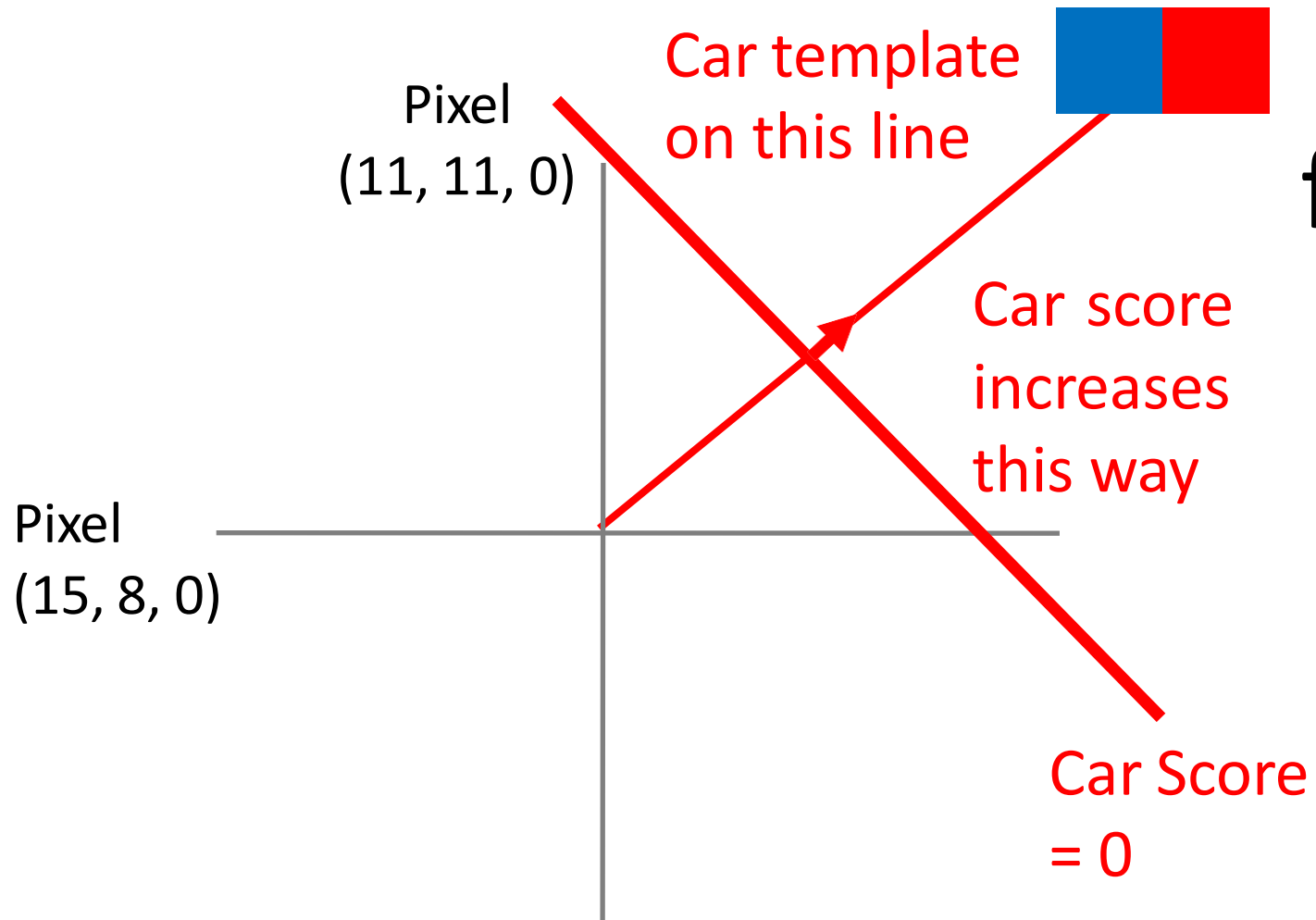


$$f(x, W) = Wx + b$$



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Geometric Interpretation: Linear Classifier

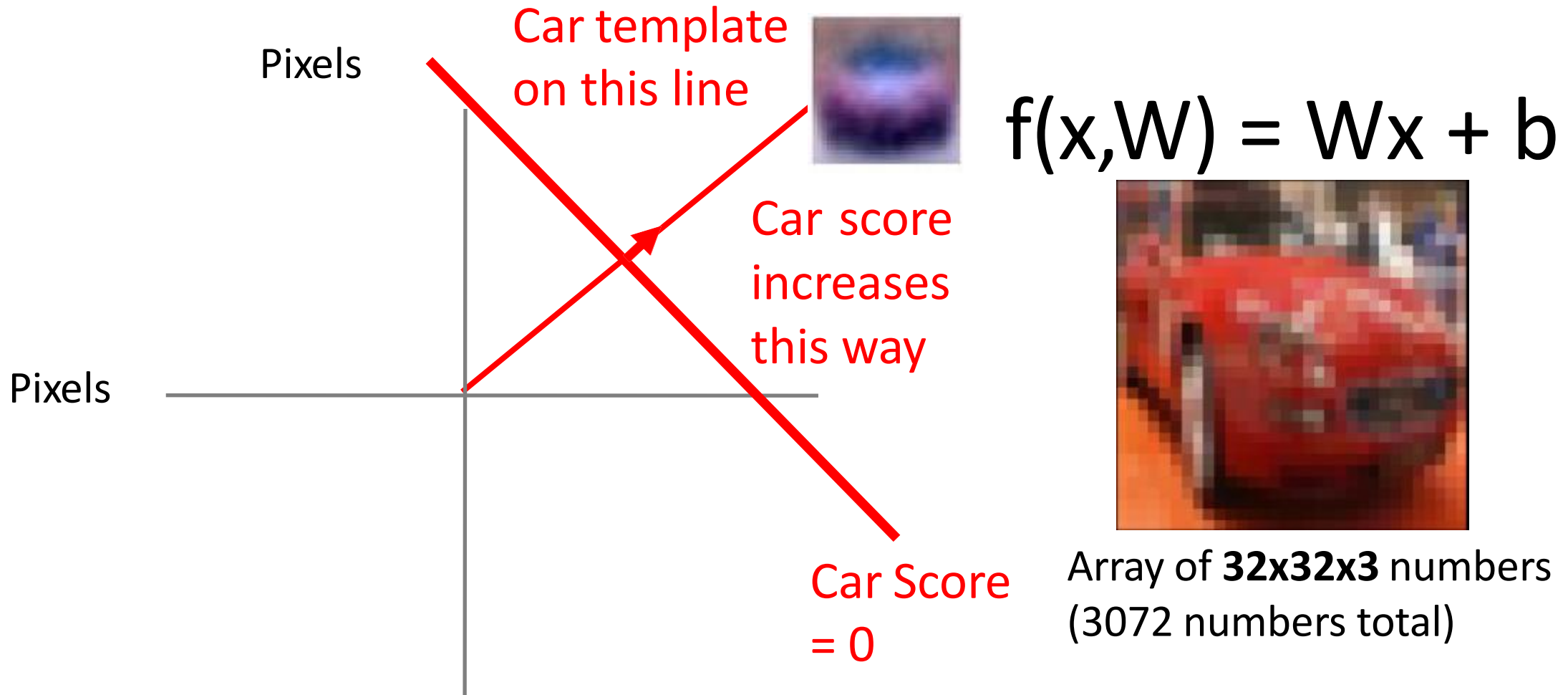


$$f(x, W) = Wx + b$$

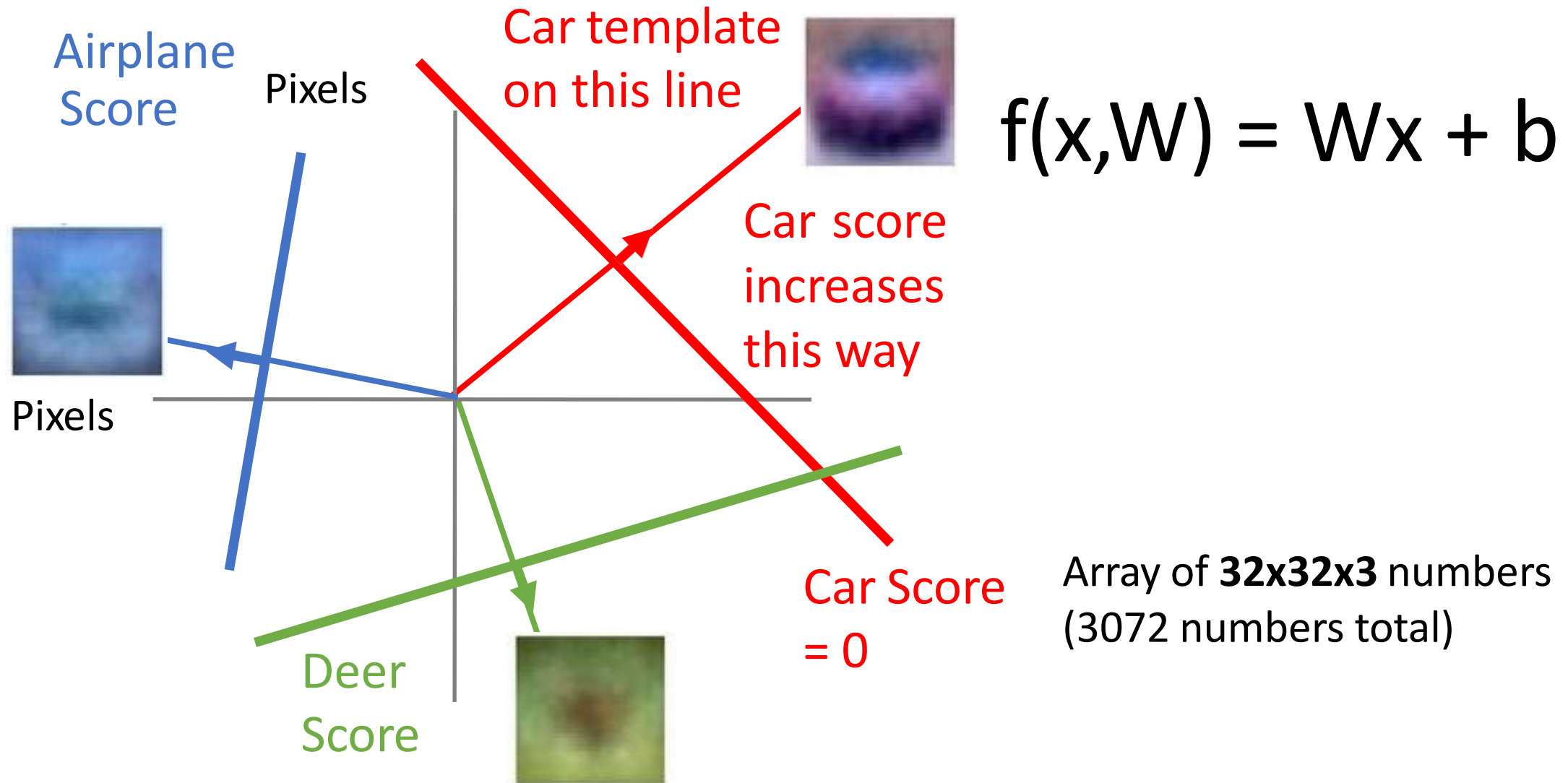


Array of **32x32x3** numbers
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Geometric Interpretation: Linear Classifier

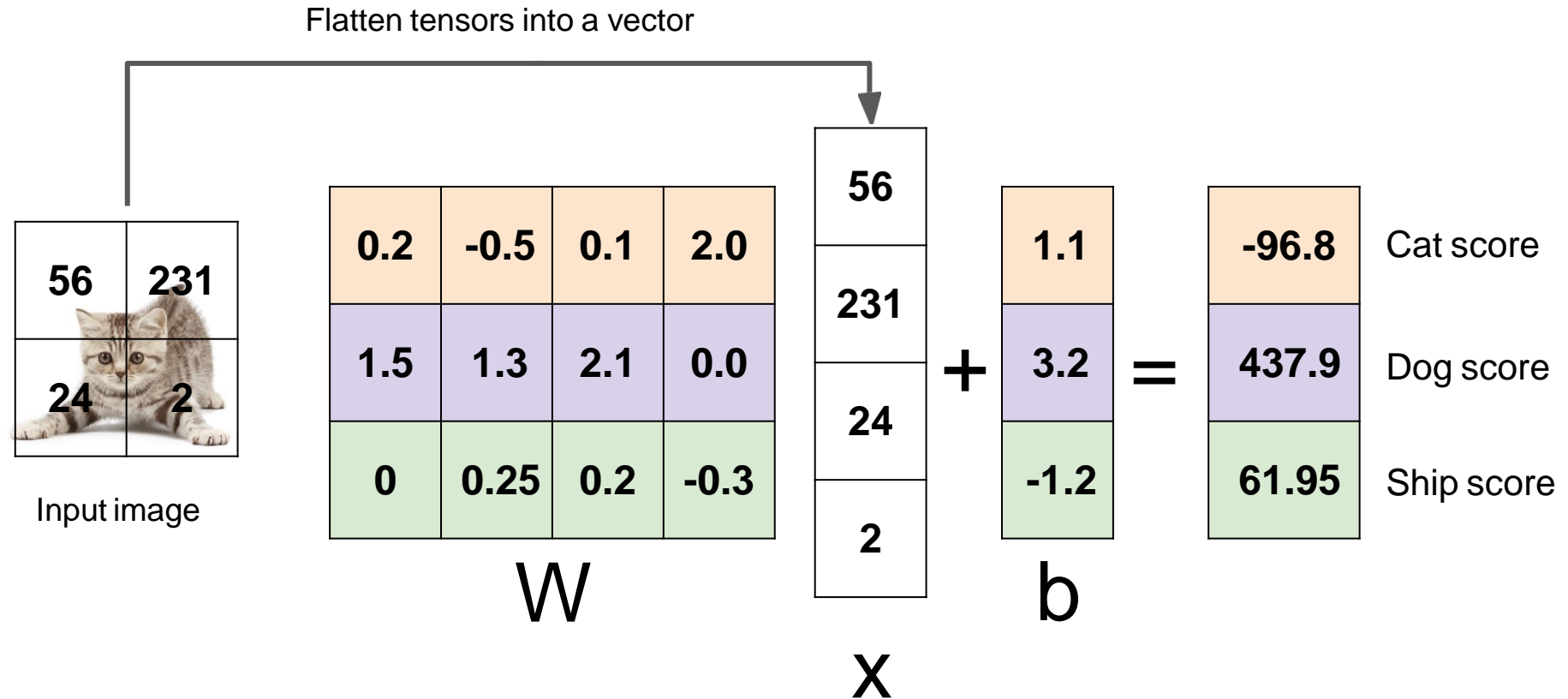


Geometric Interpretation: Linear Classifier



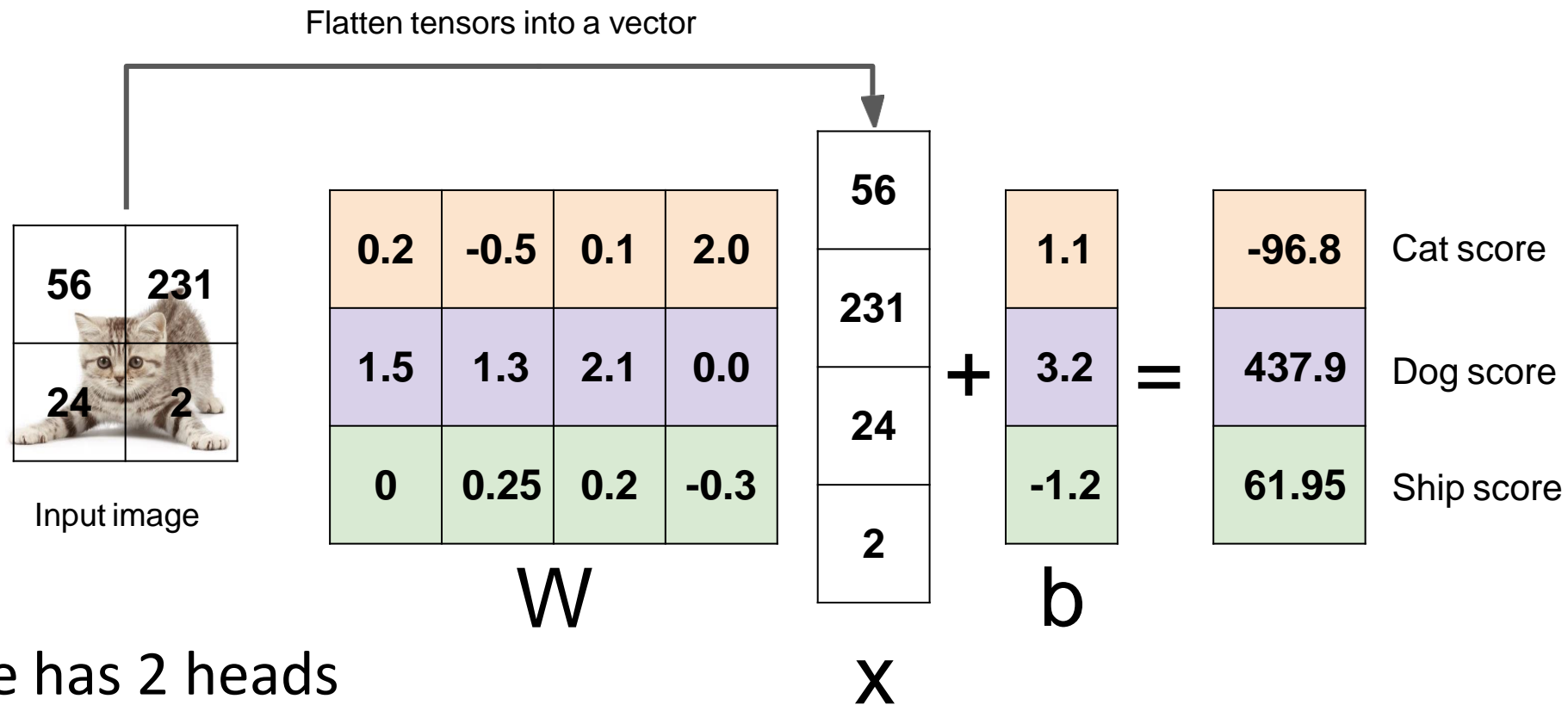
Visual Interpretation: Linear Classifier

Linear classifier has one “template” per category



Visual Interpretation: Linear Classifier

A single template cannot capture multiple modes of the data



Linear Classifiers

Algebraic Interpretation

$$f(x,W) = Wx$$

0.2	-0.5	0.1	2.0	+	56	=	1.1	-96.8	Cat score
1.5	1.3	2.1	0.0		231		3.2	437.9	Dog score
0	0.25	0.2	-0.3		24		-1.2	61.95	Ship score
W					2		b		

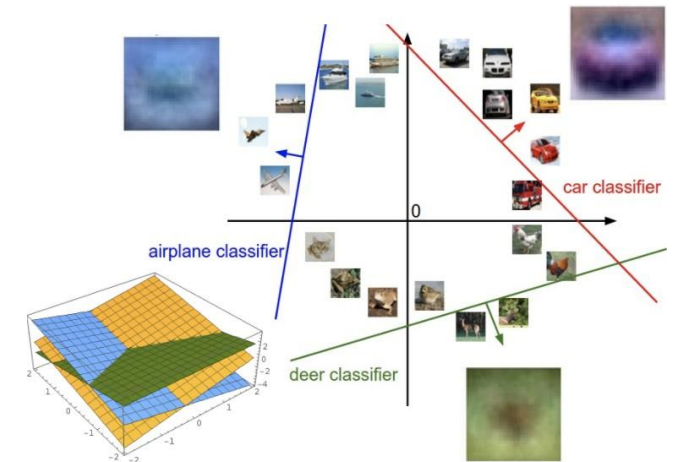
Visual Interpretation

One template
per class



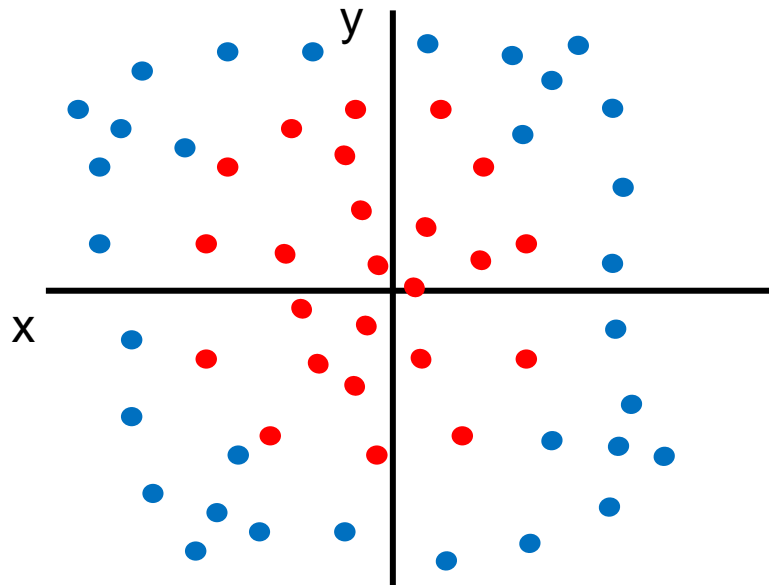
Geometric Interpretation

Hyperplanes
separating space



Linear Classifiers shortcomings

Geometric Viewpoint



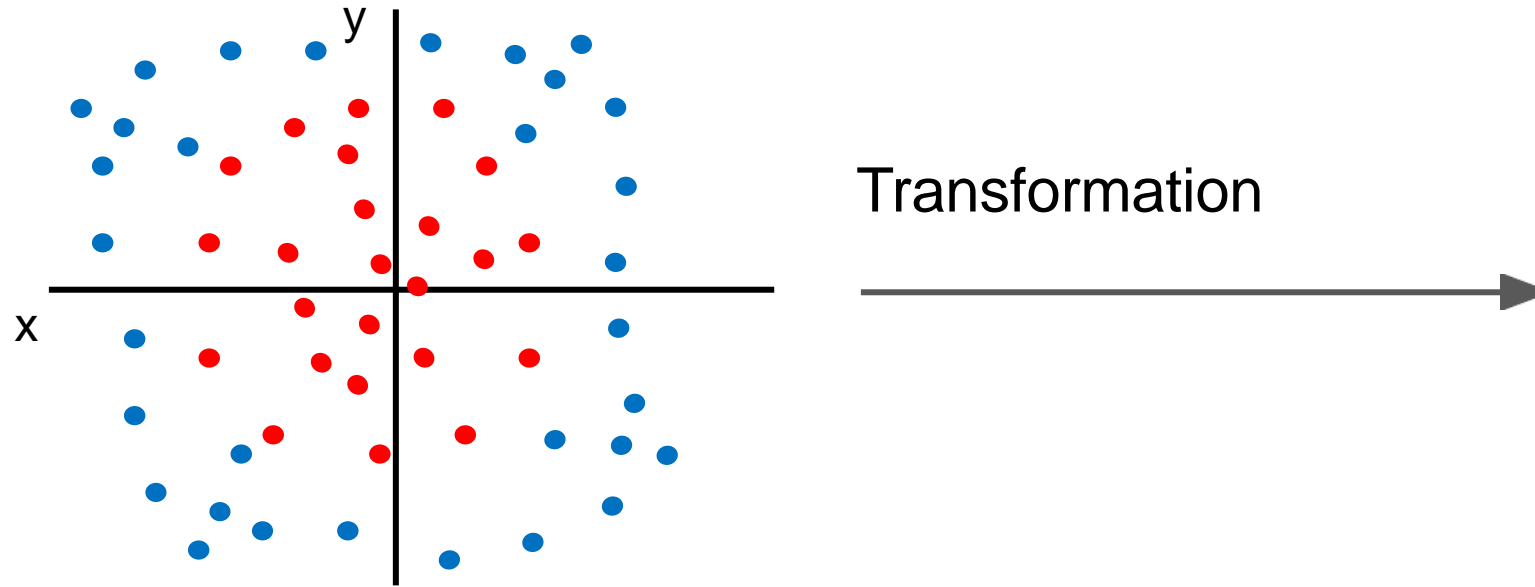
Some training data
can't be separated with
a hyperplane

Visual Viewpoint



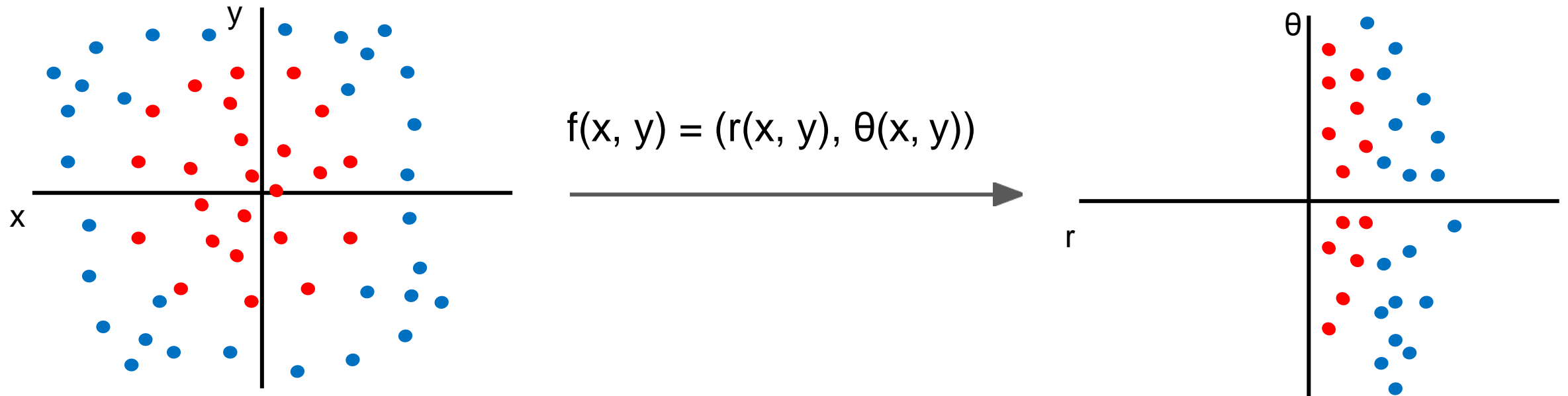
One template per class:
Can't recognize different
modes of a class

Apply Transformations



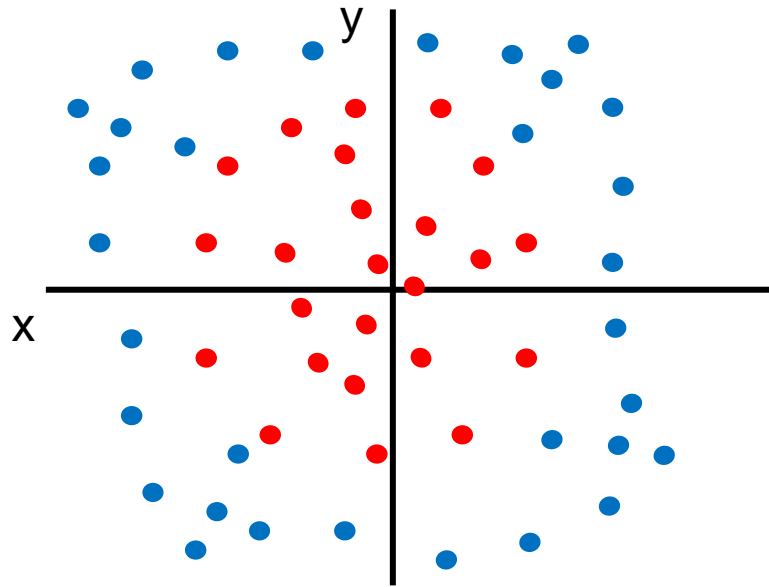
Extract features using transformations

Apply Transformations

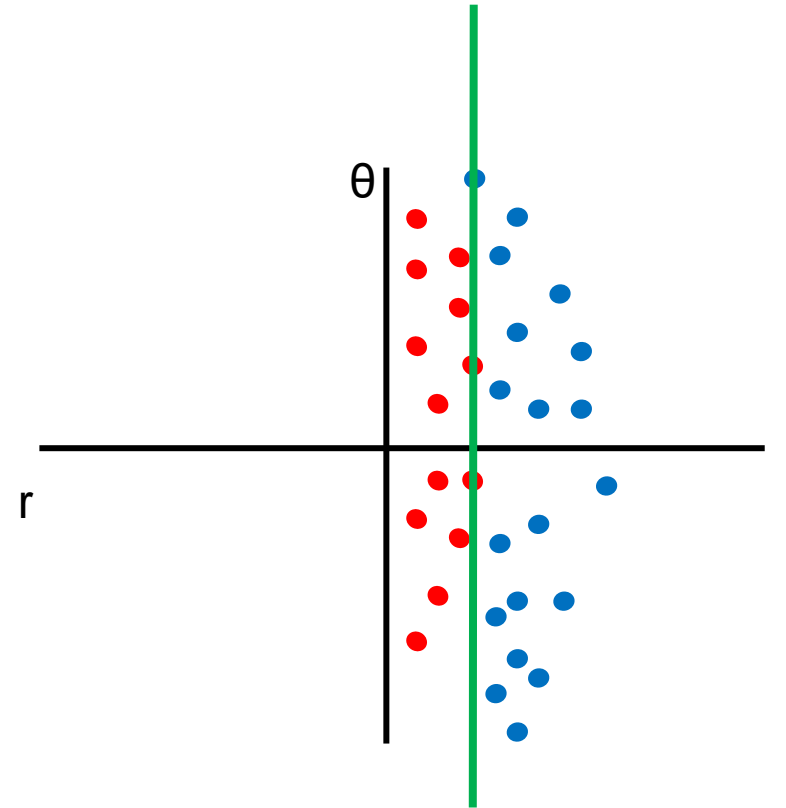


Extract features using transformations

Apply Transformations



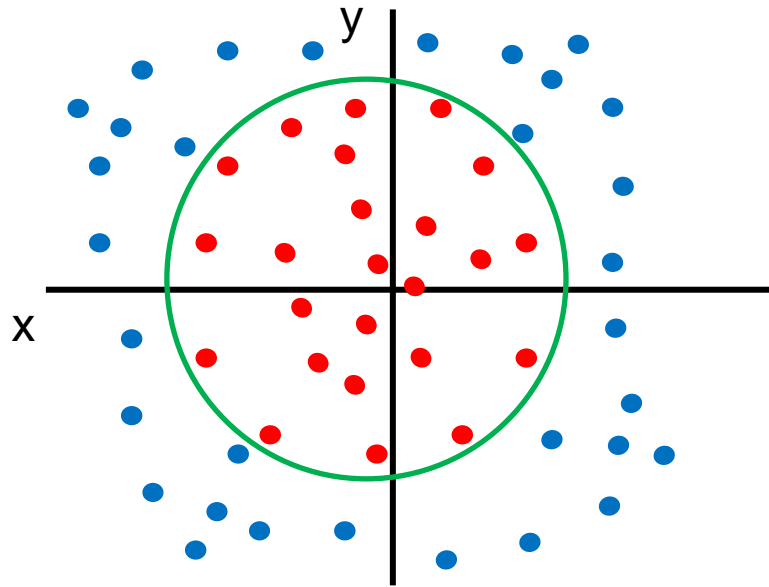
$$f(x, y) = (r(x, y), \theta(x, y))$$



Extract features using transformations

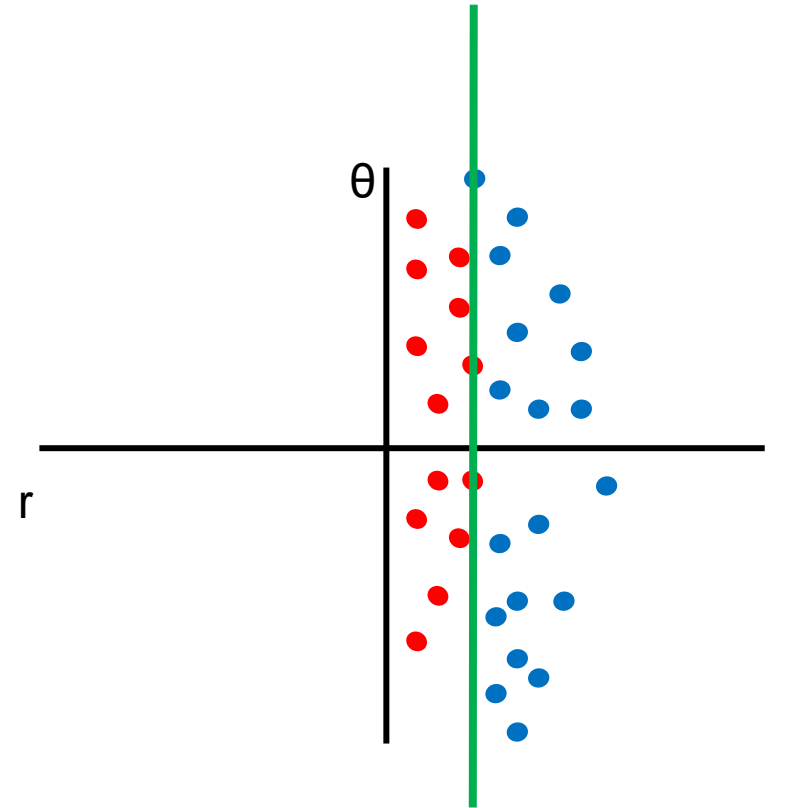
Linear classifier
in feature space

Apply Transformations



Nonlinear classifier in original space!

$$f(x, y) = (r(x, y), \theta(x, y))$$



Linear classifier in feature space



Example: Color Histogram

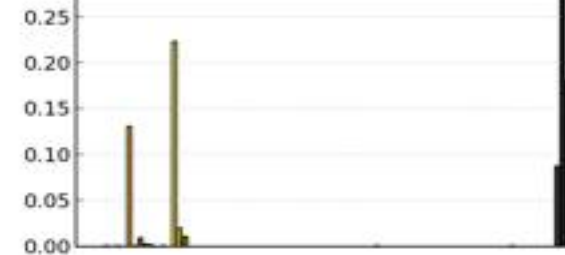
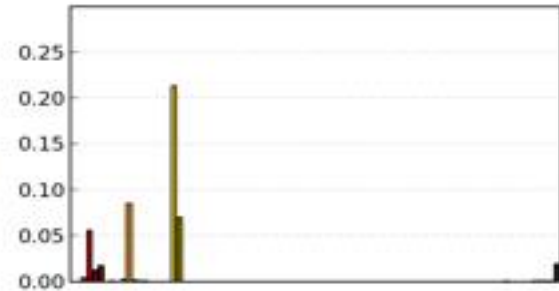
Image A



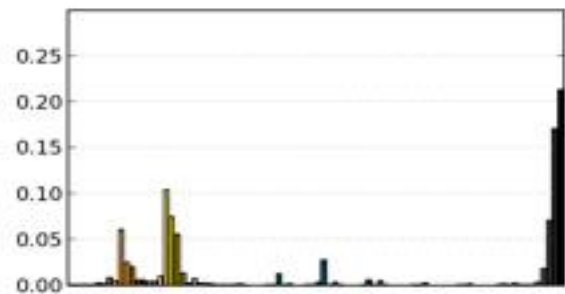
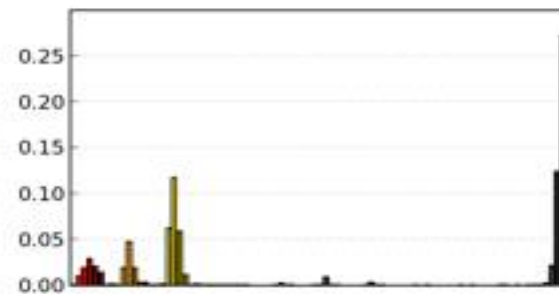
Image B



Raw Histogram



Smoothed Histogram



Example: Histogram of Oriented Gradients (HoG)

Input image



Histogram of Oriented Gradients

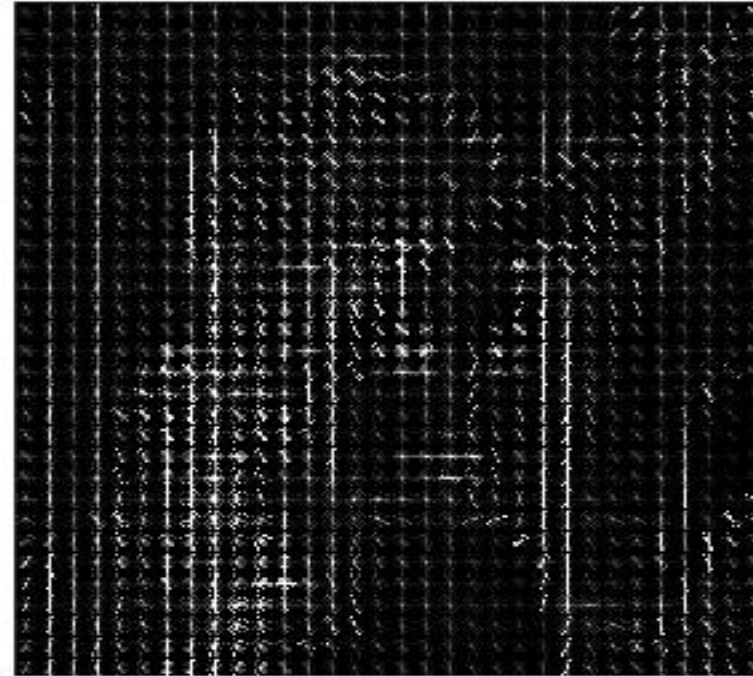


Image Feature Aggregation

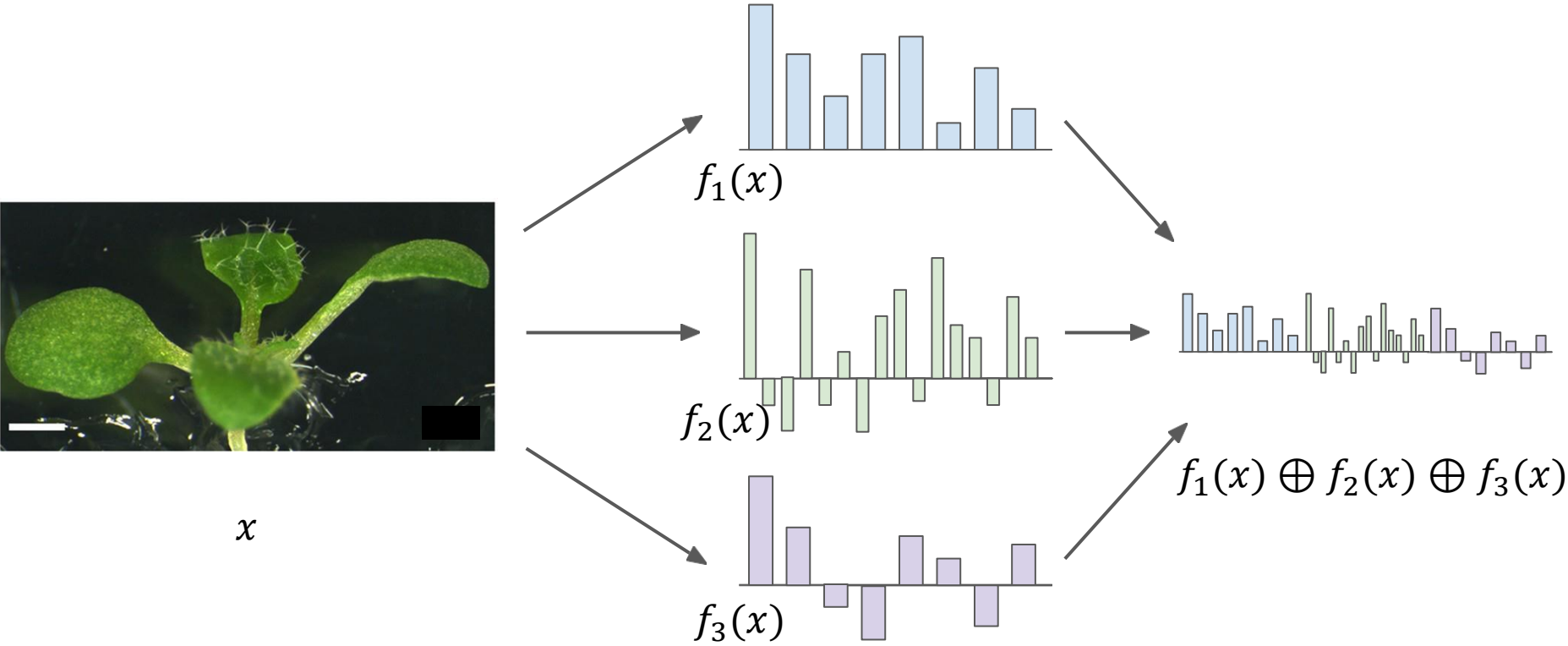
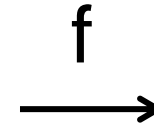
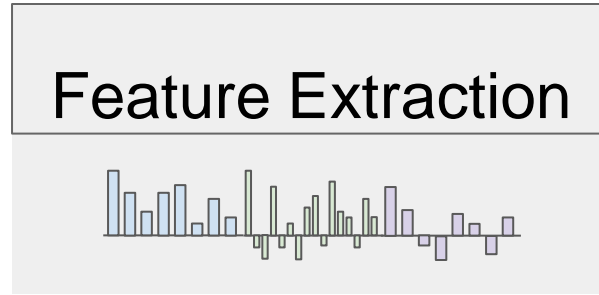


Image Features



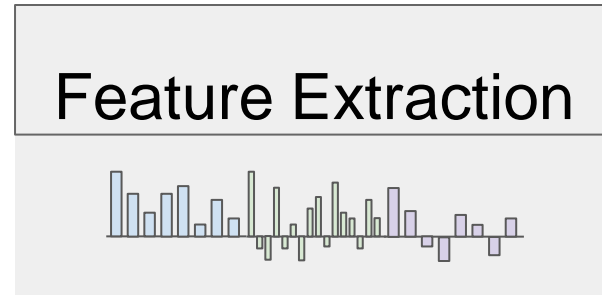
10 numbers giving scores for classes

+

Class Label



Image Features

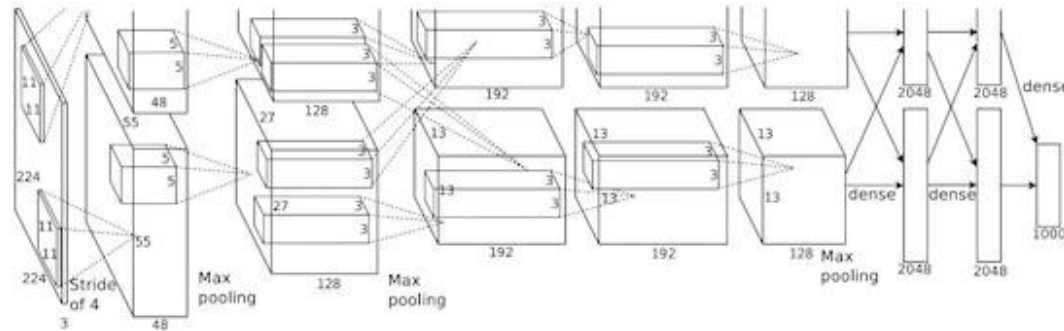


f

10 numbers giving scores for classes

+
Class Label

training



10 numbers giving scores for classes

training

Neural Networks

(Before) Linear score function:

$$f = Wx$$

$$x \in \mathbb{R}^D, W \in \mathbb{R}^{C \times D}$$

Neural Networks

(Before) Linear score function:

$$f = Wx$$

(Now) 2-layer Neural Network

$$f = W_2 \max(0, W_1 x)$$

$$W_2 \in \mathbb{R}^{C \times H} \quad W_1 \in \mathbb{R}^{H \times D} \quad x \in \mathbb{R}^D$$

Neural Networks

(Before) Linear score function:

$$f = Wx$$

(Now) 2-layer Neural Network
or 3-layer Neural Network

$$f = W_2 \max(0, W_1 x)$$

$$f = W_3 \max(0, W_2 \max(0, W_1 x))$$

$$W_3 \in \mathbb{R}^{C \times H_2} \quad W_2 \in \mathbb{R}^{H_2 \times H_1} \quad W_1 \in \mathbb{R}^{H_1 \times D} \quad x \in \mathbb{R}^D$$

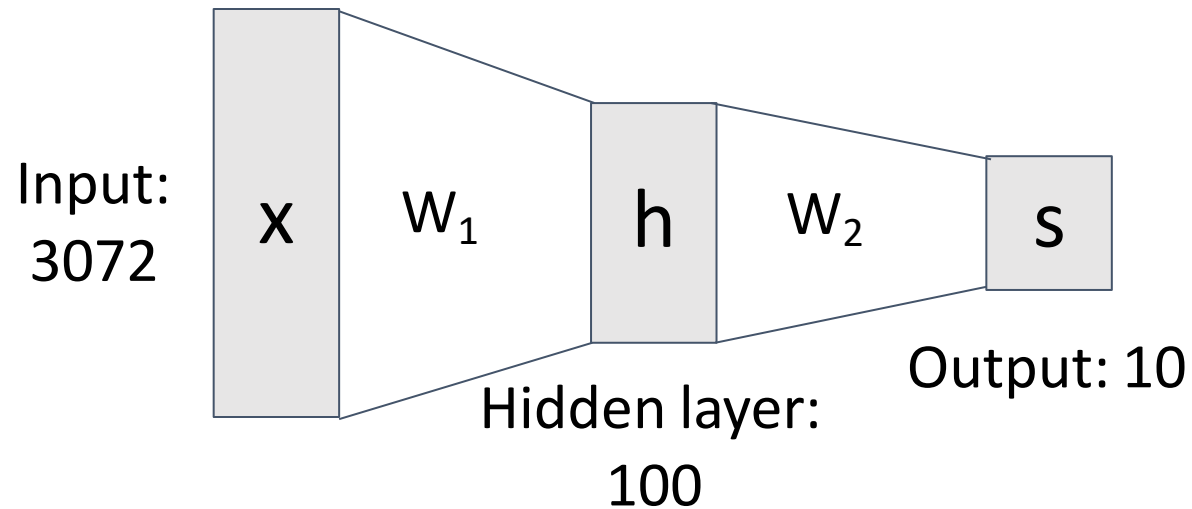
Neural Networks

$$f = Wx$$

(**Before**) Linear score function:

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(**Now**) 2-layer Neural Network



$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

Neural Networks

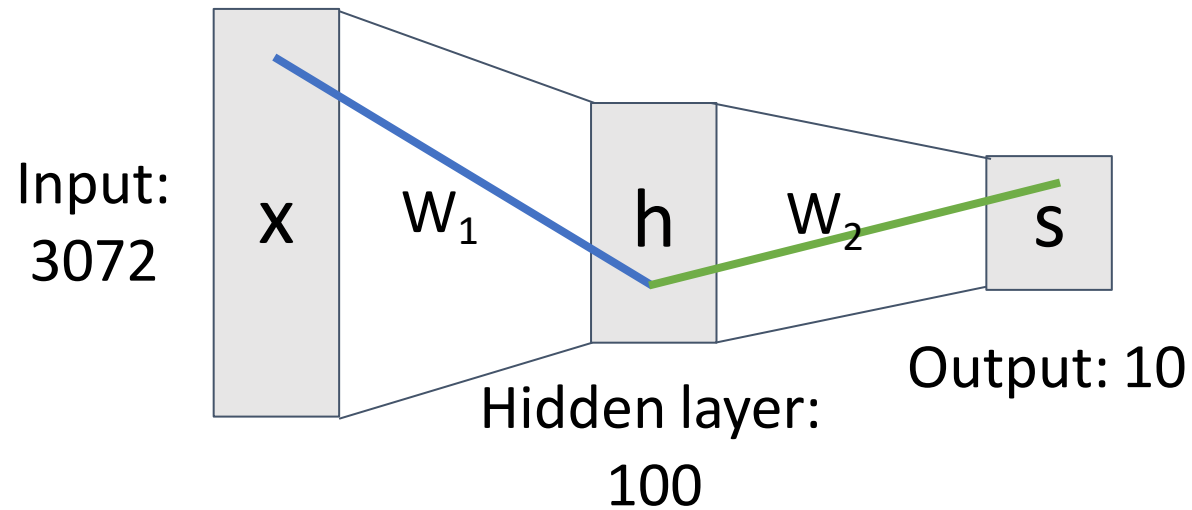
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$$f = W_2 \max(0, W_1 x)$$

Element (i, j)
of W_1 gives
the effect on
 h_i from x_j



Element (i, j)
of W_2 gives
the effect on
 s_i from h_j

$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

Neural Networks

(Before) Linear score function:

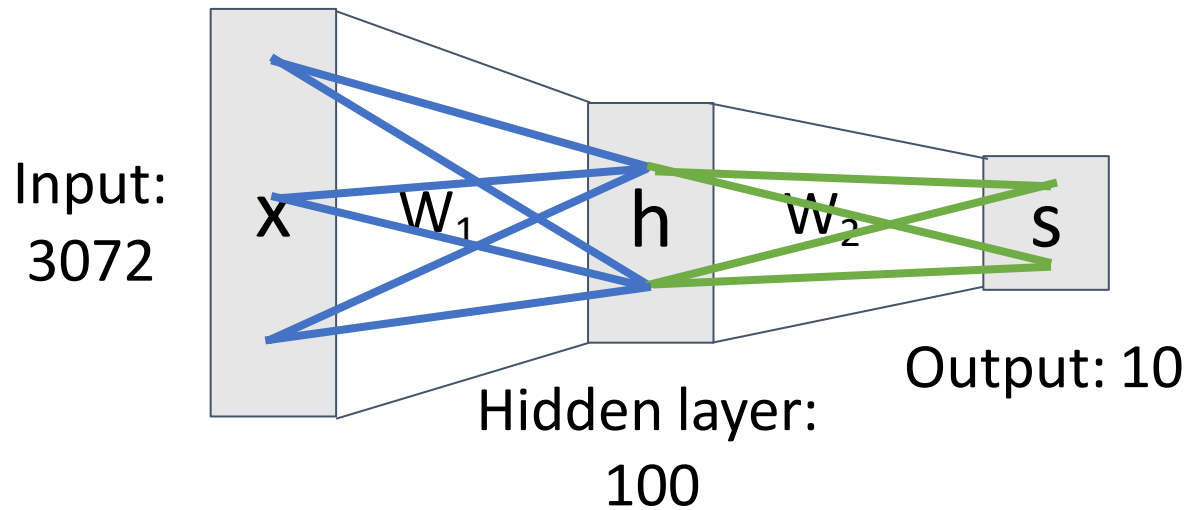
$$f = Wx$$

(Now) 2-layer Neural Network

$$f = W_2 \max(0, W_1 x)$$

Element (i, j) of W_1
gives the effect on
 h_i from x_j

All elements
of x affect all
elements of h



Element (i, j) of W_2
gives the effect on
 s_i from h_j

All elements
of h affect all
elements of s

Fully-connected neural network
Also “Multi-Layer Perceptron” (MLP)

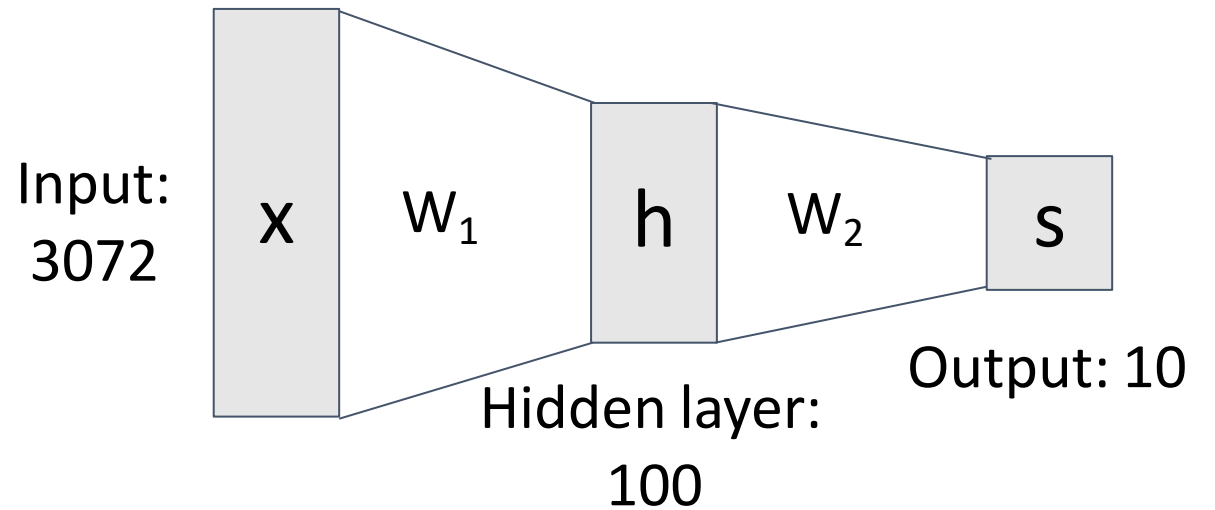
Neural Networks

Linear classifier: One template per class



(Before) Linear score function:

(Now) 2-layer Neural Network



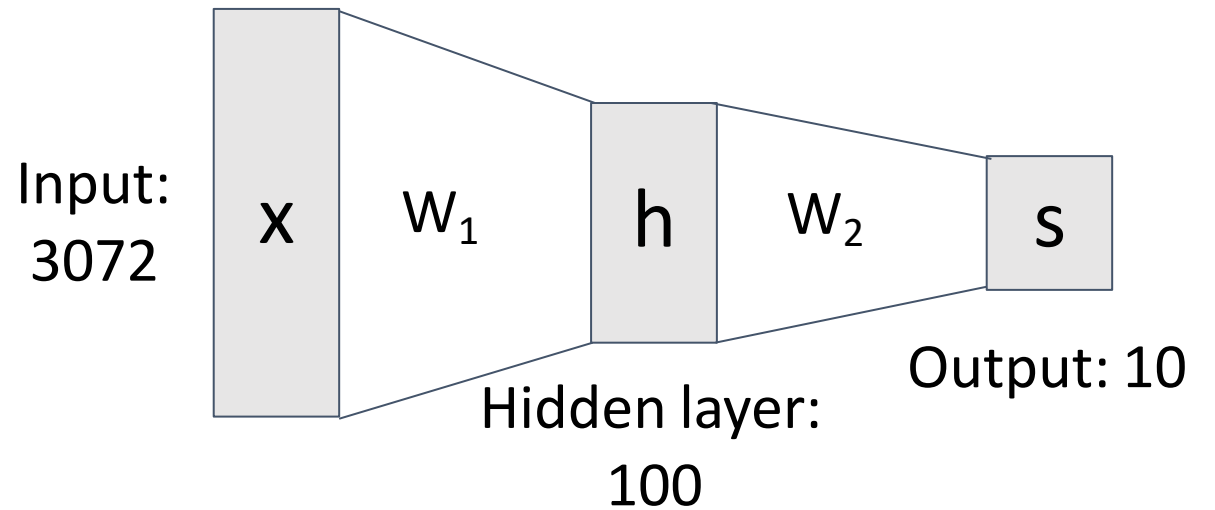
$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

Neural Networks



(Before) Linear score function:

(Now) 2-layer Neural Network



$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

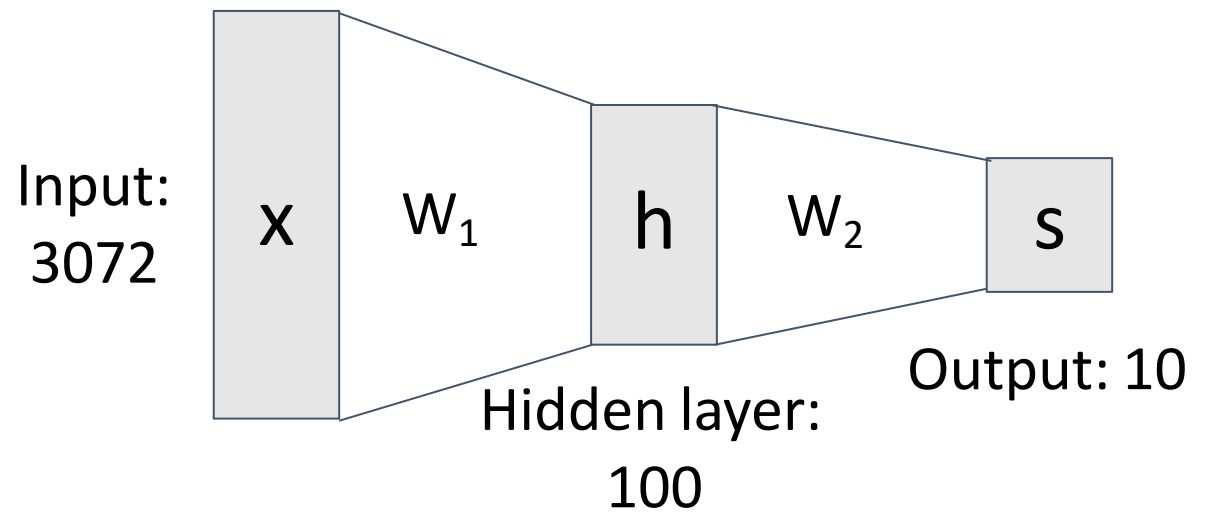
Neural Networks

Can use different templates to cover multiple modes of a class



(Before) Linear score function:

(Now) 2-layer Neural Network



$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

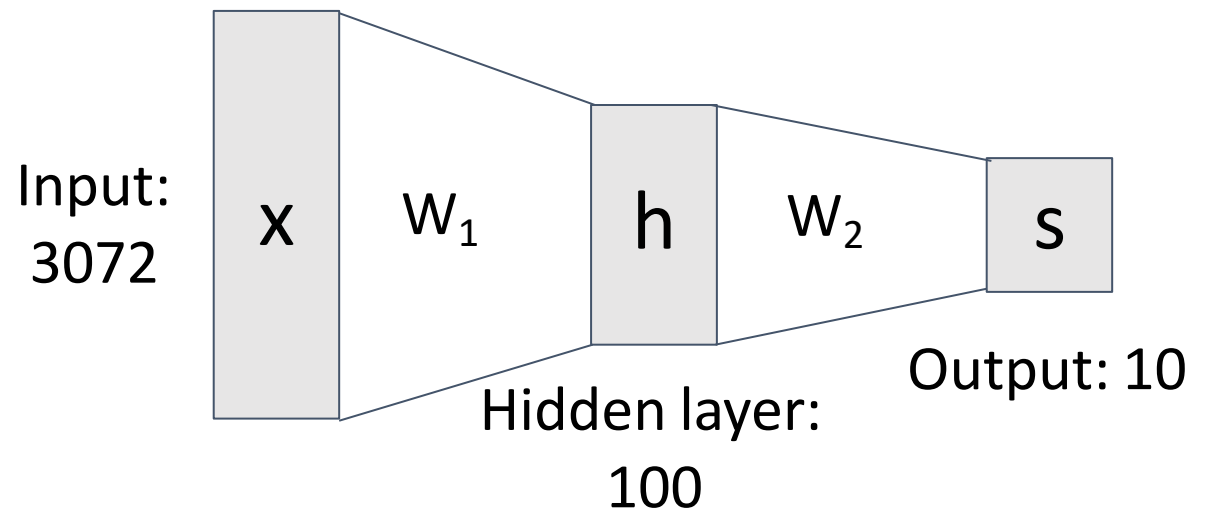
Neural Networks

Most templates not interpretable



(Before) Linear score function:

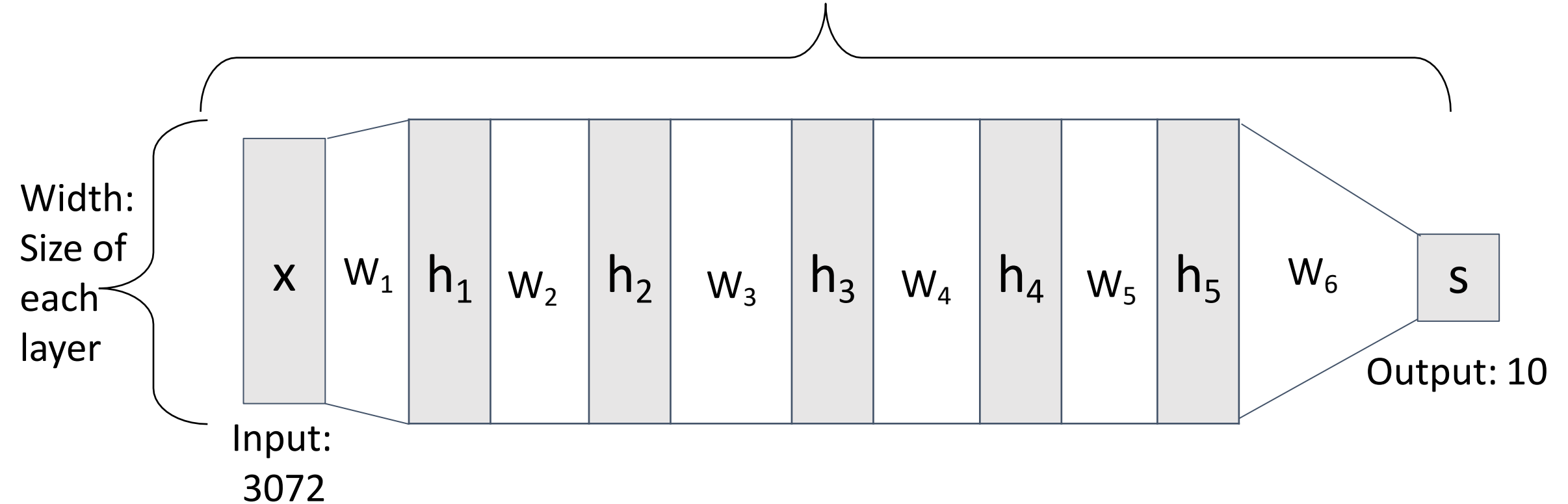
(Now) 2-layer Neural Network



$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

Deep Neural Networks

Depth = number of layers

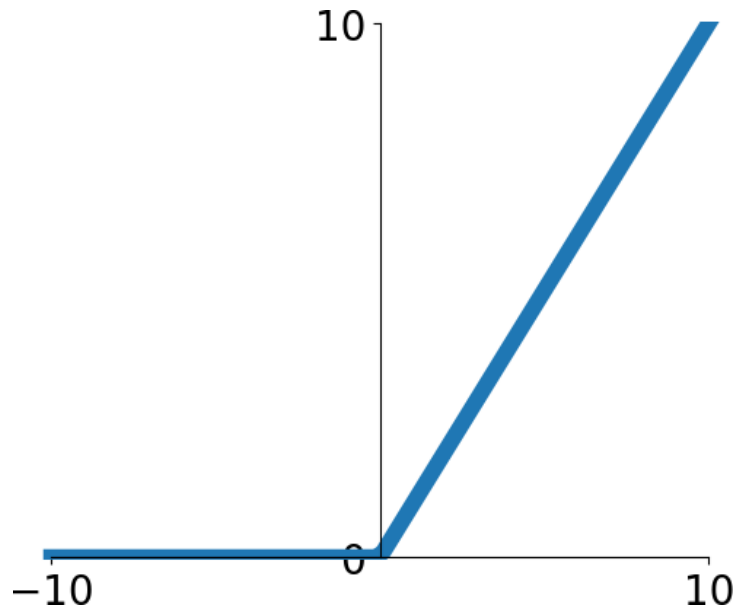


$$s = W_6 \max(0, W_6 \max(0, W_5 \max(0, W_4 \max(0, W_3 \max(0, W_2 \max(0, W_1 x))))))$$

Activation Functions

2-layer Neural Network

The function $ReLU(z) = \max(0, z)$ is called “Rectified Linear Unit”



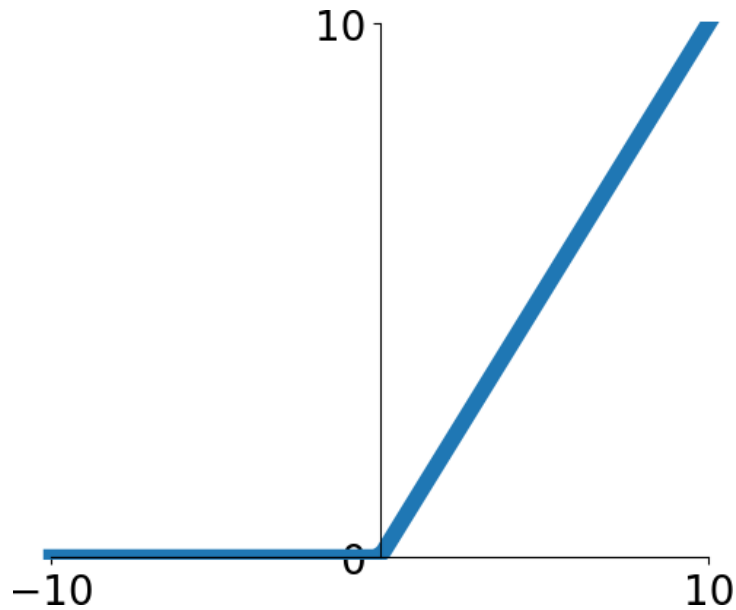
$$f = W_2 \max(0, W_1 x)$$

This is called the **activation function** of the neural network

Activation Functions

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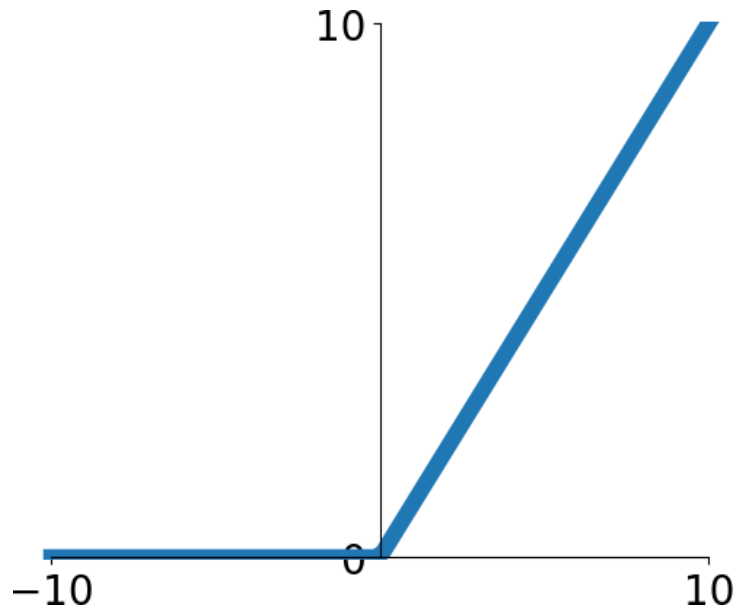
Without activation function:

$$s = W_2 W_1 x$$

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Without activation function:

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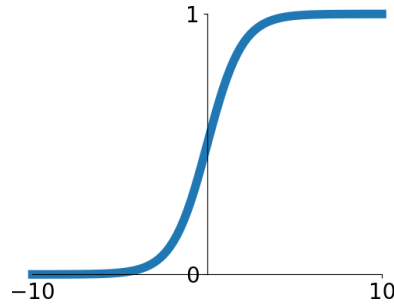
$$W_3 = W_2 W_1 \in \mathbb{R}^{C \times H} \quad s = W_3 x$$

→ Linear classifier

Activation Functions

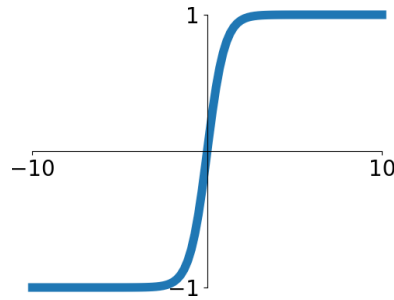
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



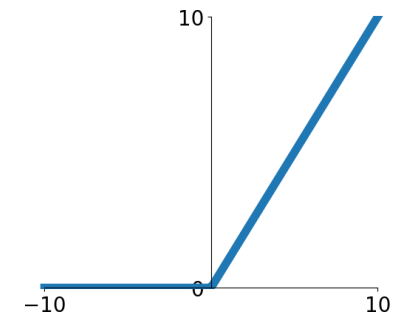
tanh

$$\tanh(x)$$



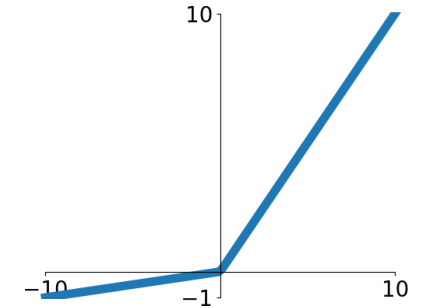
ReLU

$$\max(0, x)$$



Leaky ReLU

$$\max(0.1x, x)$$

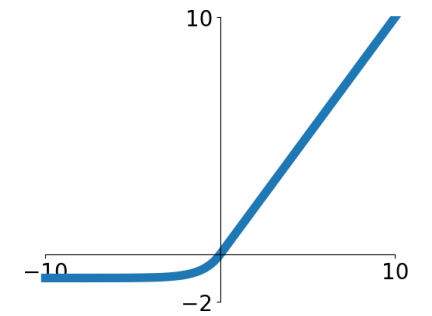


Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

ELU

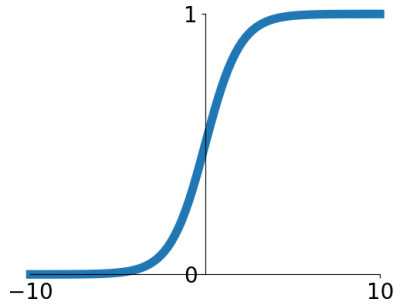
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Activation Functions

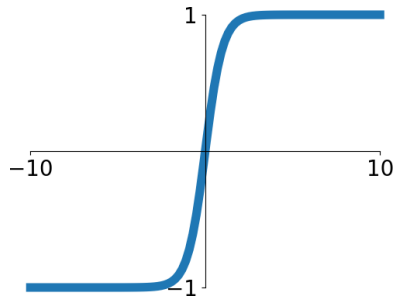
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



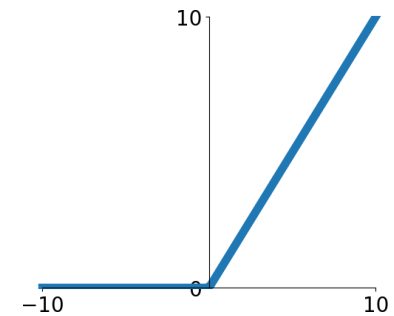
tanh

$$\tanh(x)$$



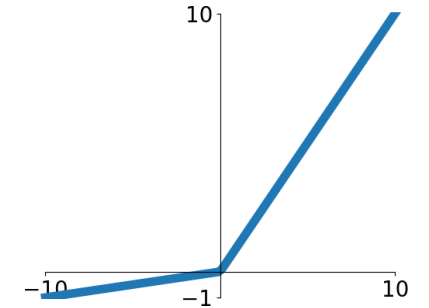
ReLU

$$\max(0, x)$$



Leaky ReLU

$$\max(0.1x, x)$$

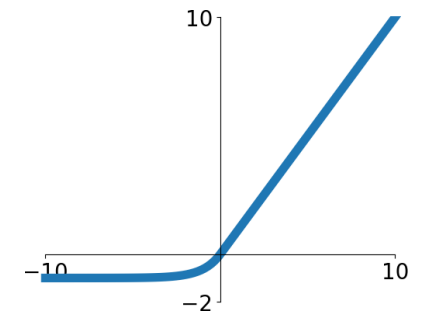


Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

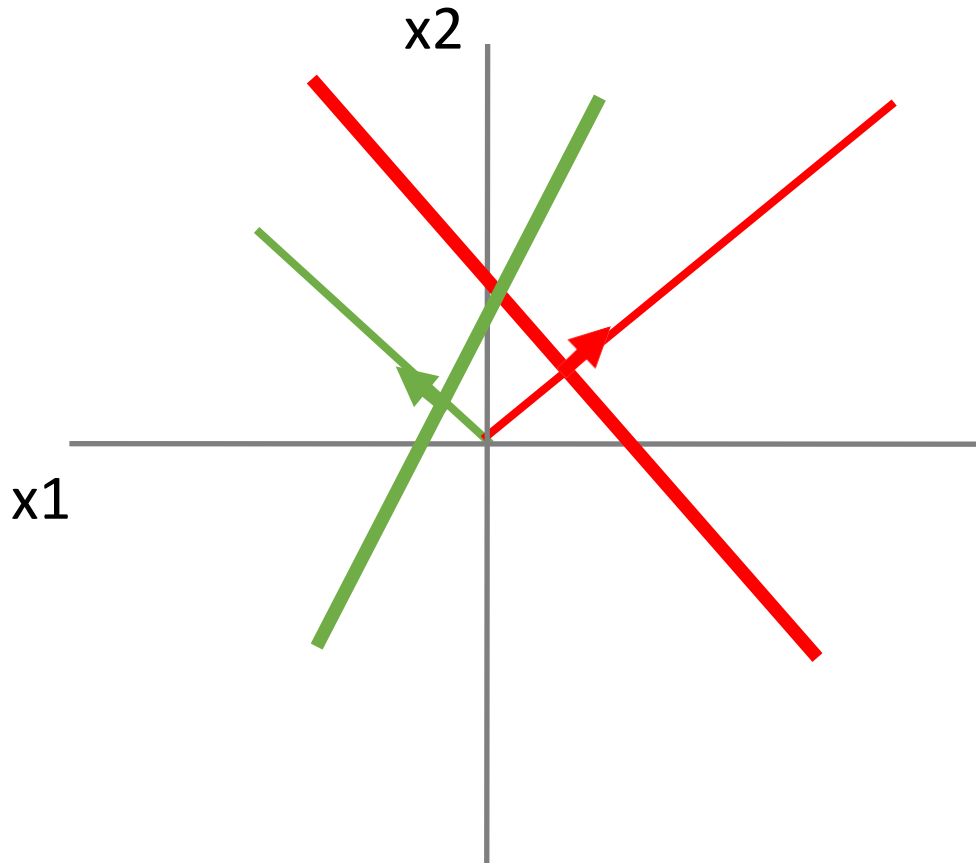
ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



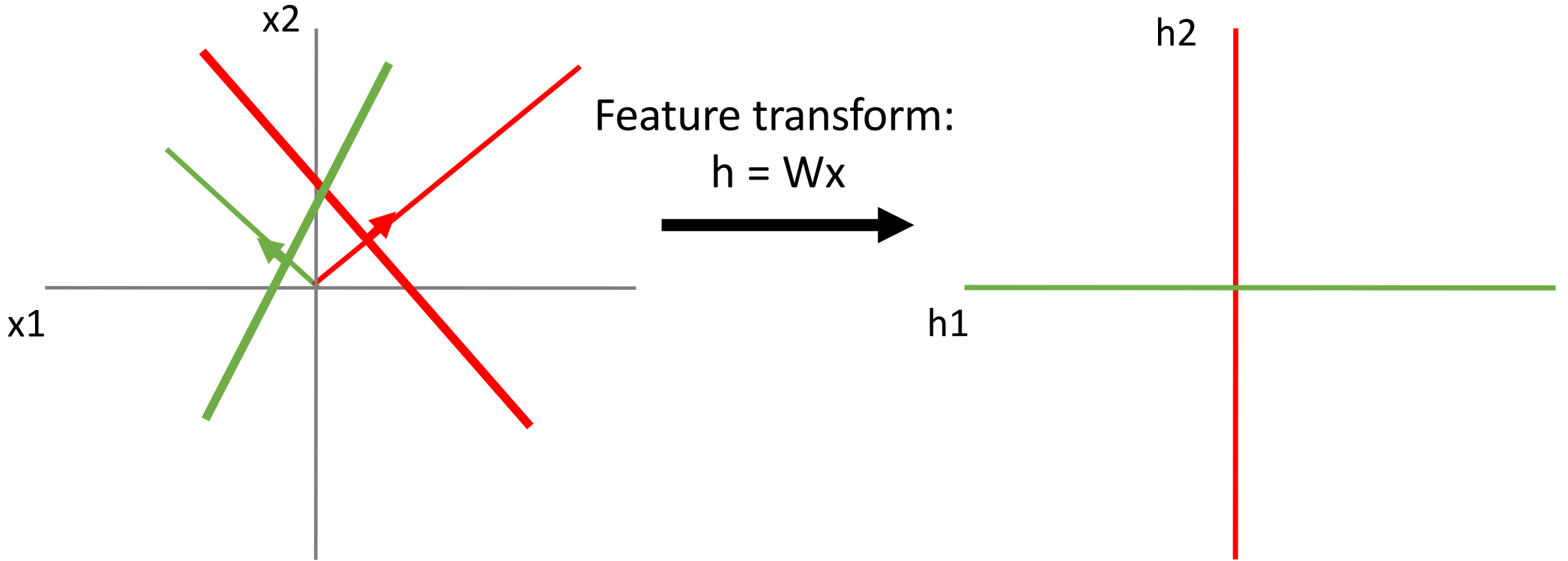
Feature Transform

Consider a linear transform: $h = Wx$
Where x, h are both 2-dimensional



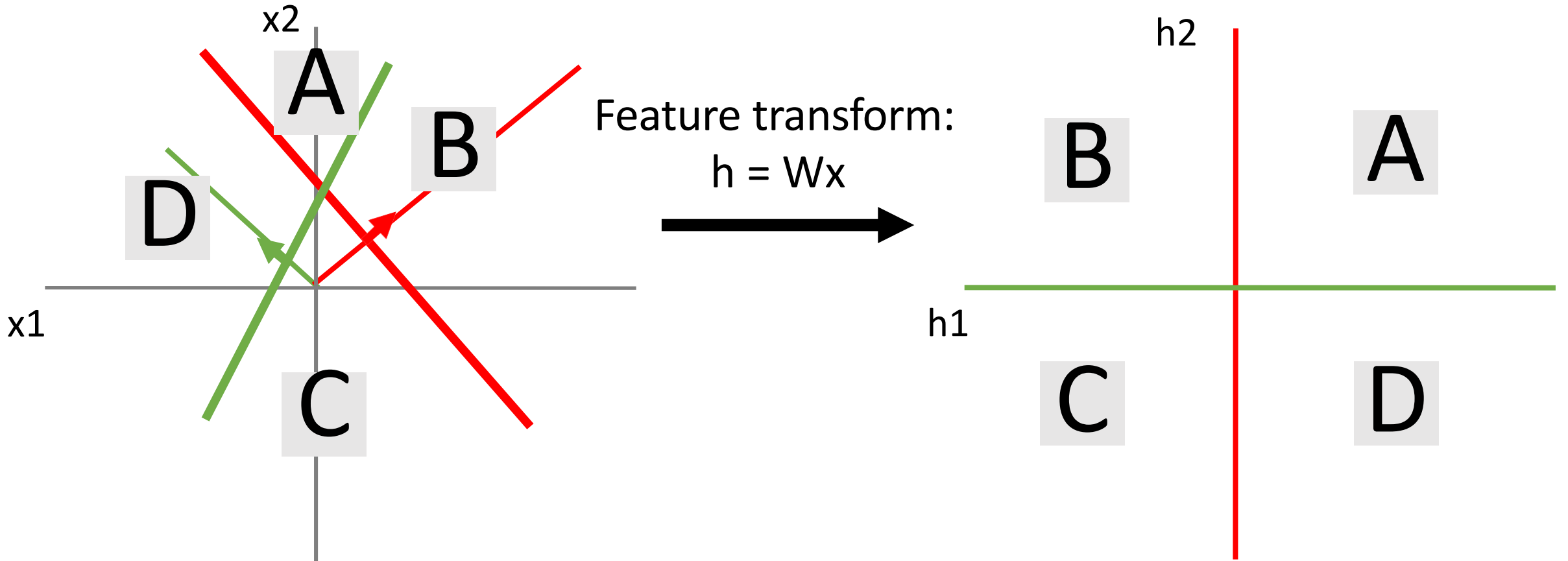
Feature Transform

Consider a linear transform: $h = Wx$
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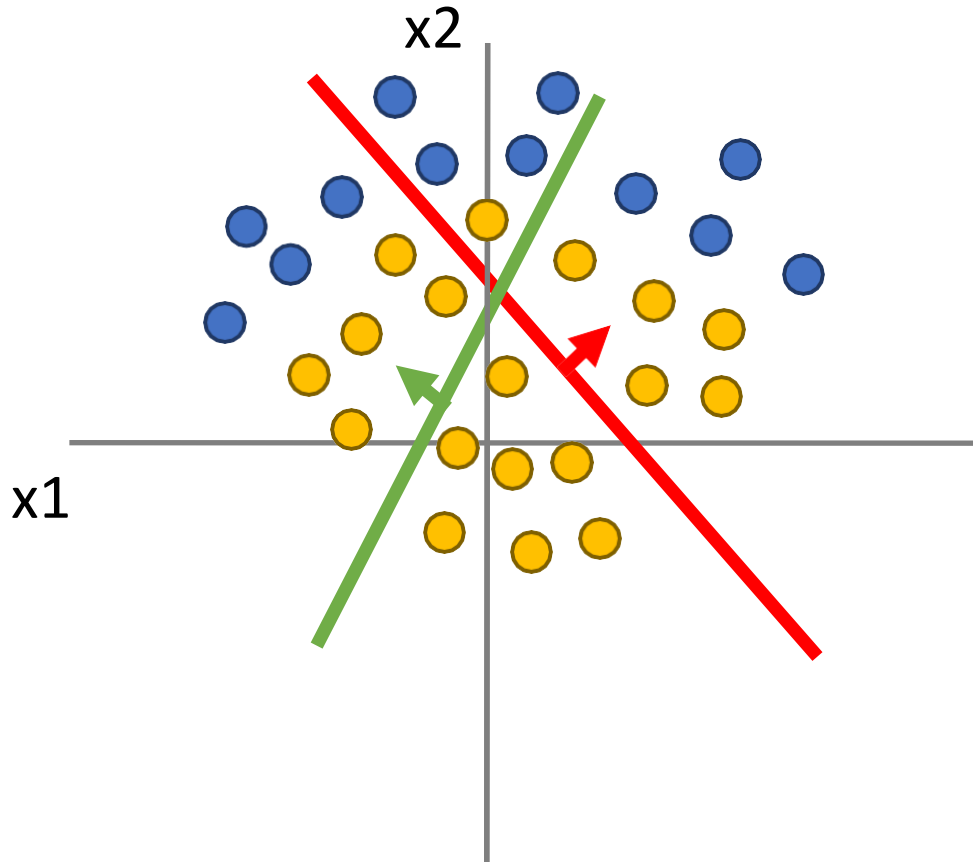
Feature Transform

Consider a linear transform: $h = Wx$
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Feature Transform

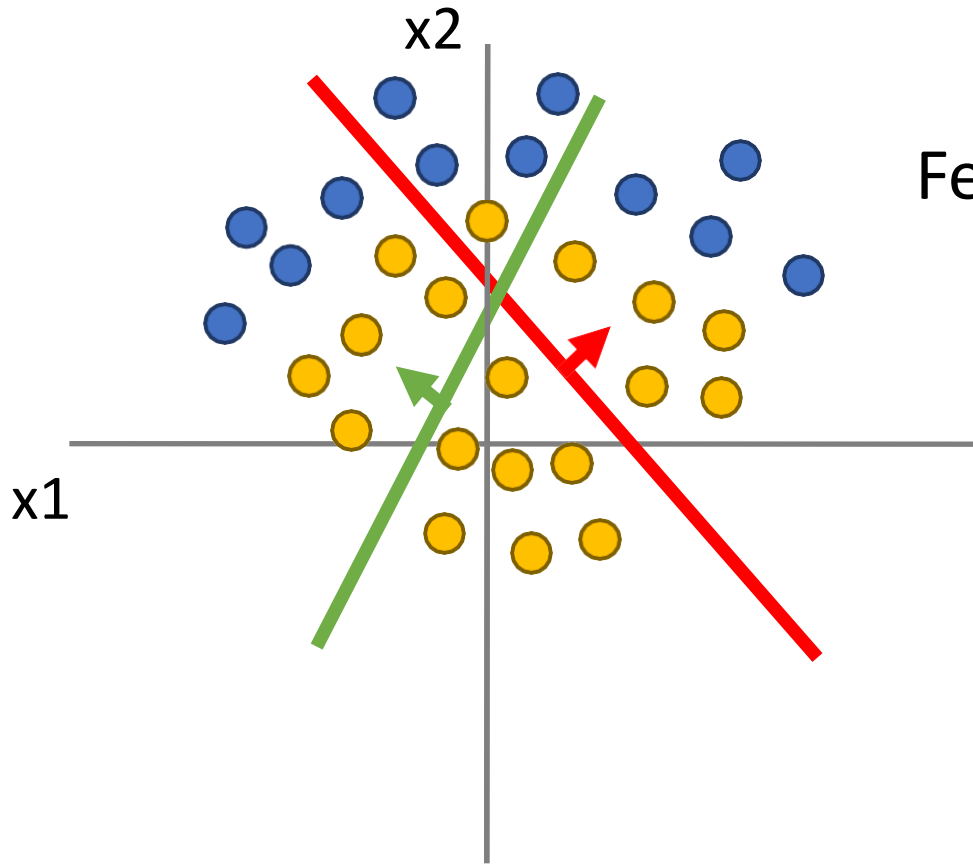
Points not linearly separable in original space



Consider a linear transform: $h = Wx$
Where x, h are both 2-dimensional

Feature Transform

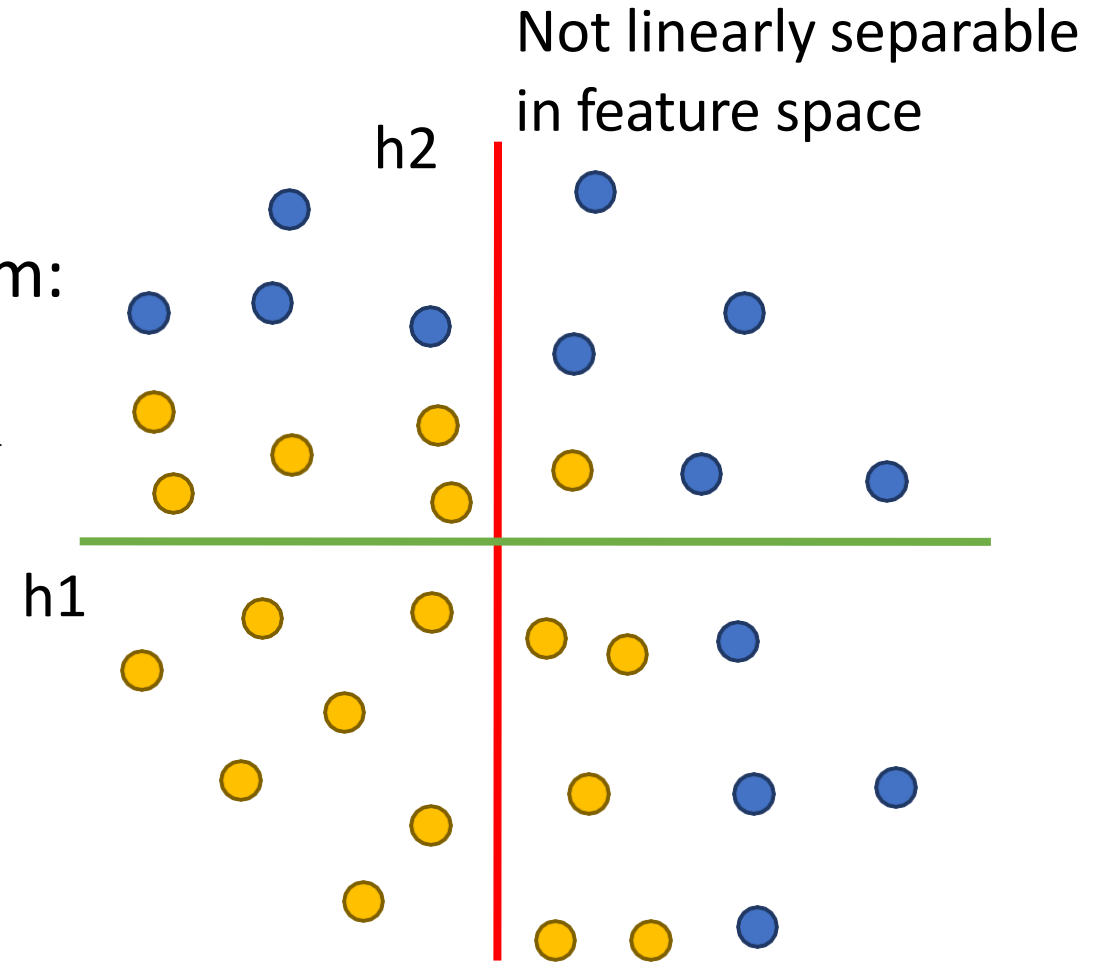
Points not linearly separable in original space



Feature transform:
 $h = Wx$

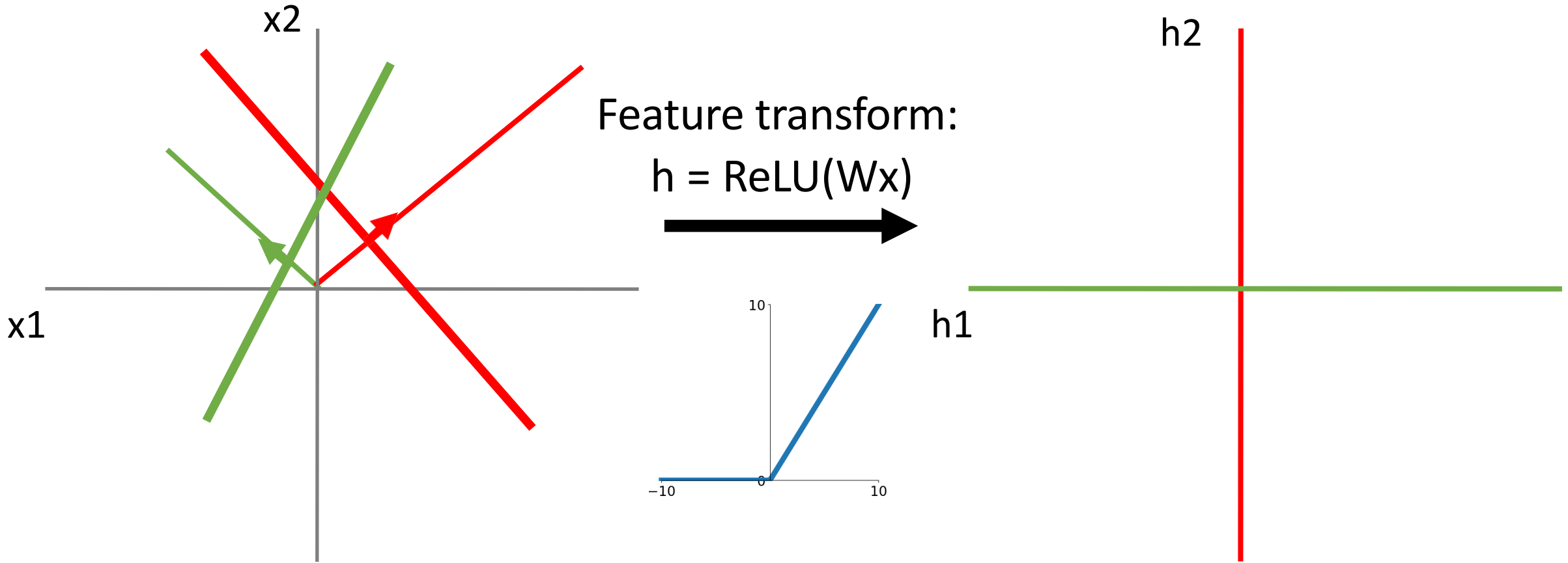


Consider a linear transform: $h = Wx$
Where x, h are both 2-dimensional



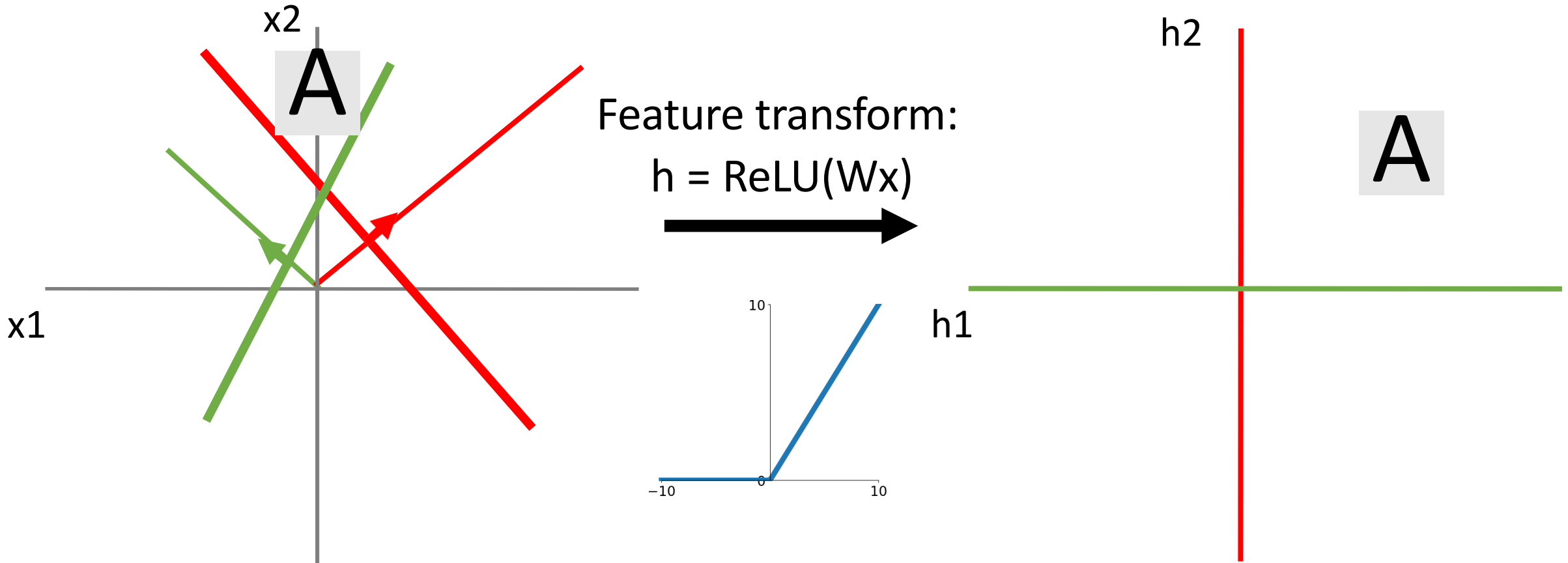
Feature Transform

Consider a neural net hidden layer:
 $h = \text{ReLU}(Wx) = \max(0, Wx)$
Where x, h are both 2-dimensional



Feature Transform

Consider a neural net hidden layer:
 $h = \text{ReLU}(Wx) = \max(0, Wx)$
Where x, h are both 2-dimensional

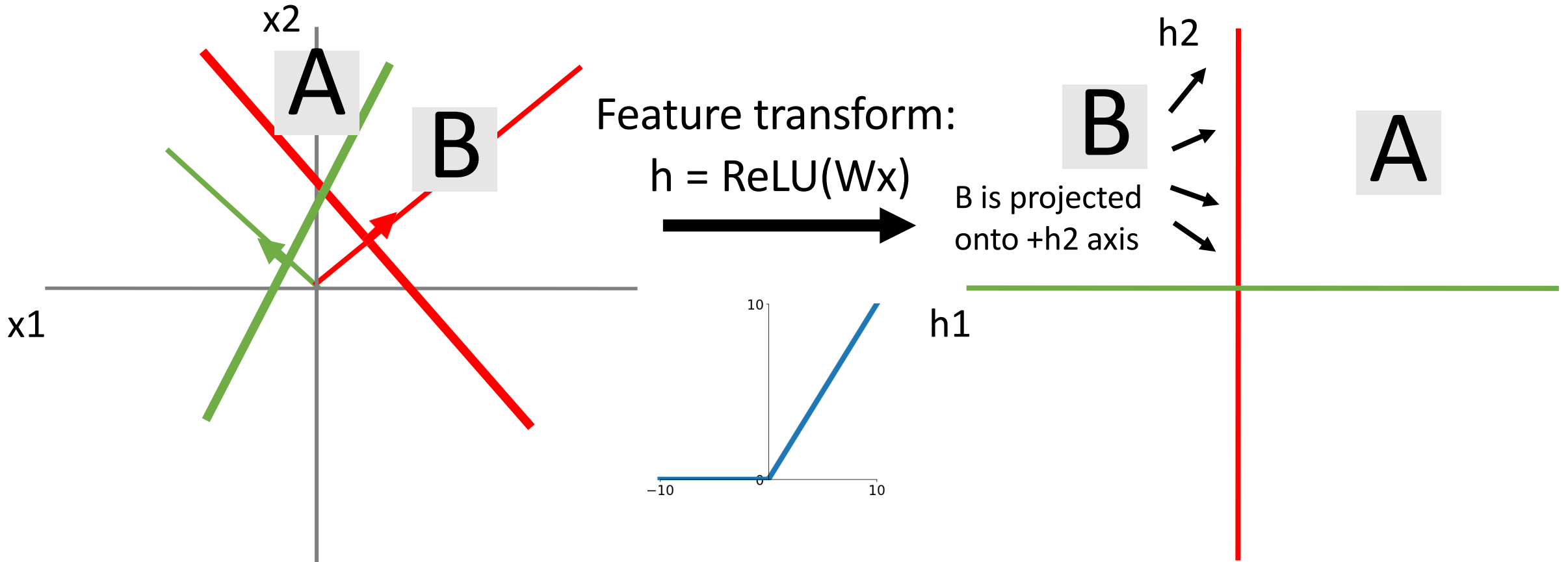


Feature Transform

Consider a neural net hidden layer:

$$h = \text{ReLU}(Wx) = \max(0, Wx)$$

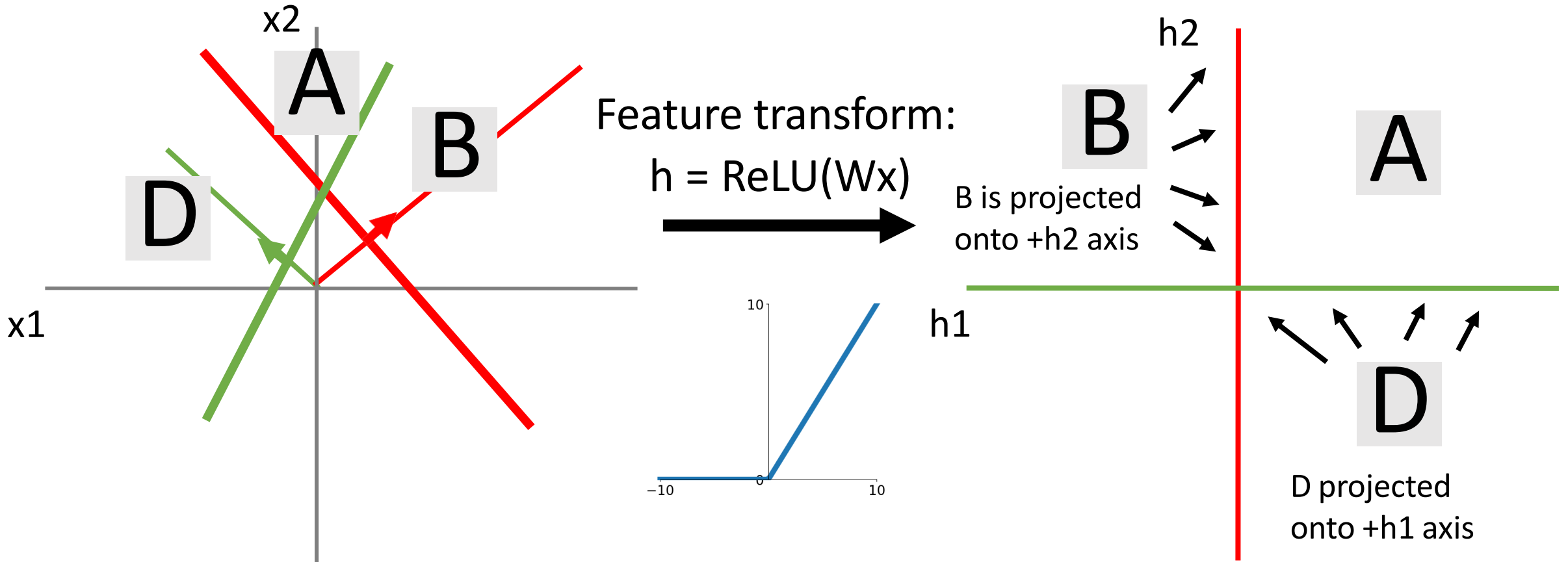
Where x, h are both 2-dimensional



Feature Transform

Consider a neural net hidden layer:
 $h = \text{ReLU}(Wx) = \max(0, Wx)$

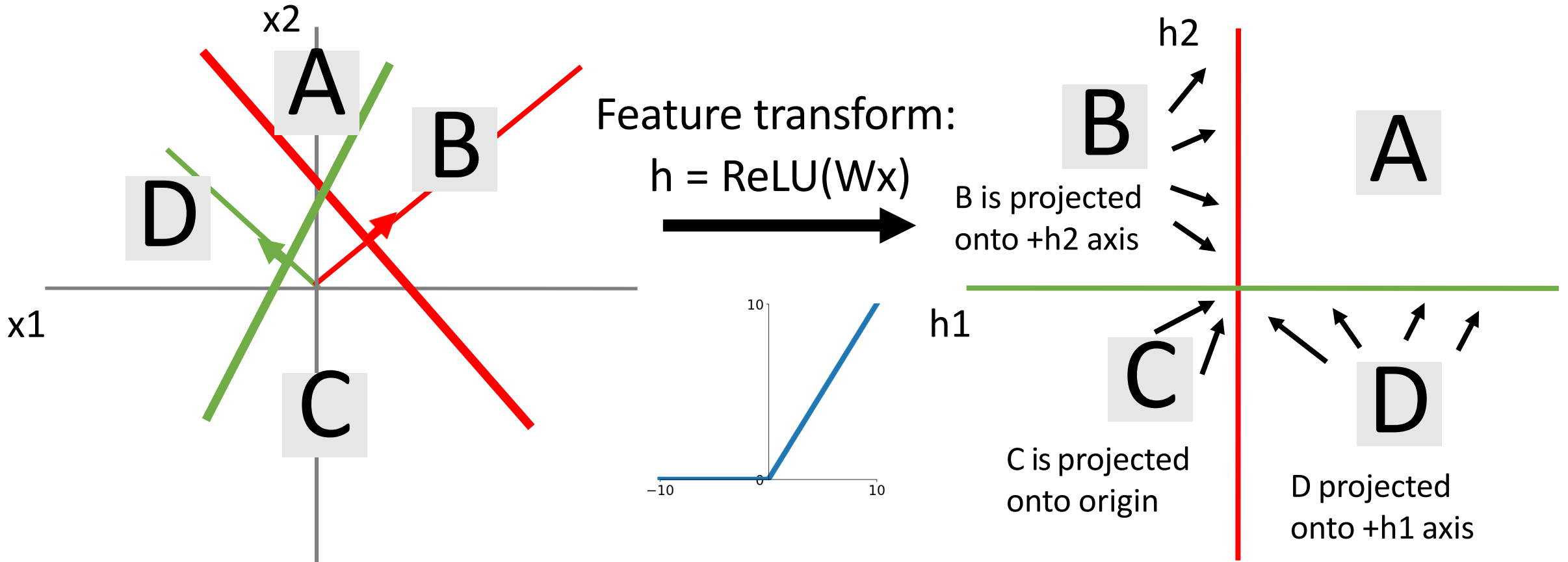
Where x, h are both 2-dimensional



Feature Transform

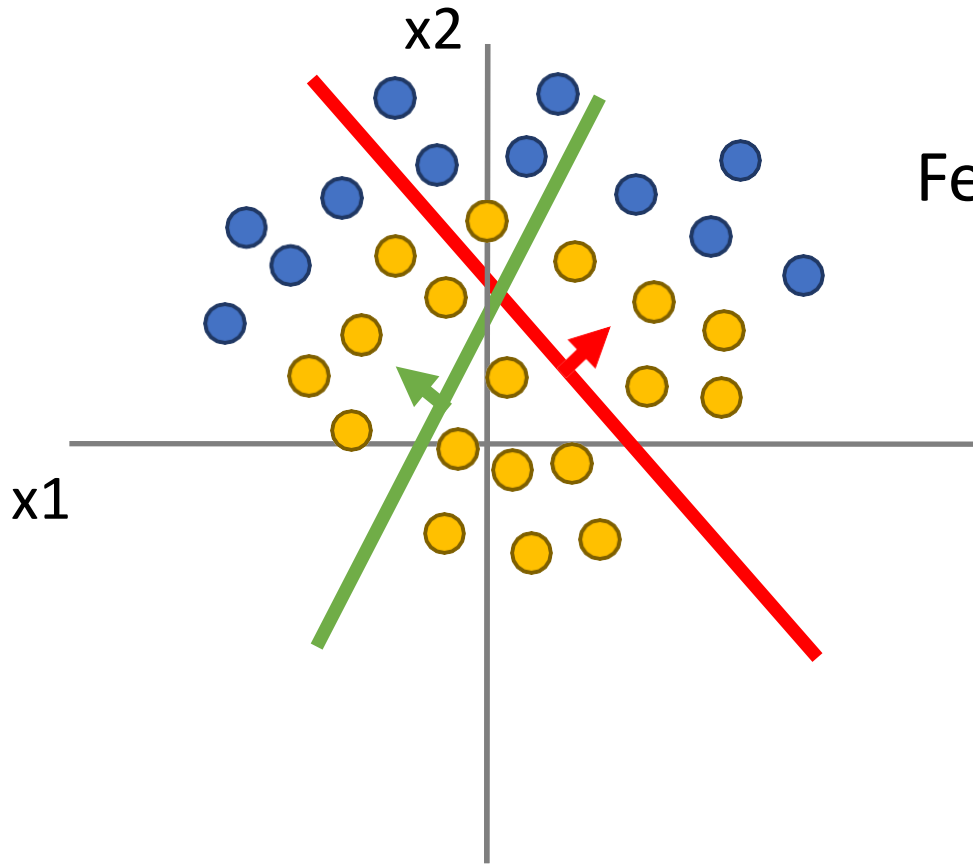
Consider a neural net hidden layer:
 $h = \text{ReLU}(Wx) = \max(0, Wx)$

Where x, h are both 2-dimensional



Feature Transform

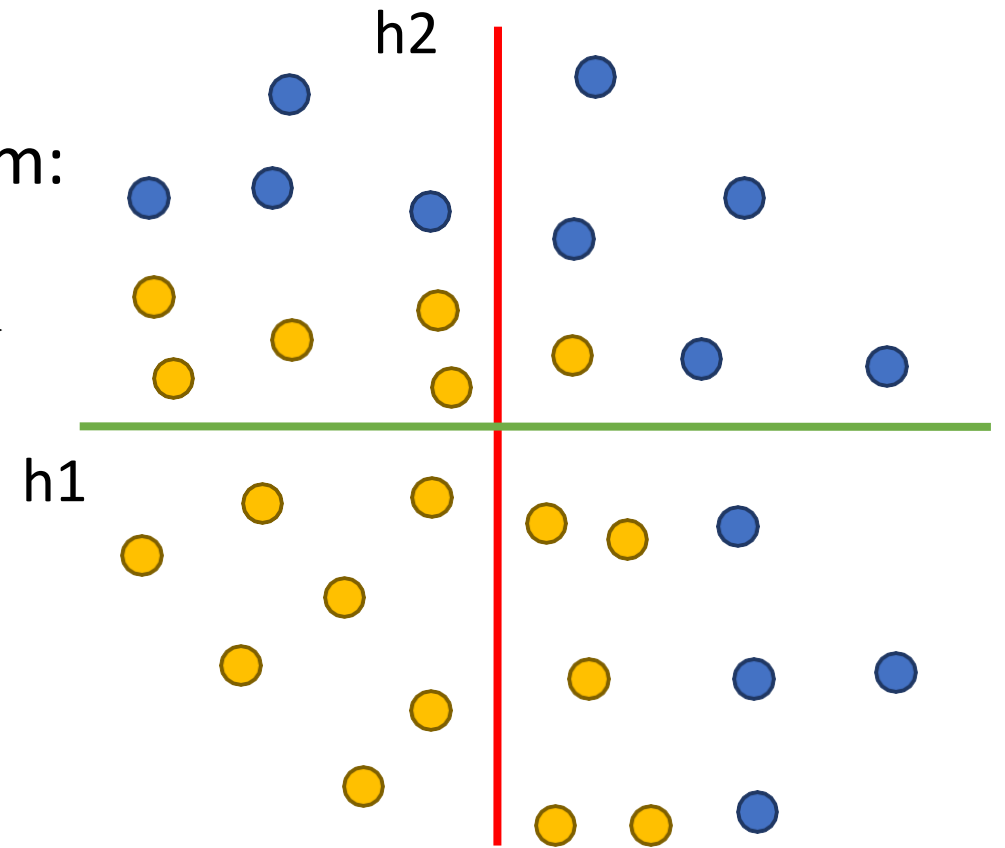
Points not linearly separable in original space



Feature transform:
 $h = Wx$

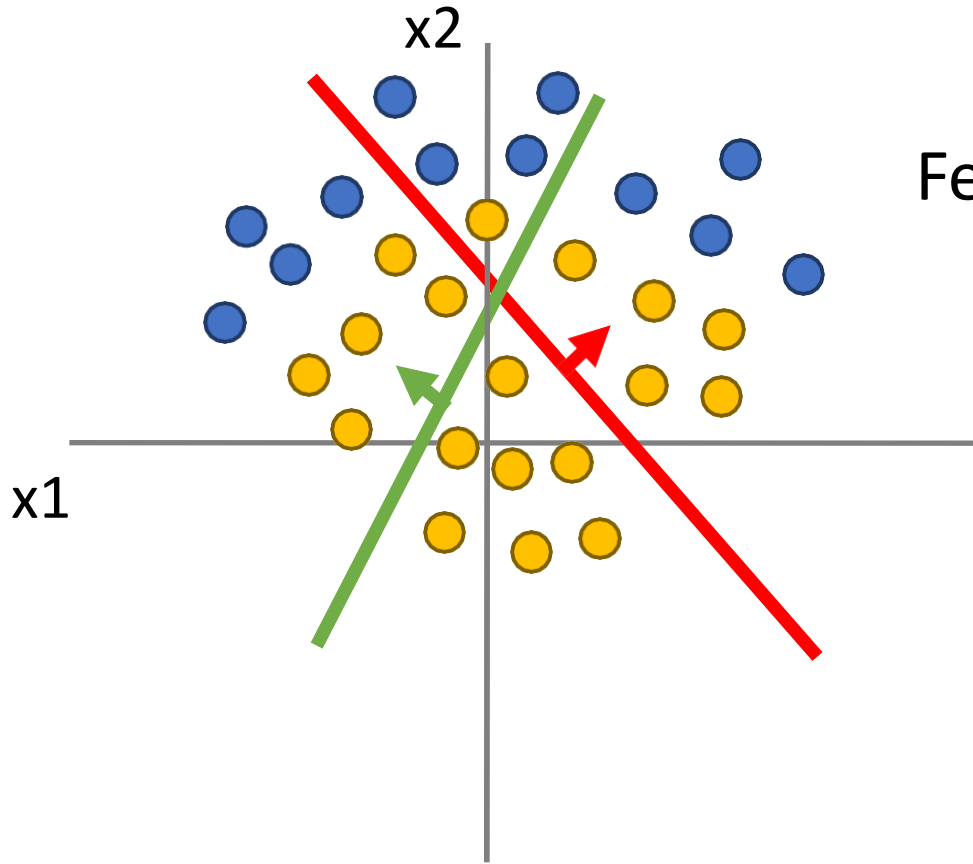


Consider a neural net hidden layer:
 $h = \text{ReLU}(Wx) = \max(0, Wx)$
Where x, h are both 2-dimensional

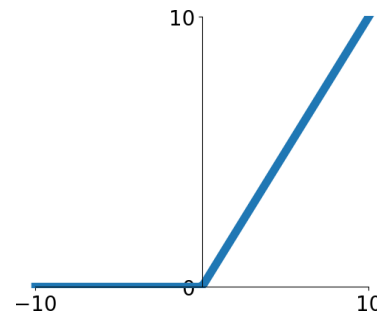


Feature Transform

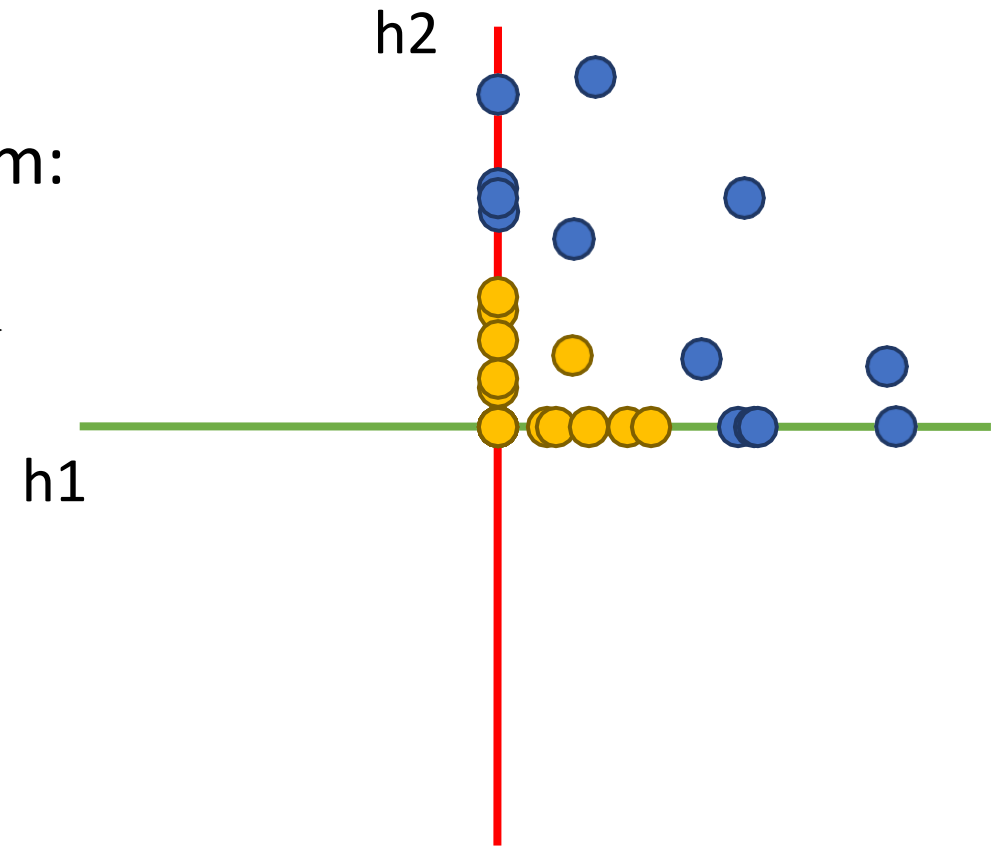
Points not linearly separable in original space



Feature transform:
 $h = \text{ReLU}(Wx)$

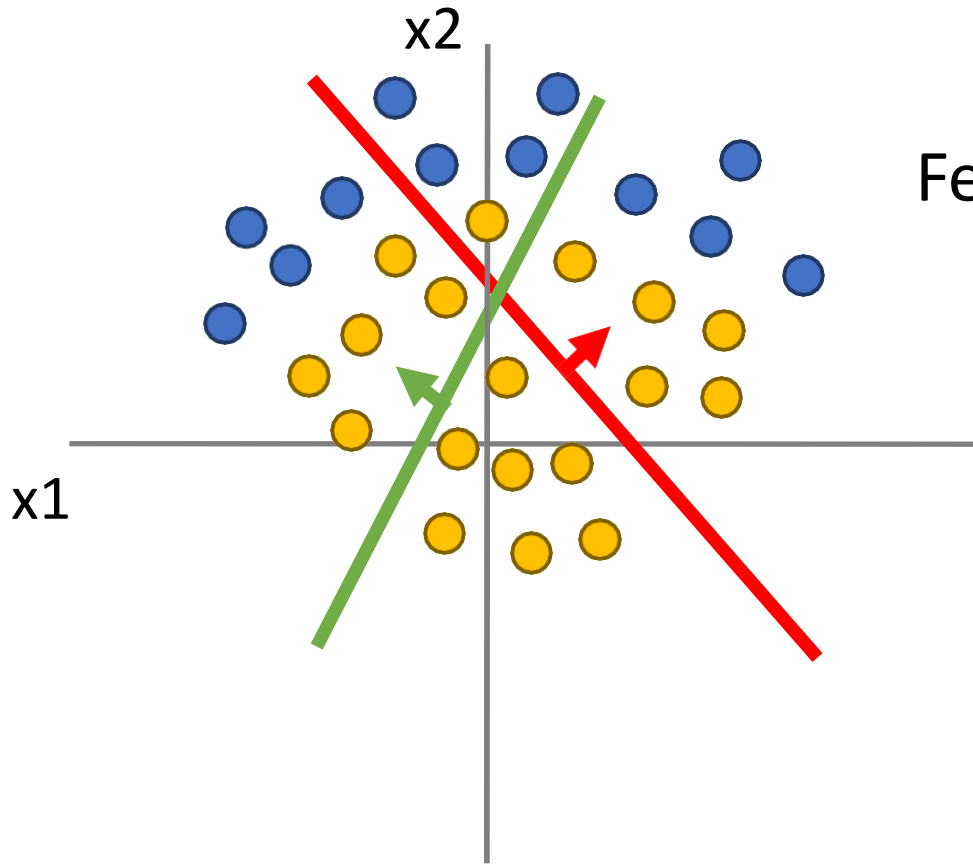


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Where x, h are both 2-dimensional

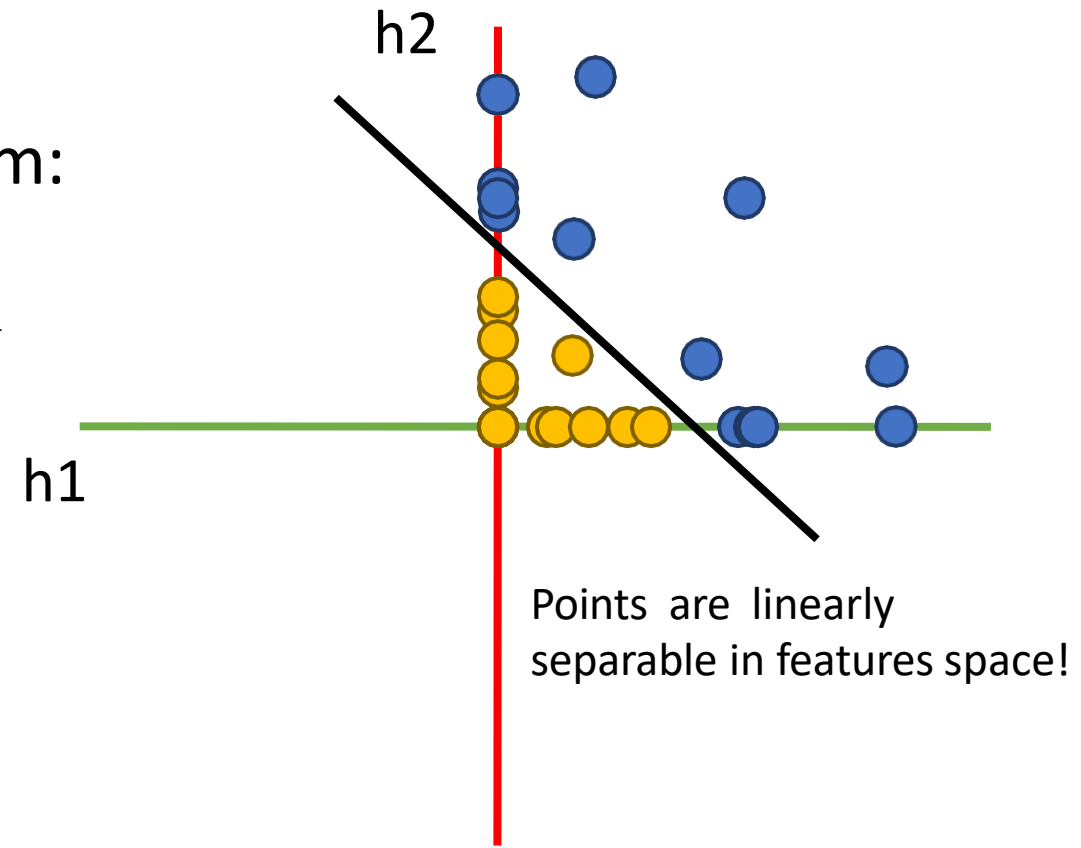
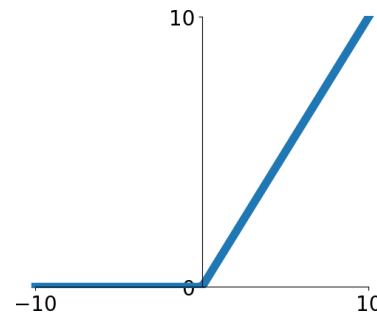


Feature Transform

Points not linearly separable in original space



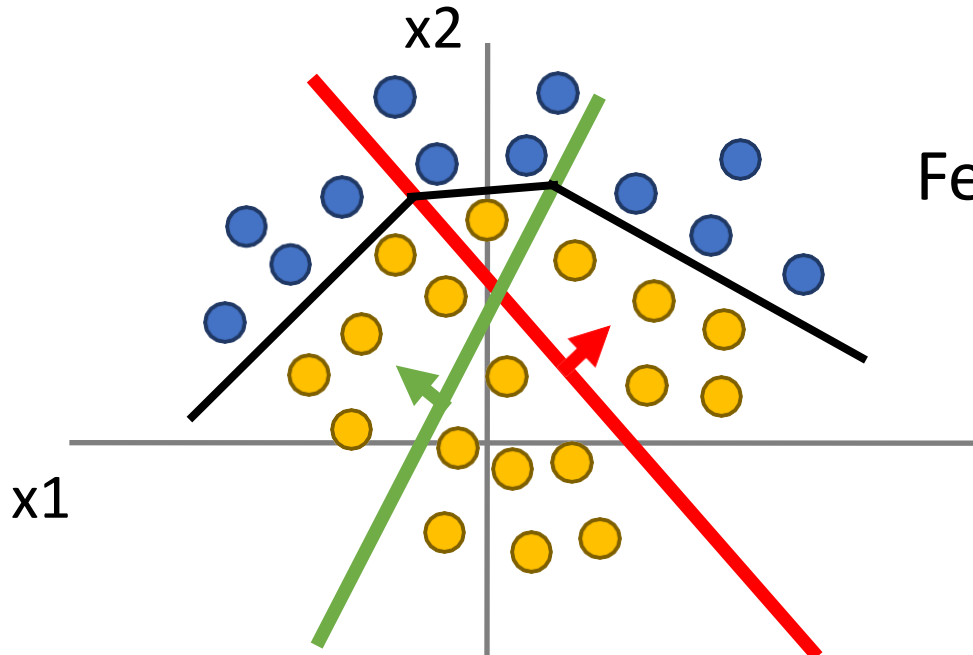
Feature transform:
 $h = \text{ReLU}(Wx)$



Consider a neural net hidden layer:
 $h = \text{ReLU}(Wx) = \max(0, Wx)$
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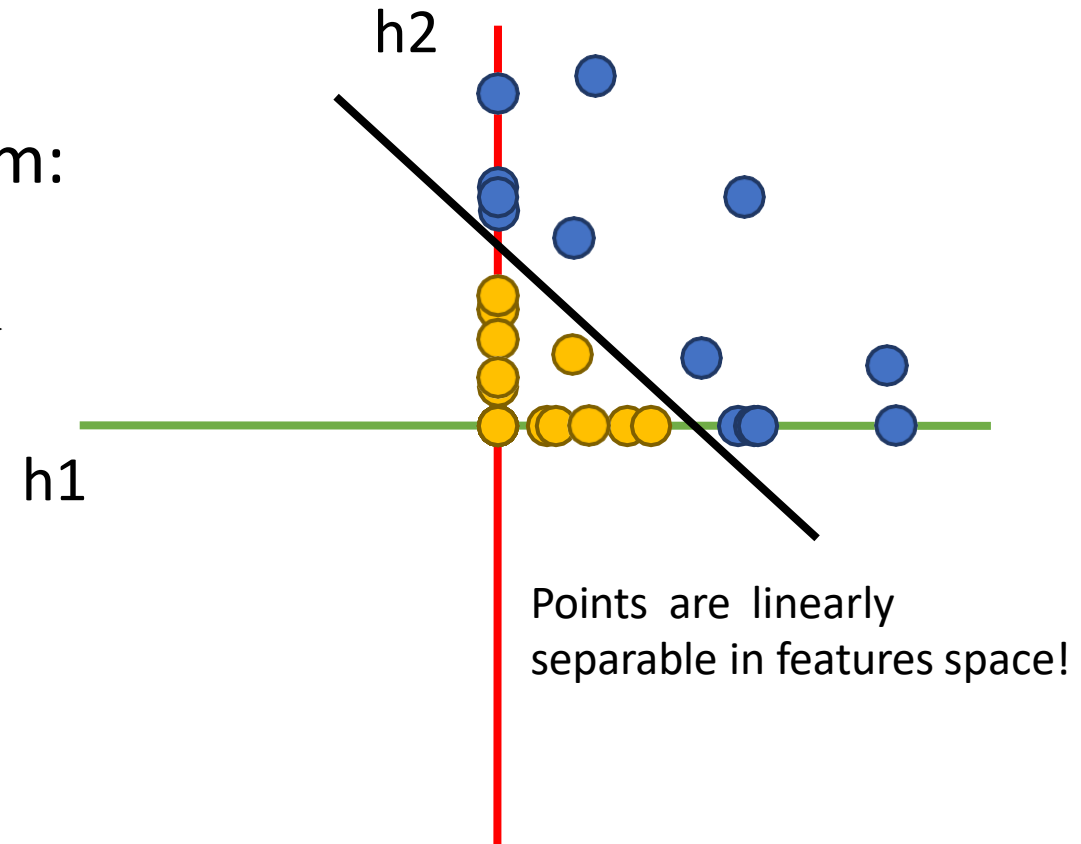
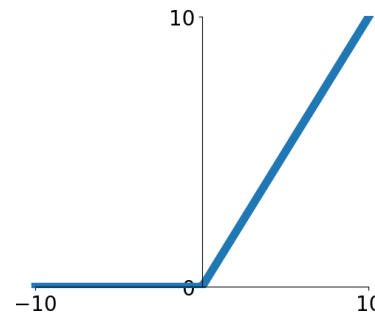
Feature Transform

Points not linearly separable in original space



Linear classifier in feature space gives nonlinear classifier in original space

Feature transform:
 $h = \text{ReLU}(Wx)$

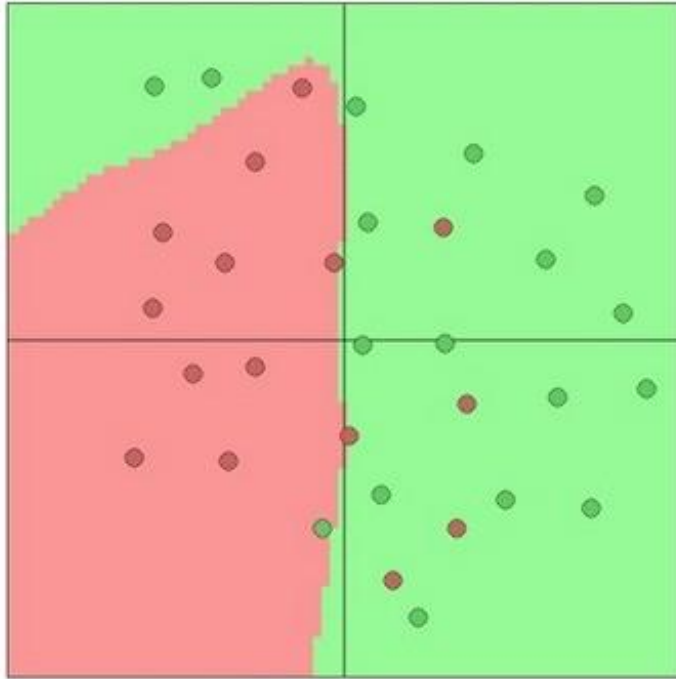


Points are linearly separable in features space!

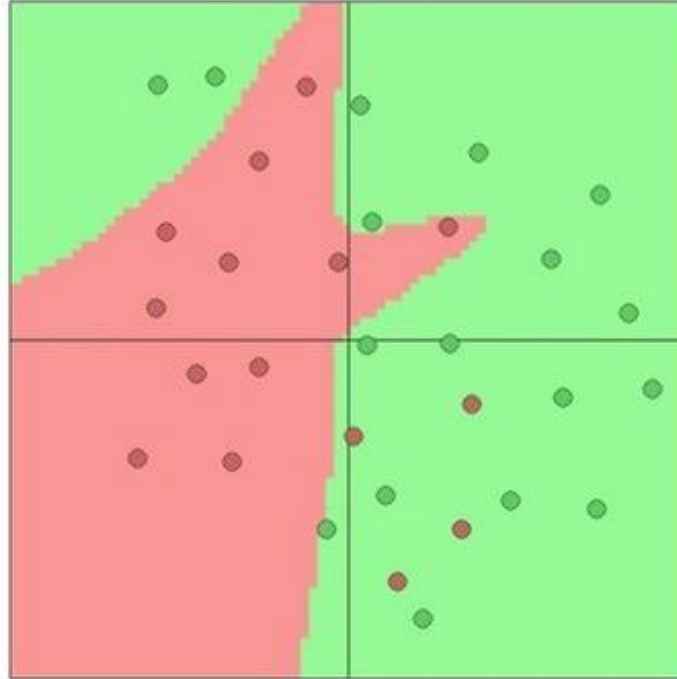
Consider a neural net hidden layer:
 $h = \text{ReLU}(Wx) = \max(0, Wx)$
Where x, h are both 2-dimensional

Setting the number of layers and their sizes

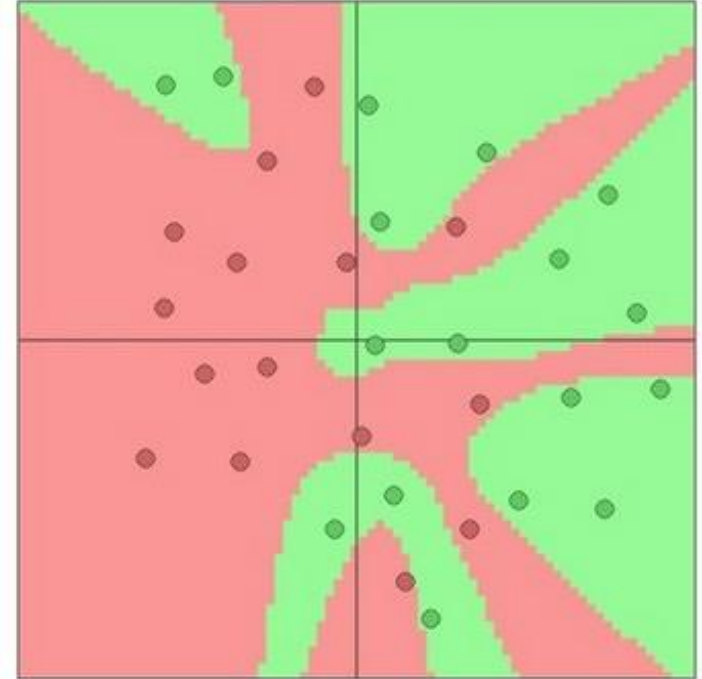
3 hidden units



6 hidden units



20 hidden units



More hidden units = more capacity

Regularization

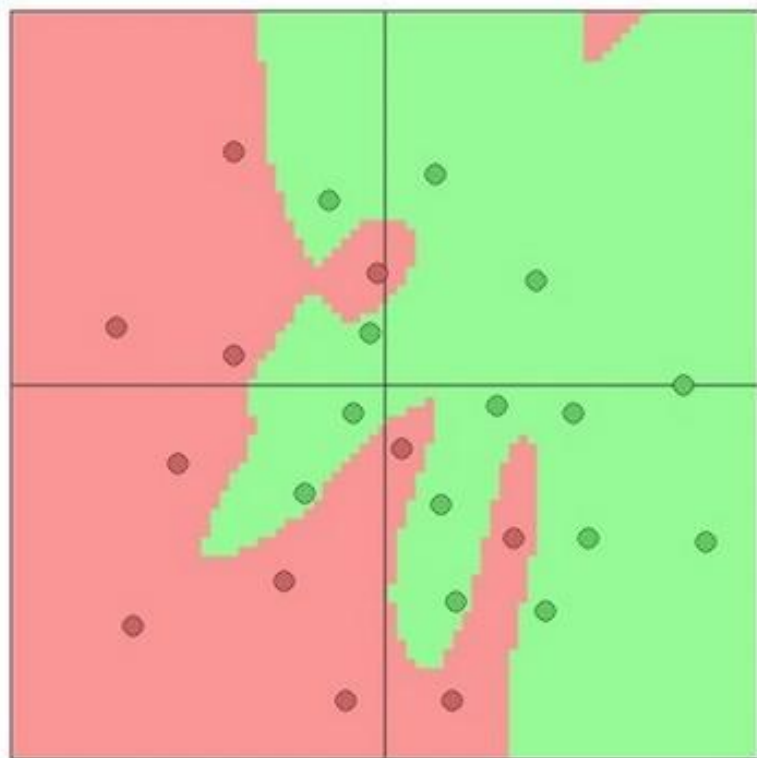
$$L(W) = \underbrace{\frac{1}{N} \sum_{i=1}^N L_i(f(x_i, W), y_i)}_{\text{Data loss}} + \underbrace{\lambda R(W)}_{\text{Regularization}}$$

Data loss: Model predictions should match training data

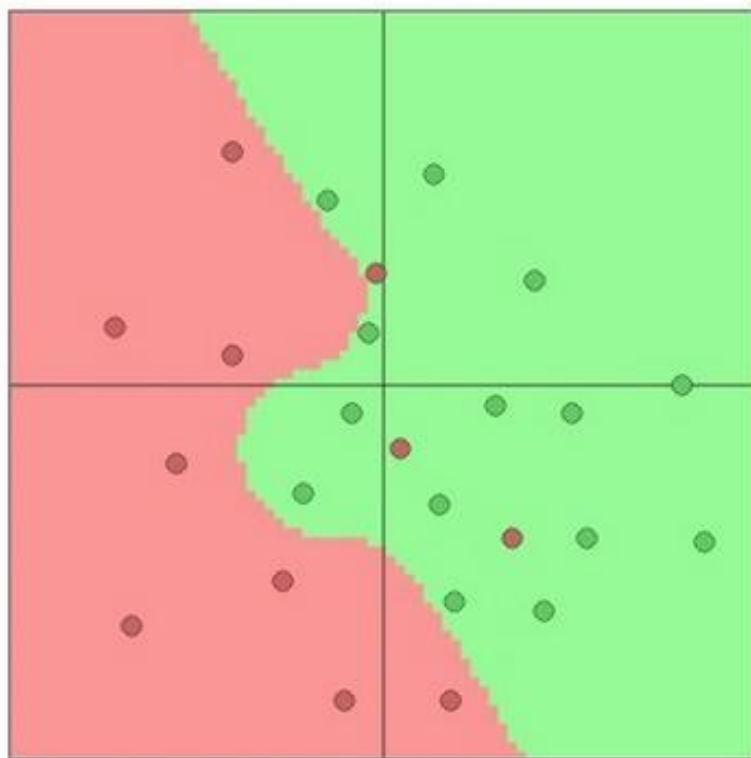
Regularization: Prevent the model from doing *too well* on training data

Regularization with constant number of layers

$\lambda = 0.001$



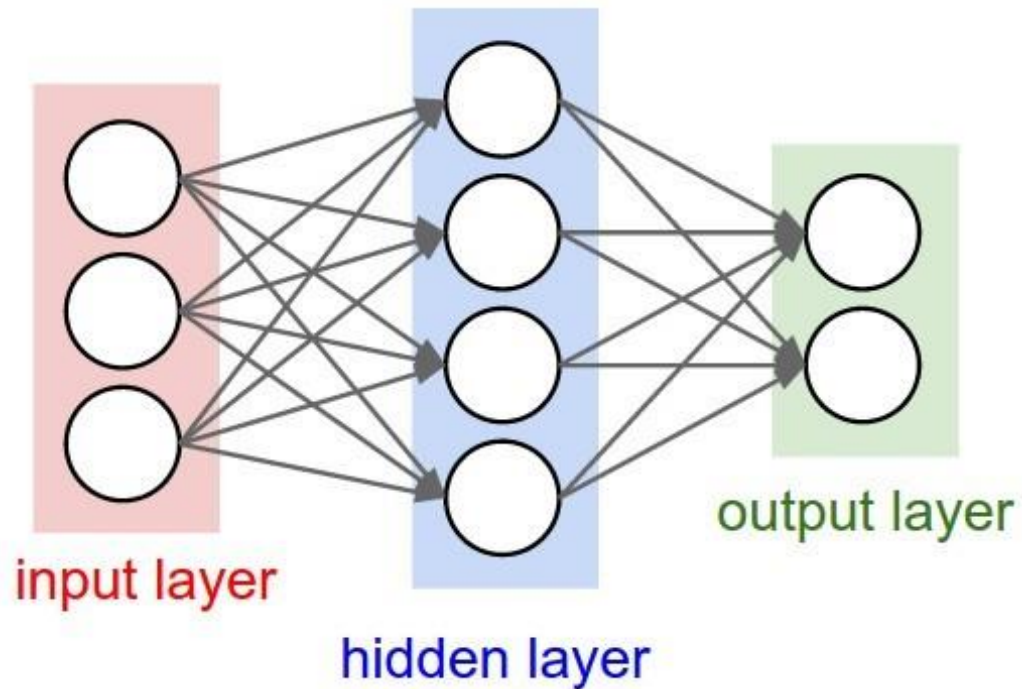
$\lambda = 0.01$



$\lambda = 0.1$

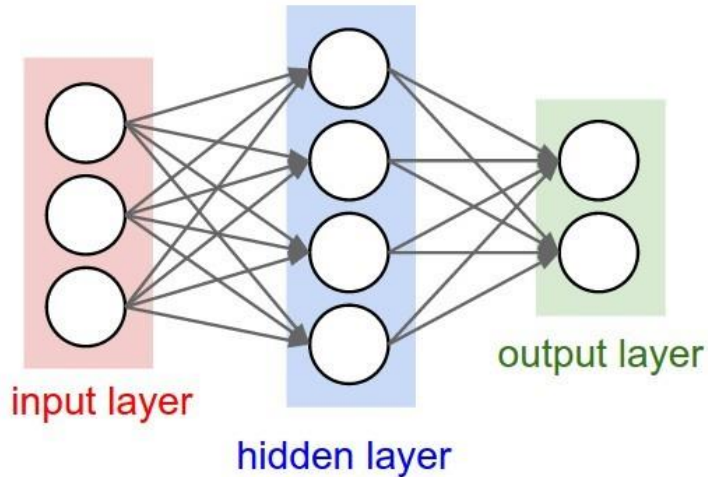


Neural Net sample code

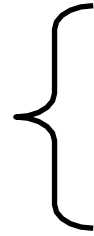


```
1 import numpy as np
2 from numpy.random import randn
3
4 N, Din, H, Dout = 64, 1000, 100, 10
5 x, y = randn(N, Din), randn(N, Dout)
6 w1, w2 = randn(Din, H), randn(H, Dout)
7 for t in range(10000):
8     h = 1.0 / (1.0 + np.exp(-x.dot(w1)))
9     y_pred = h.dot(w2)
10    loss = np.square(y_pred - y).sum()
11    dy_pred = 2.0 * (y_pred - y)
12    dw2 = h.T.dot(dy_pred)
13    dh = dy_pred.dot(w2.T)
14    dw1 = x.T.dot(dh * h * (1 - h))
15    w1 -= 1e-4 * dw1
16    w2 -= 1e-4 * dw2
```

Neural Net sample code

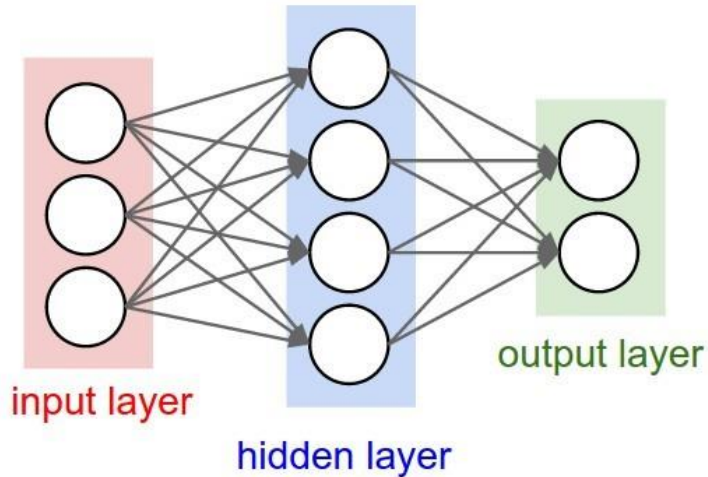


Initialize weights
and data



```
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Neural Net sample code

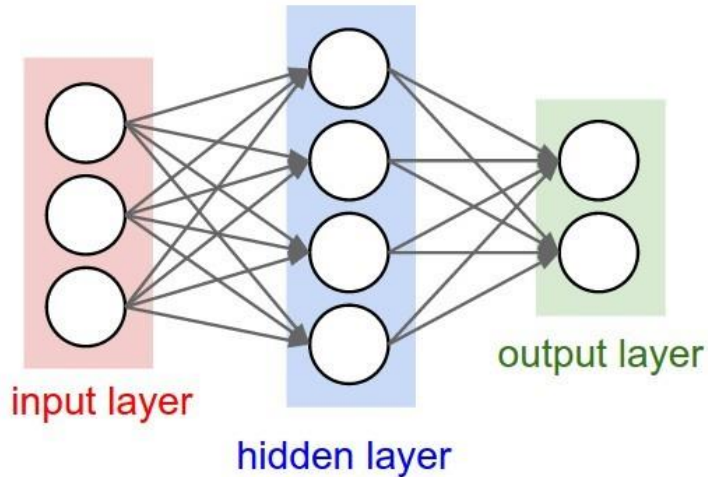


Initialize weights
and data

Compute loss
(sigmoid activation,
L2 loss)

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```

Neural Net sample code



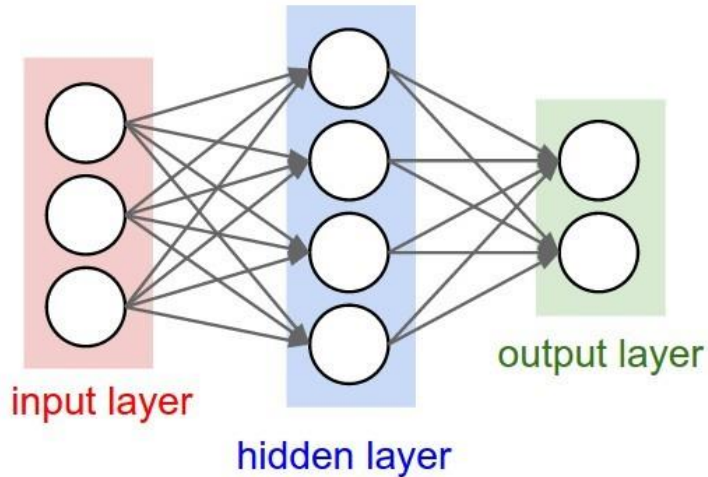
Initialize weights
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Compute
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```

Neural Net sample code



Initialize weights
and data

Compute loss
(sigmoid activation,
L2 loss)

Compute
gradients

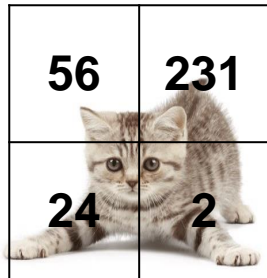
SGD
step

```
1 import numpy as np
2 from numpy.random import randn
3
4 N, Din, H, Dout = 64, 1000, 100, 10
5 x, y = randn(N, Din), randn(N, Dout)
6 w1, w2 = randn(Din, H), randn(H, Dout)
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```


Spatial Information

$$f = W_2 \max(0, W_1 x)$$

Flatten lattice into vector



Input image
(2, 2)



(4,)

Histogram of Oriented Gradients

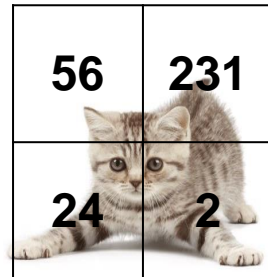


Spatial Information

$$f = W_2 \max(0, W_1 x)$$

Flatten lattice into vector

56	231
24	2



Input image
(2, 2)

Problem: So far our neural networks don't respect the spatial structure of images

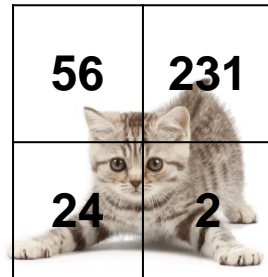
56
231
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2

(4,)

Spatial Information

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Input image
(2, 2)

Problem: So far our neural networks don't respect the spatial structure of images

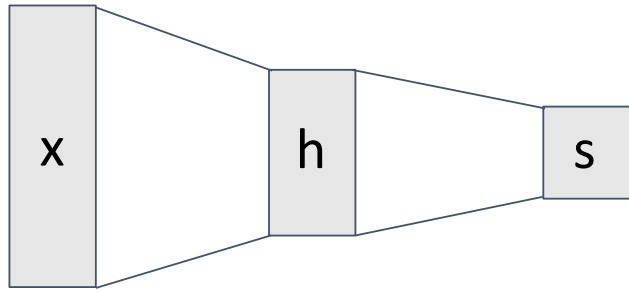
Solution: Define new computational operators



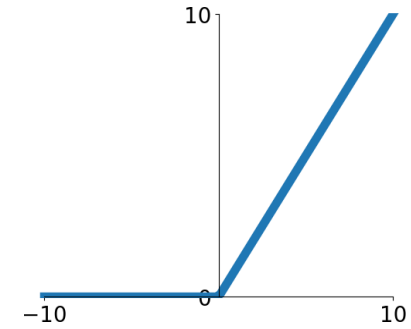
(4,)

Components of a Fully-Connected Network

Fully-Connected Layers

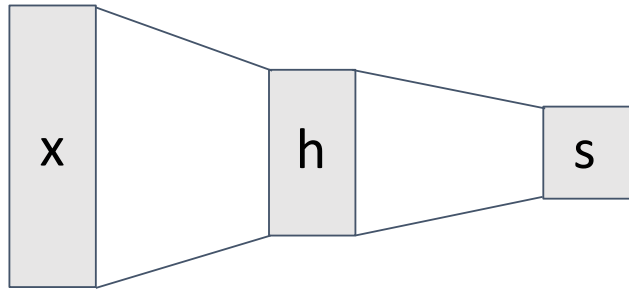


Activation Function

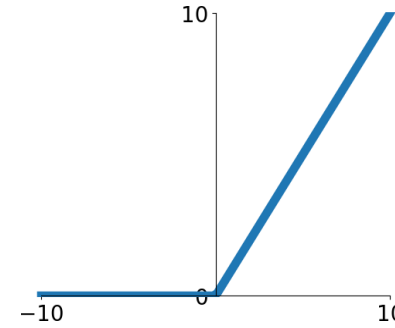


Components of a Convolutional Network

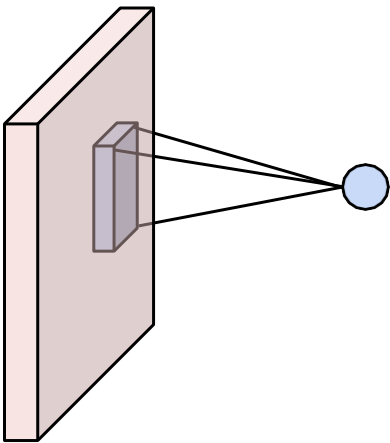
Fully-Connected Layers



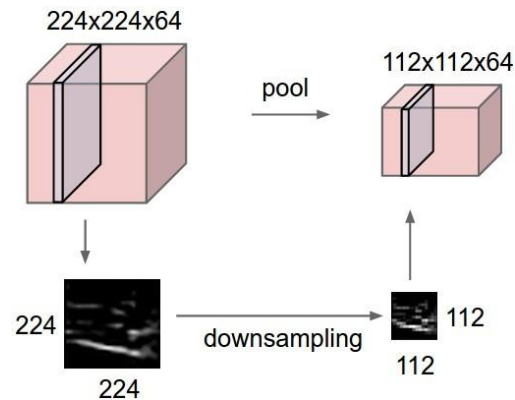
Activation Function



Convolution Layers



Pooling Layers

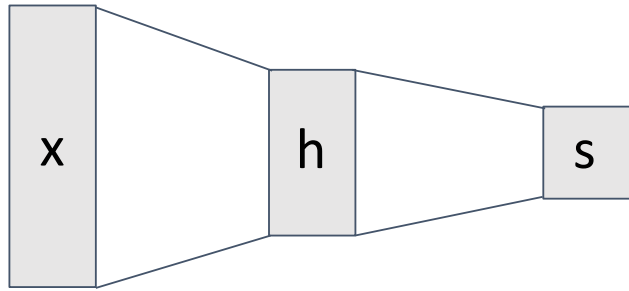


Normalization

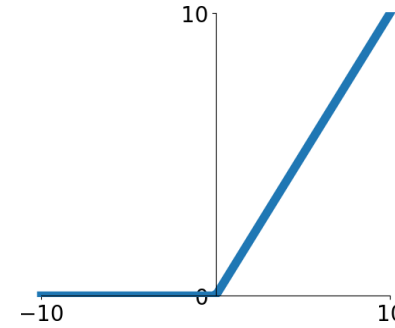
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$

Components of a Convolutional Network

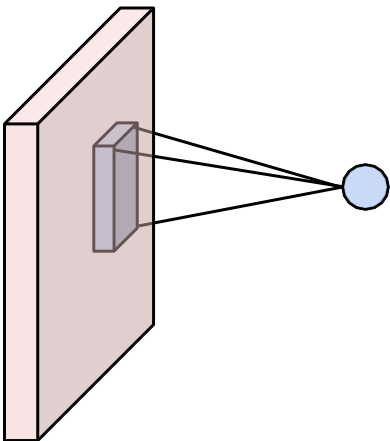
Fully-Connected Layers



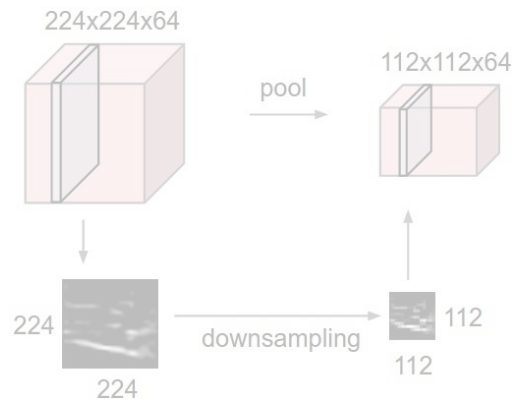
Activation Function



Convolution Layers



Pooling Layers

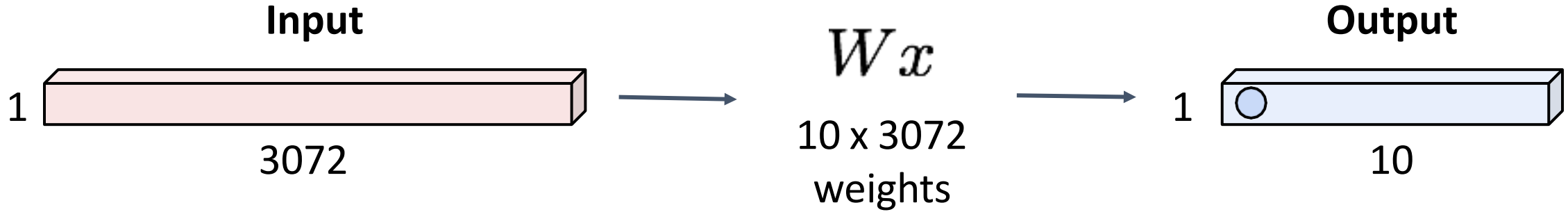


Normalization

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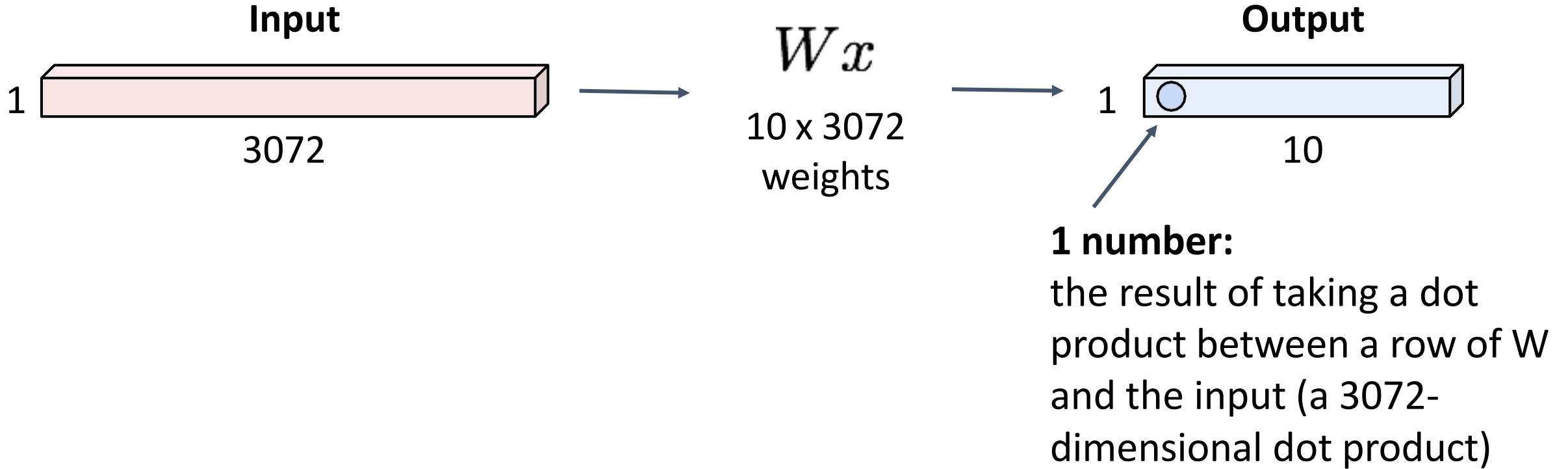
Fully-Connected Layer

32x32x3 image -> flatten to 3072 x 1



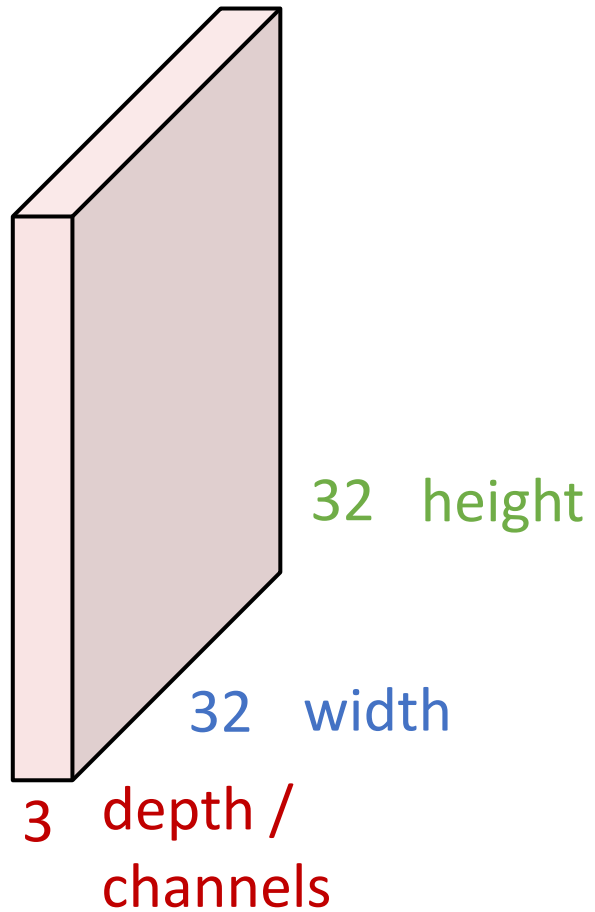
Fully-Connected Layer

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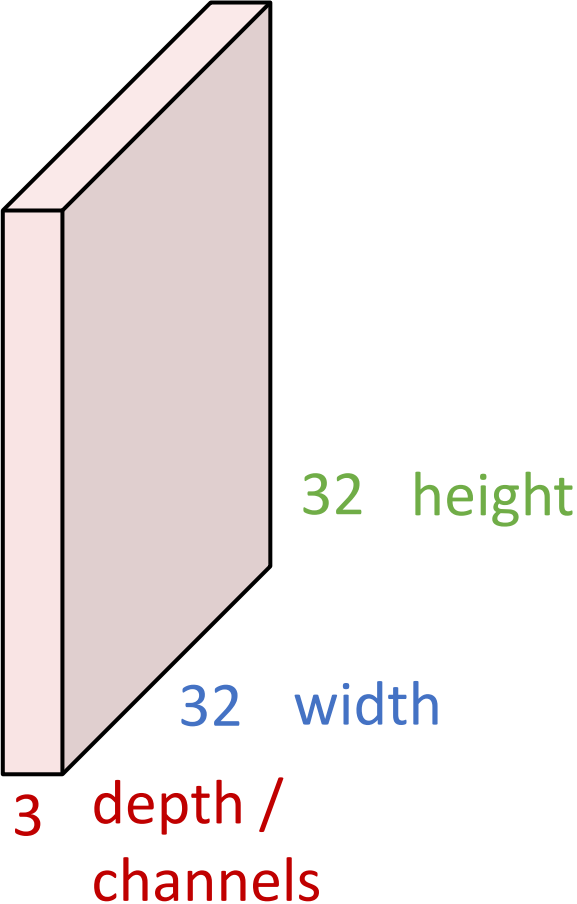
Convolution Layer

3x32x32 image: preserve structure



Convolution Layer

3x32x32 image



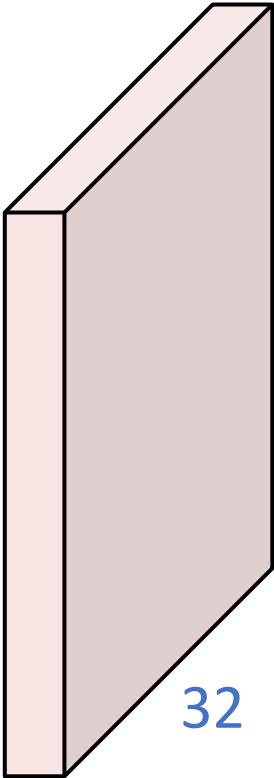
3x5x5 filter



Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

Convolution Layer

3x32x32 image



32 height

32 width

3 depth / channels

Filters always extend the full depth of the input volume

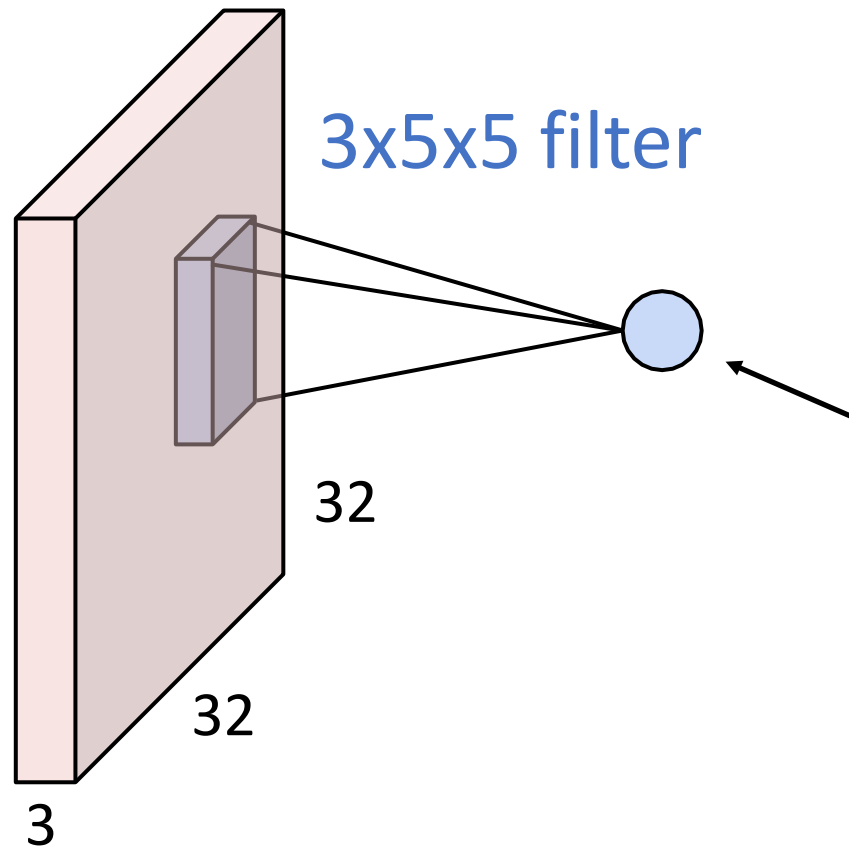
3x5x5 filter



Convolve the filter with the image
i.e. “slide over the image spatially,
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Convolution Layer

3x32x32 image



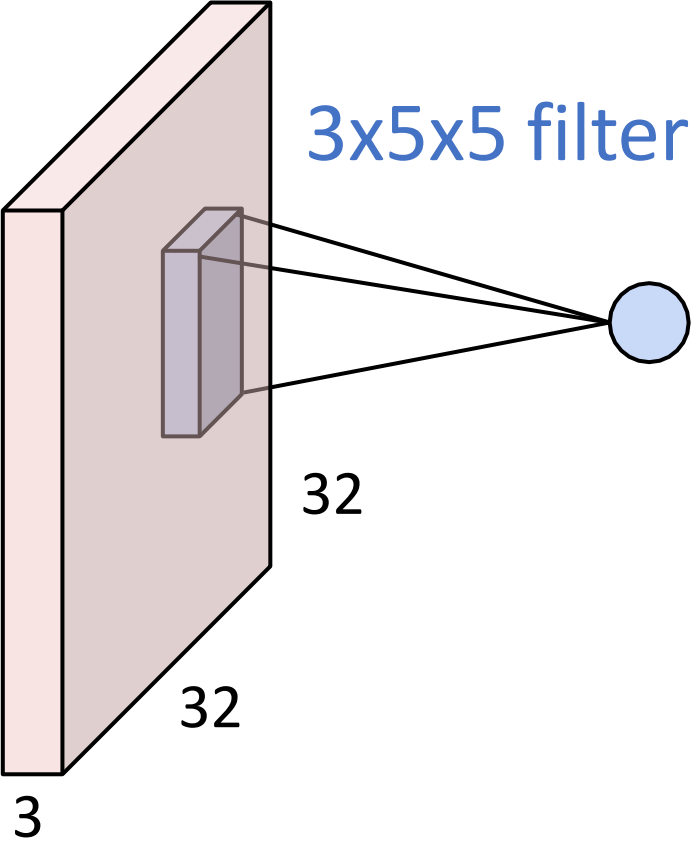
1 number:

the result of taking a dot product between the filter and a small 3x5x5 chunk of the image (i.e. $3*5*5 = 75$ -dimensional dot product + bias)

$$w^T x + b$$

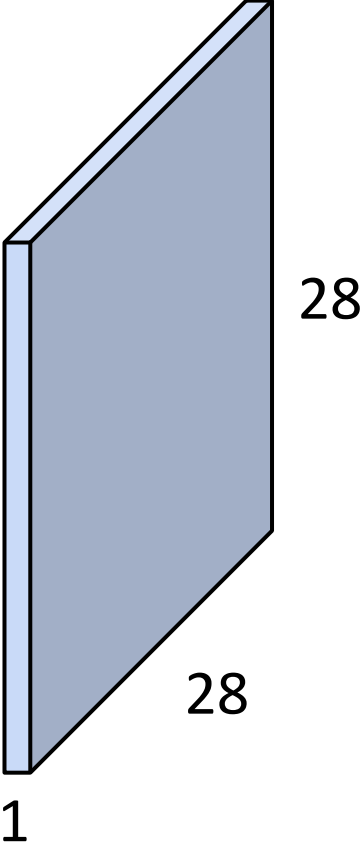
Convolution Layer

3x32x32 image



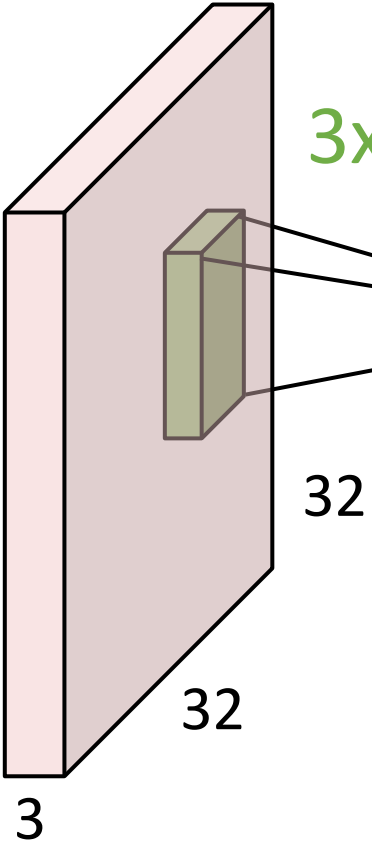
convolve (slide) over all spatial locations

1x28x28
activation map

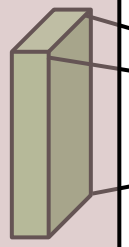


Convolution Layer

3x32x32 image



3x5x5 filter

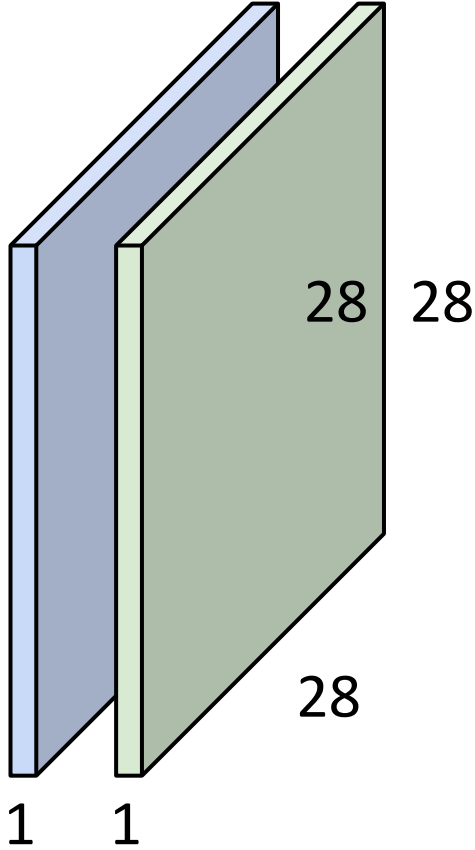


Consider repeating with a second (green) filter:



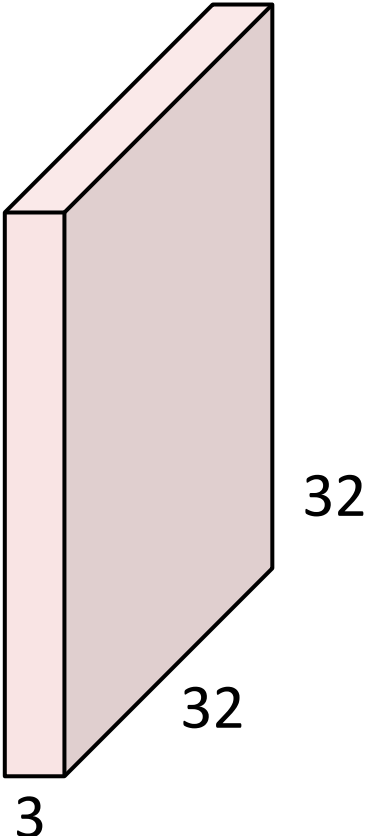
convolve (slide) over all spatial locations

two 1x28x28 activation map

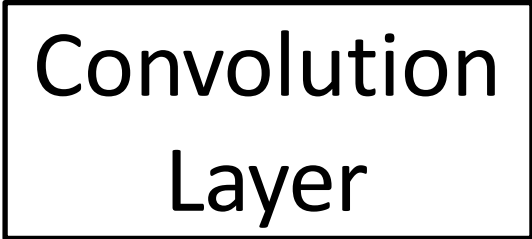


Convolution Layer

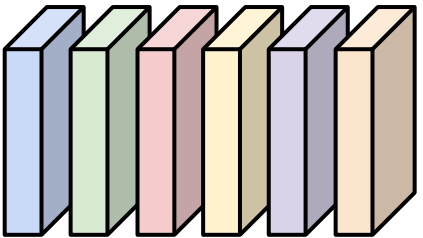
3x32x32 image



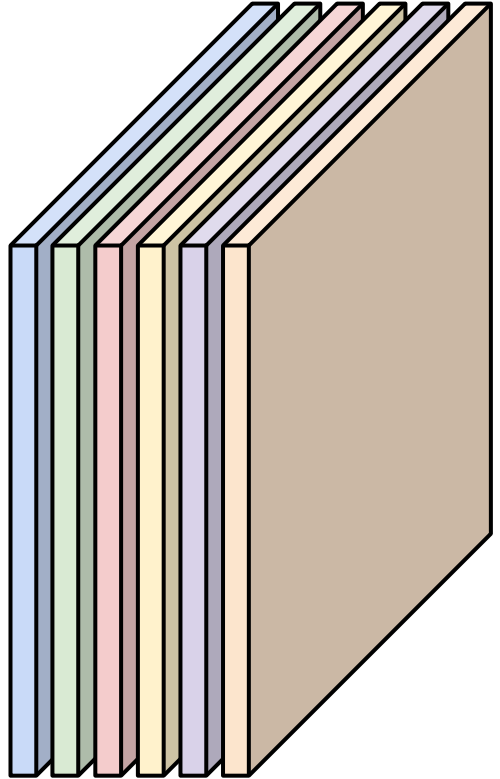
Consider 6 filters, each 3x5x5



6x3x5x5 filters



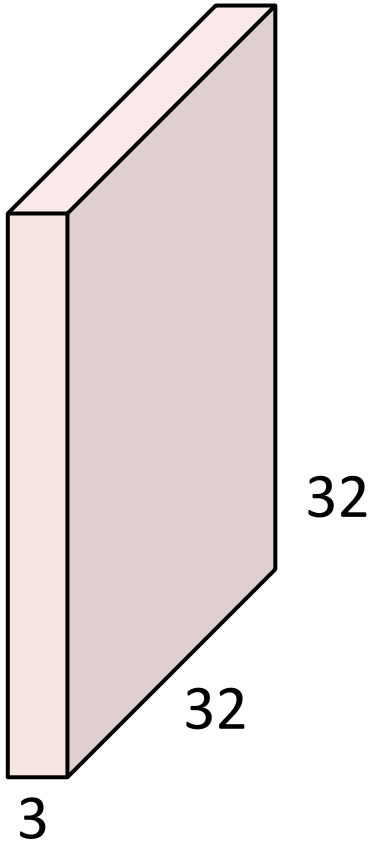
28x28 grid, at each point a 6-dim vector



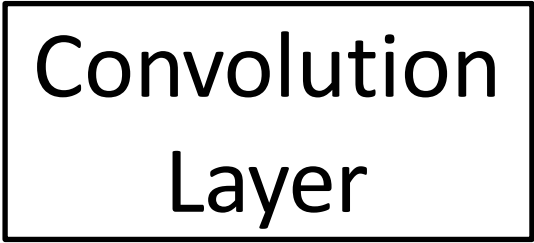
Stack activations to get a 6x28x28 output image

Convolution Layer

3x32x32 image



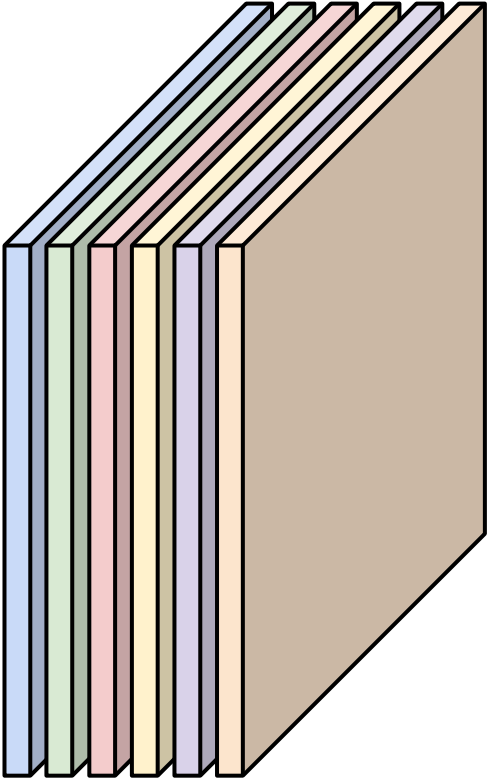
Also 6-dim bias vector:



6x3x5x5 filters



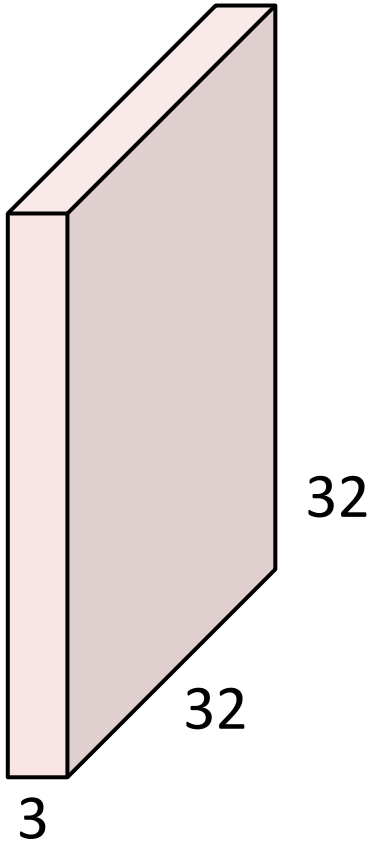
28x28 grid, at each point a 6-dim vector



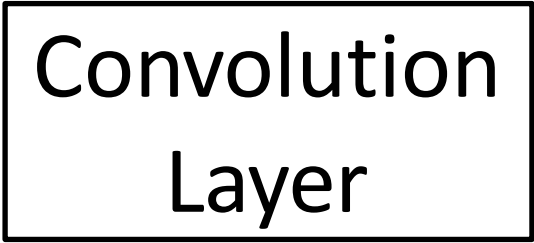
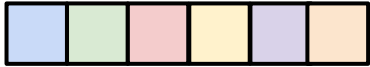
Stack activations to get a 6x28x28 output image

Convolution Layer

3x32x32 image



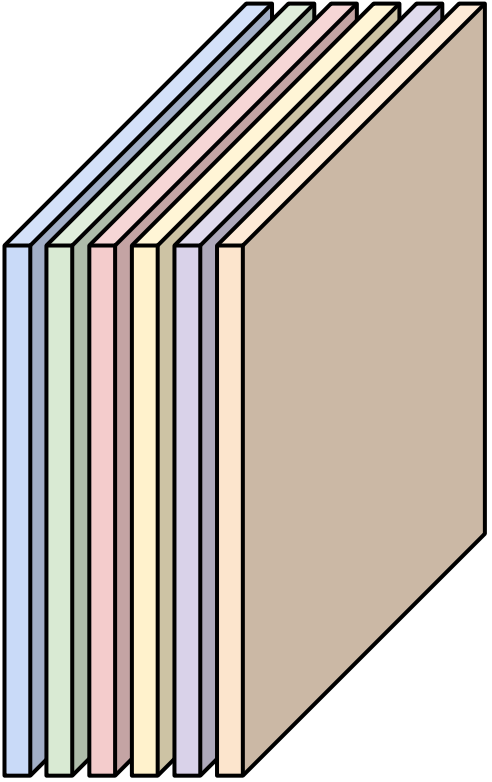
Also 6-dim bias vector:



6x3x5x5 filters



6 activation maps, each 1x28x28

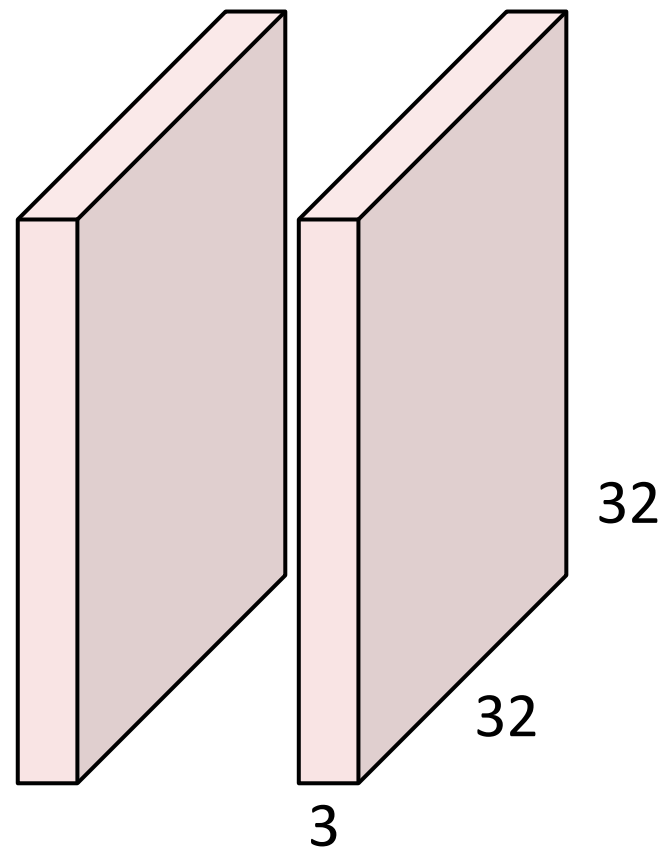


Stack activations to get a 6x28x28 output image

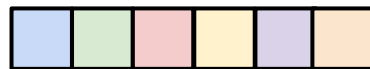
Convolution Layer

$2 \times 3 \times 32 \times 32$

Batch of images

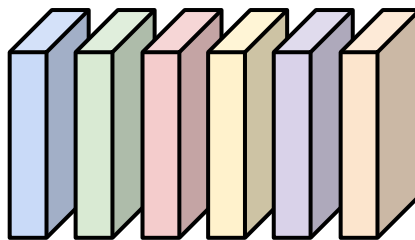


Also 6-dim bias vector:

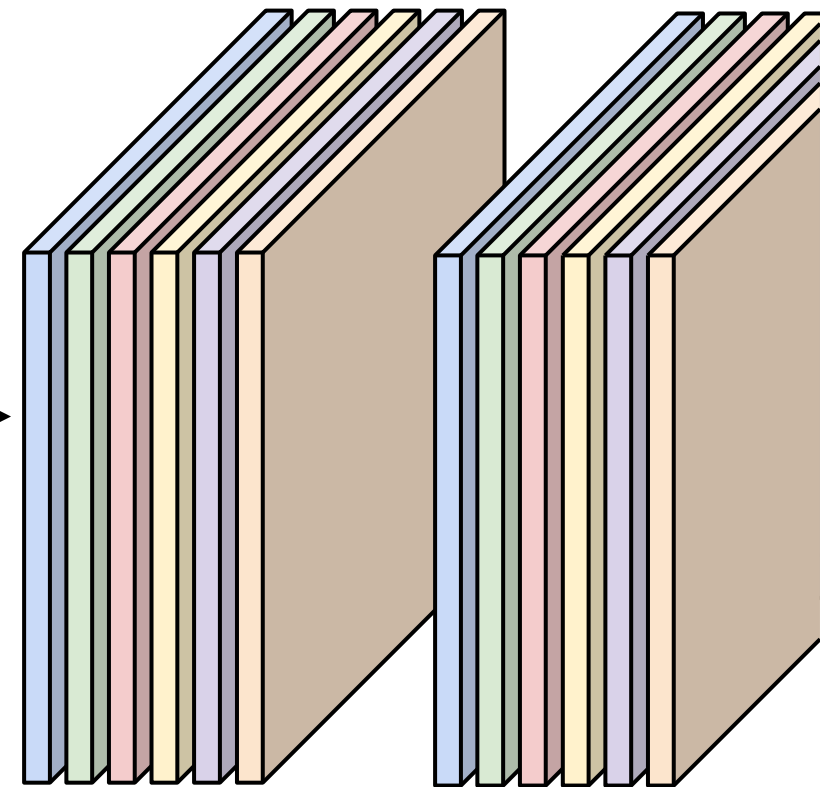


Convolution Layer

$6 \times 3 \times 5 \times 5$
filters

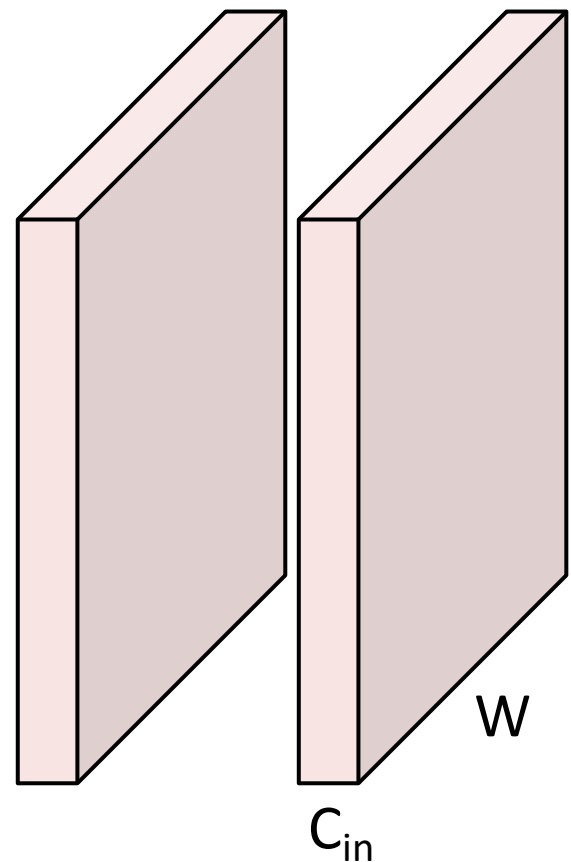


$2 \times 6 \times 28 \times 28$
Batch of outputs

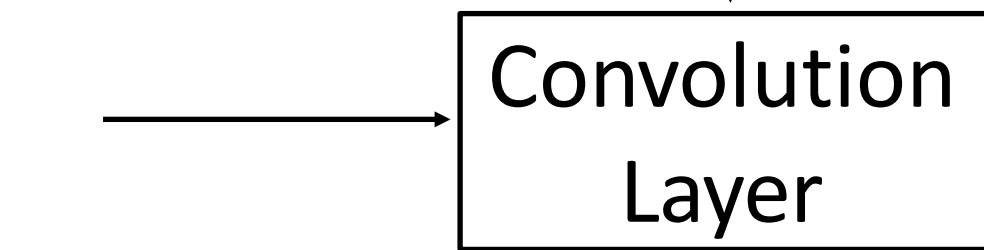


Convolution Layer

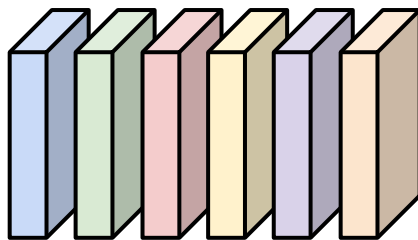
$N \times C_{in} \times H \times W$
Batch of images



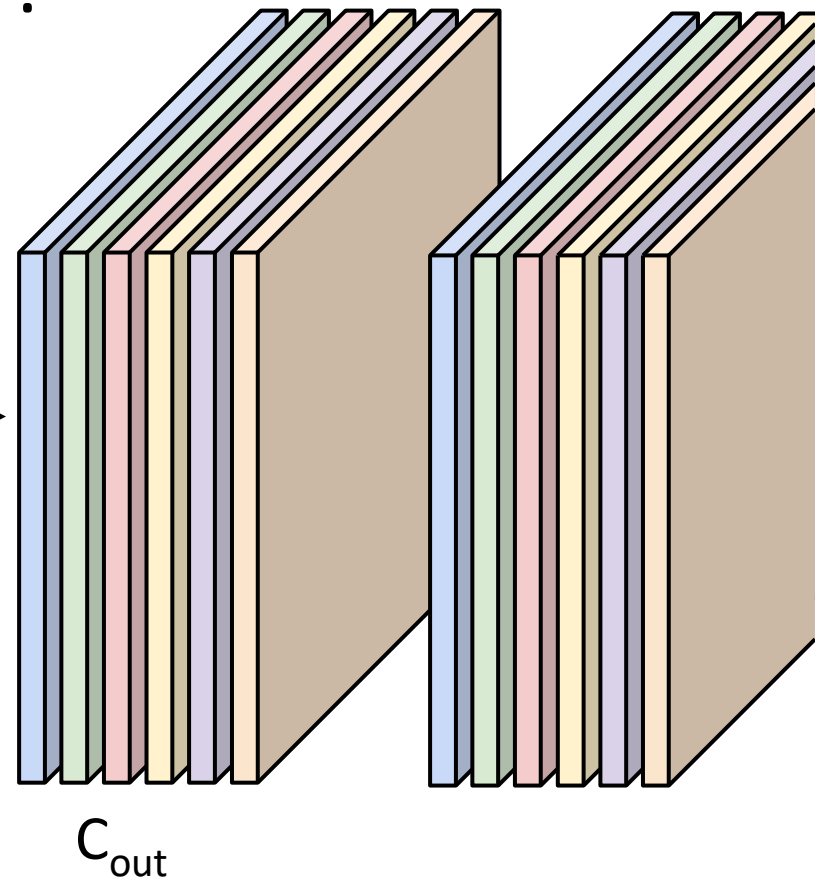
Also C_{out} -dim bias vector:



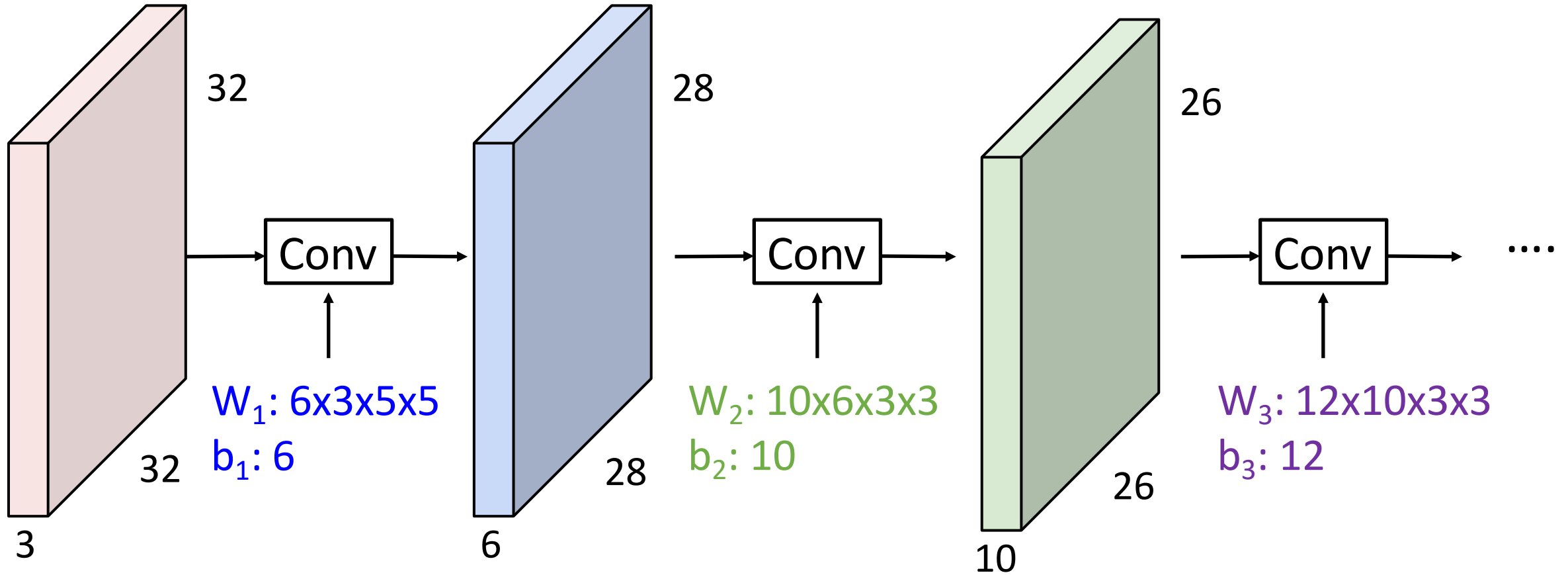
$C_{out} \times C_{in} \times K_w \times K_h$
filters



$N \times C_{out} \times H' \times W'$
Batch of outputs



Stacking Convolutions



Input:

$N \times 3 \times 32 \times 32$

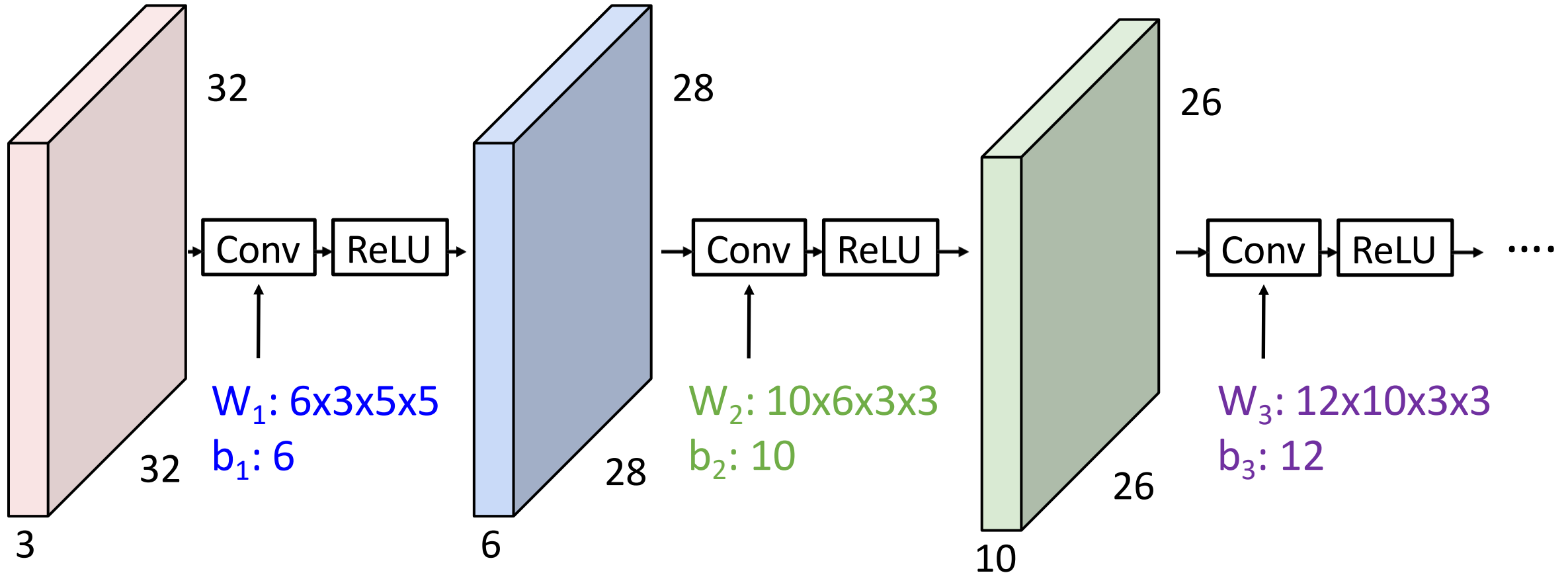
First hidden layer:

$N \times 6 \times 28 \times 28$

Second hidden layer:

$N \times 10 \times 26 \times 26$

Stacking Convolutions: Add Non-linearity



Input:

$N \times 3 \times 32 \times 32$

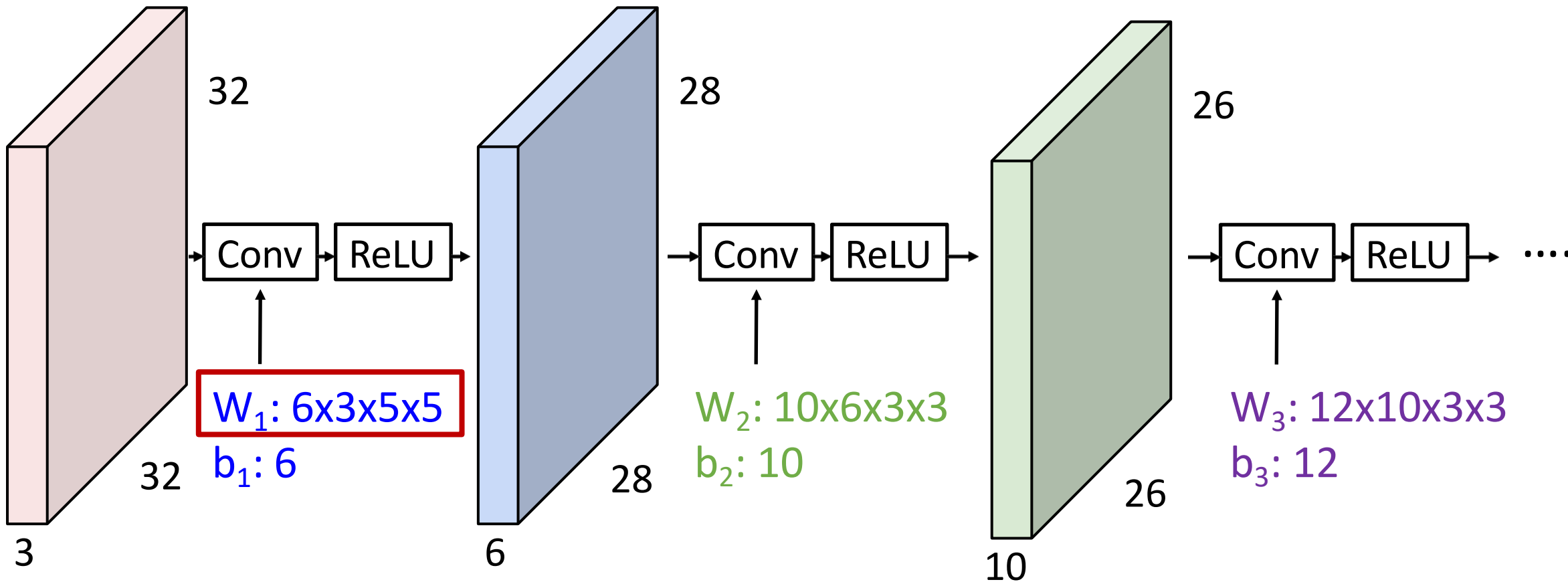
First hidden layer:

$N \times 6 \times 28 \times 28$

Second hidden layer:

$N \times 10 \times 26 \times 26$

What do convolutional filters learn?

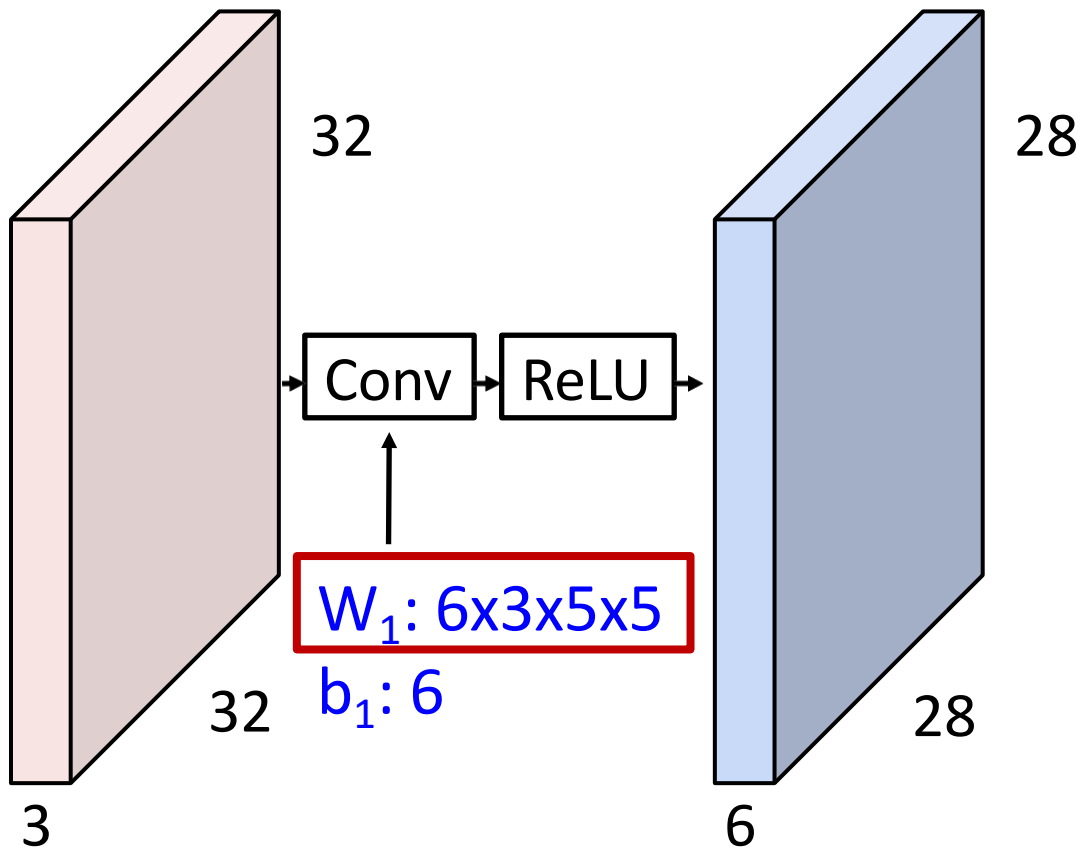


Input:
 $N \times 3 \times 32 \times 32$

First hidden layer:
 $N \times 6 \times 28 \times 28$

Second hidden layer:
 $N \times 10 \times 26 \times 26$

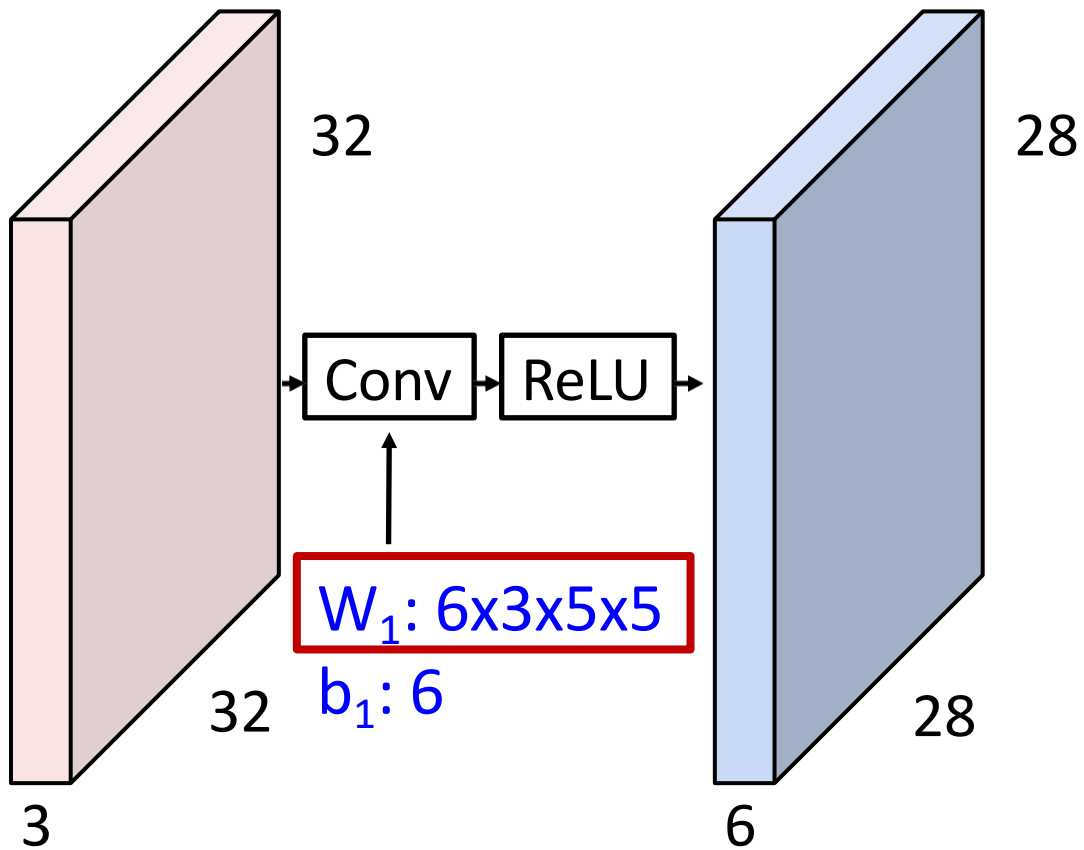
What do convolutional filters learn?



Linear classifier: One template per class



What do convolutional filters learn?



Input:

$N \times 3 \times 32 \times 32$

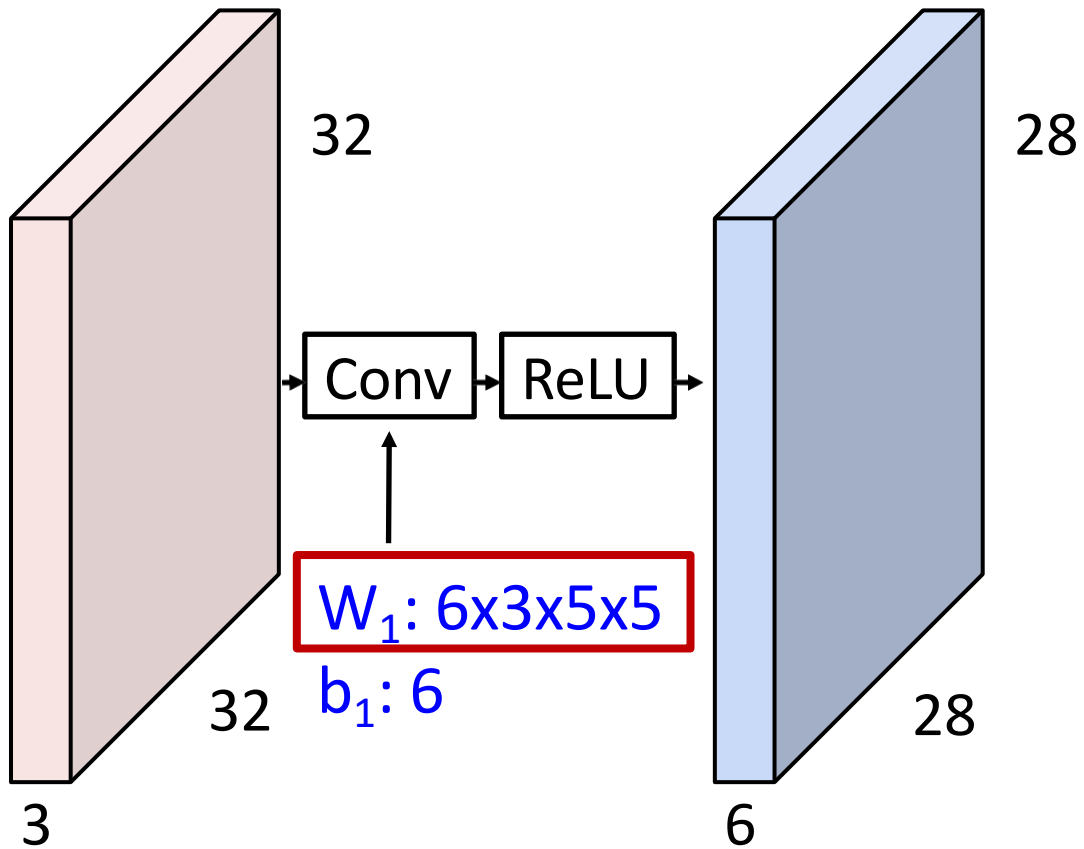
First hidden layer:

$N \times 6 \times 28 \times 28$

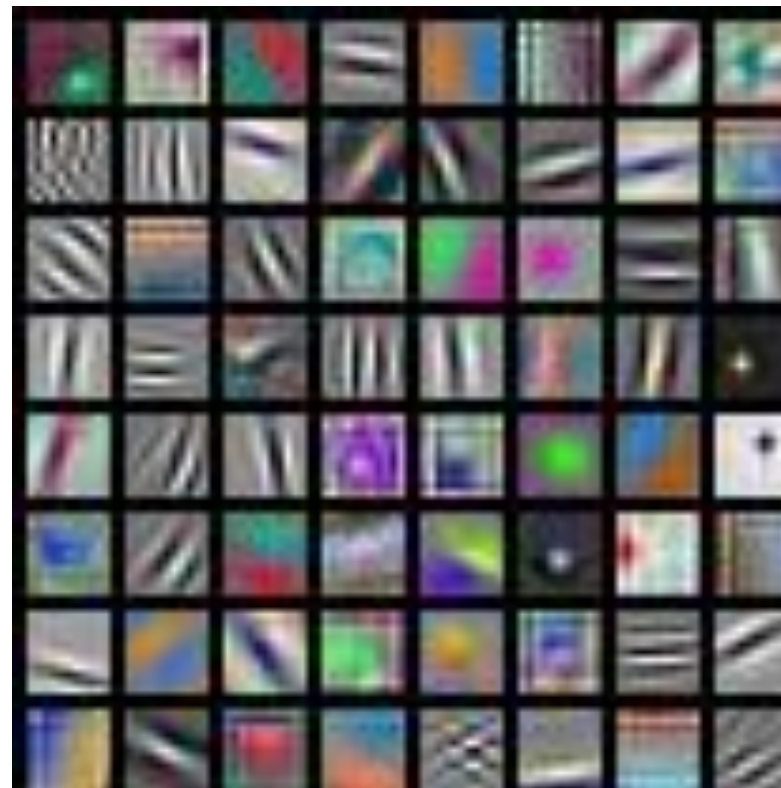
MLP: Bank of whole-image templates



What do convolutional filters learn?



First-layer conv filters: local image templates
(Often learns oriented edges, opposing colors)



AlexNet: 64 filters, each $3 \times 11 \times 11$