Deep Learning

Summary

1. Use **Linear Models** for image classification problems

$$s = f(x; W) = Wx$$

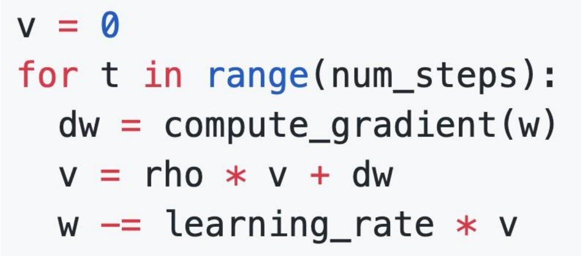
2. Use **Loss Functions** to express preferences over different selection of weights

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$ Softmax

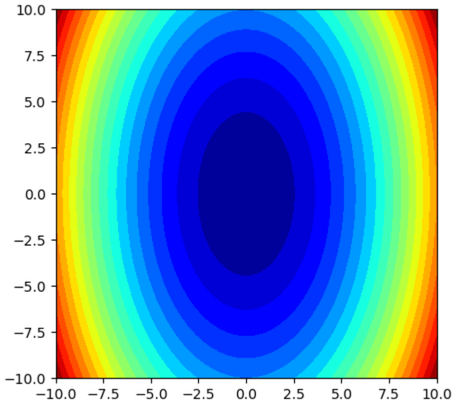
$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$ SVM

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i + R(W)$$

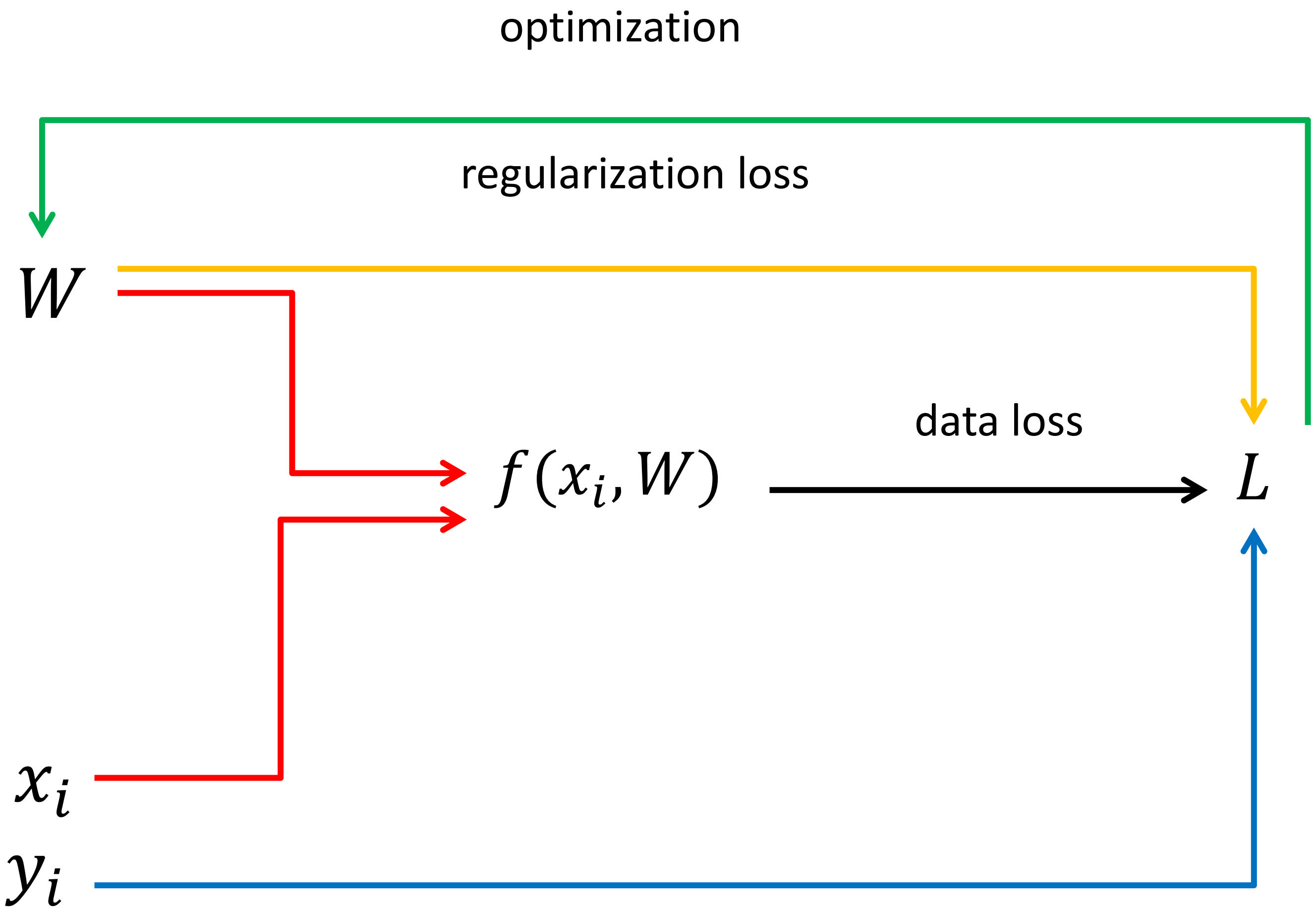
3. Use **Stochastic Gradient Descent** to minimize our loss functions and train the model



```
v = 0
for t in range(num_steps):
  dw = compute_gradient(w)
  v = rho * v + dw
  w -= learning_rate * v
```



Linear Classifier



optimization

regularization loss

data loss

W

$f(x_i, W)$

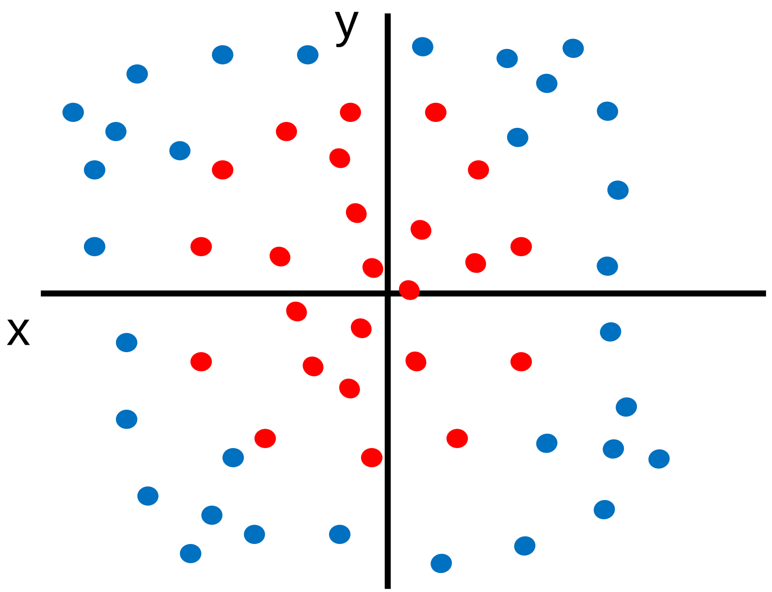
L

x_i

y_i

Linear Classifiers shortcomings

Geometric Viewpoint



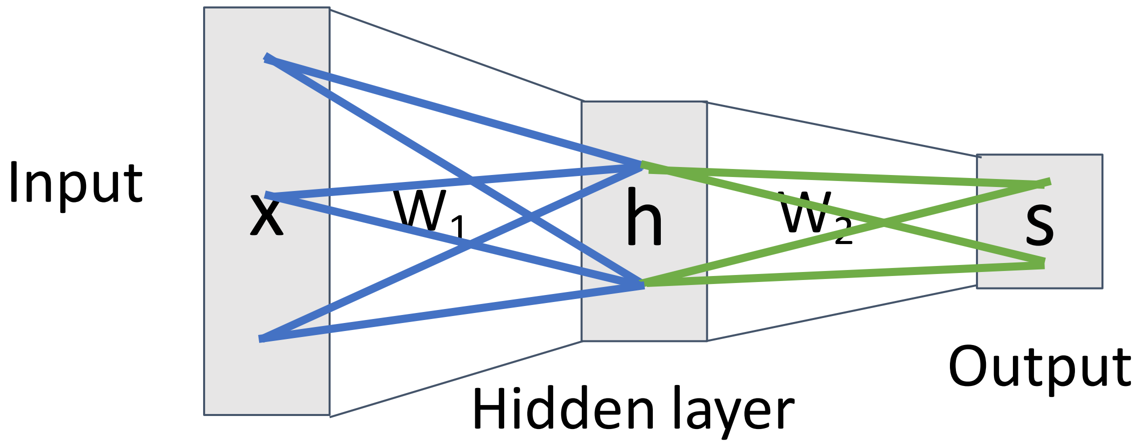
Some training data can't be separated with a hyperplane

Visual Viewpoint

One template per class: Can't recognize different modes of a class

Multi-Layer Perceptrons

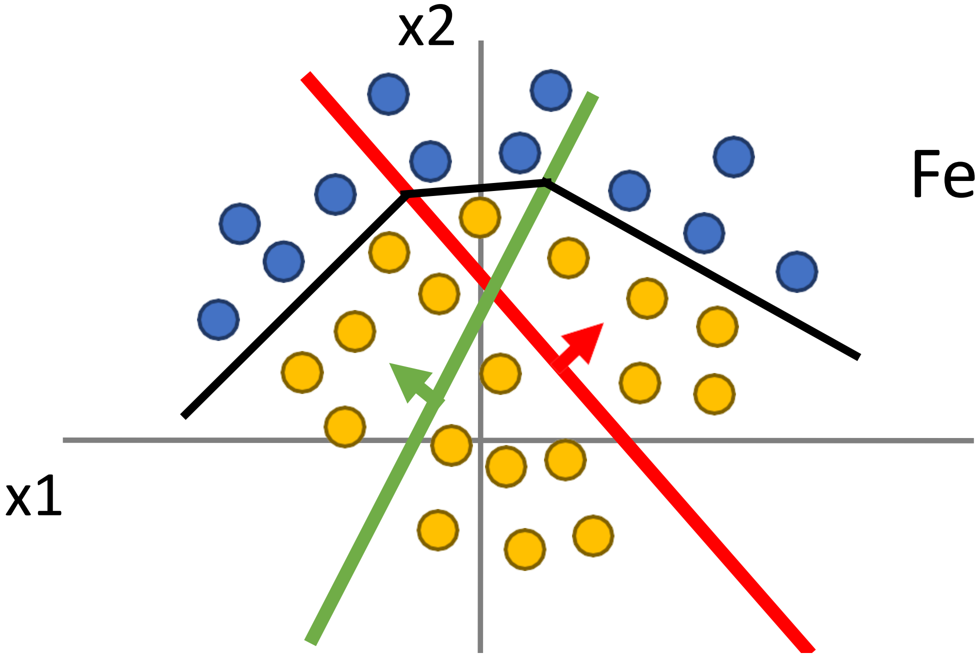
$$f = W_2 \max(0, W_1 x)$$



Feature Transform

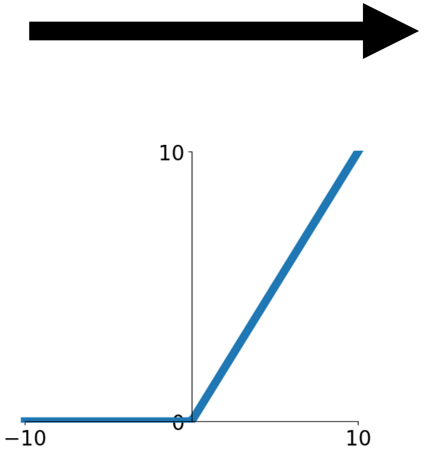
Consider a neural net hidden layer:
h = ReLU(Wx) = max(0, Wx)
Where x, h are both 2-dimensional

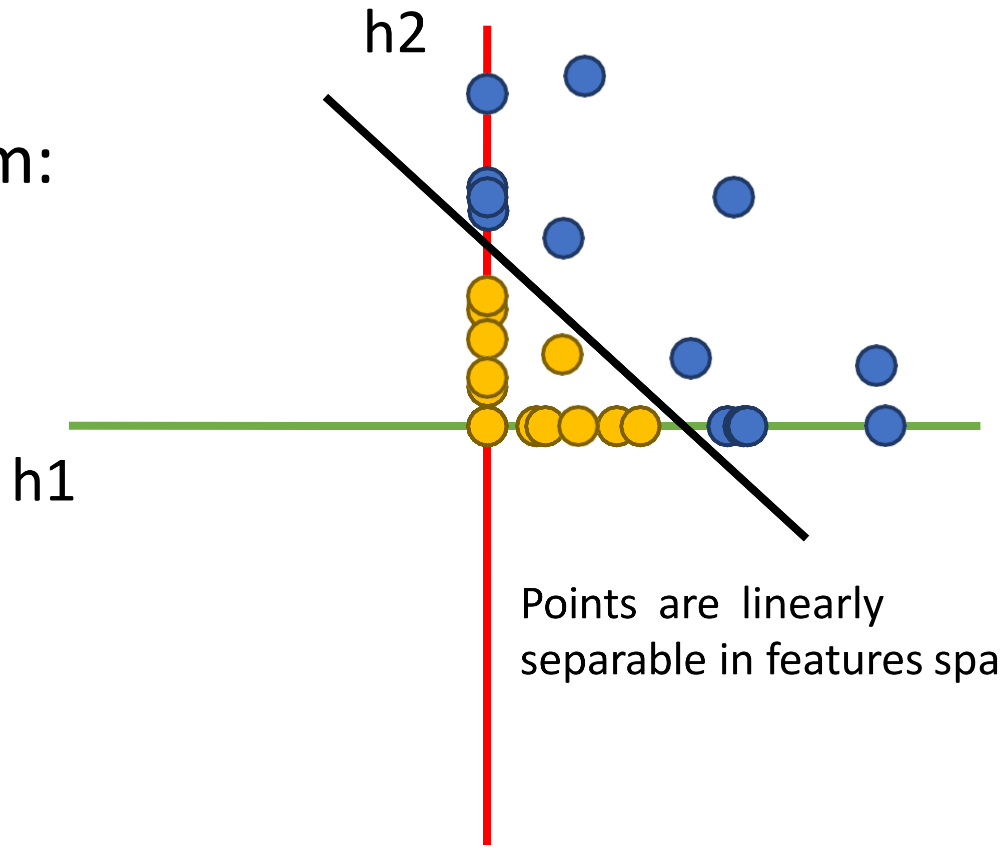
Points not linearly separable in original space



Linear classifier in feature space gives nonlinear classifier in original space

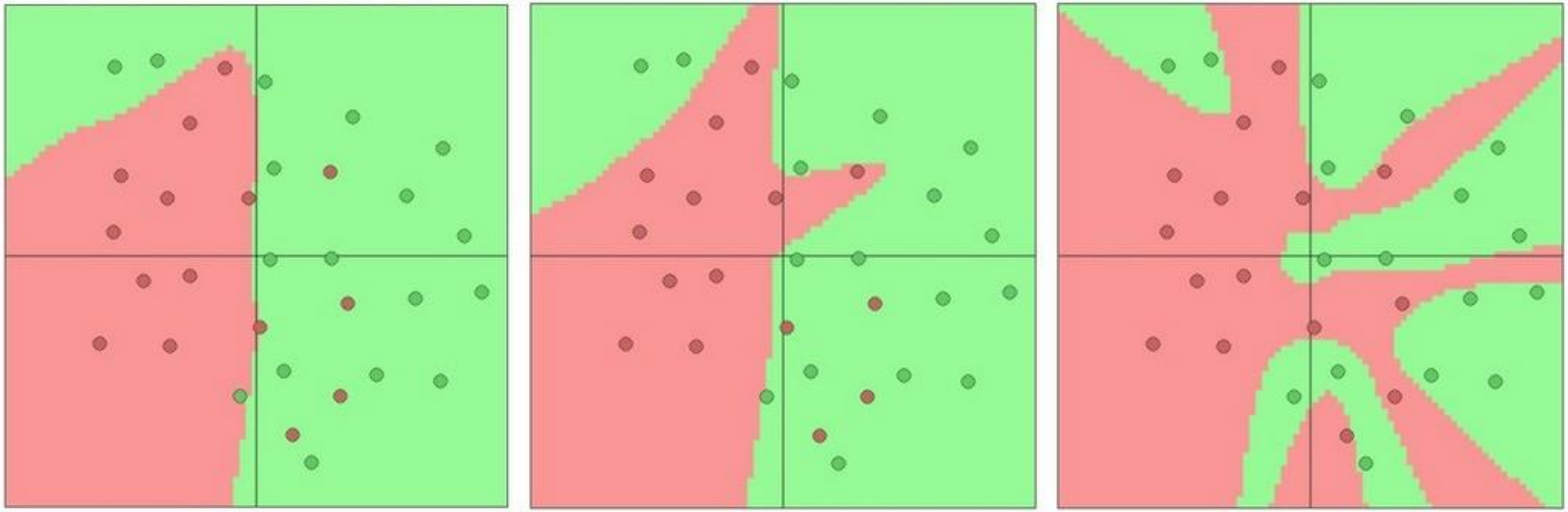
Feature transform:
h = ReLU(Wx)





Points are linearly separable in features space!

Setting the number of layers and their sizes



3 hidden units

6 hidden units

20 hidden units

More hidden units = more capacity

Spatial Information

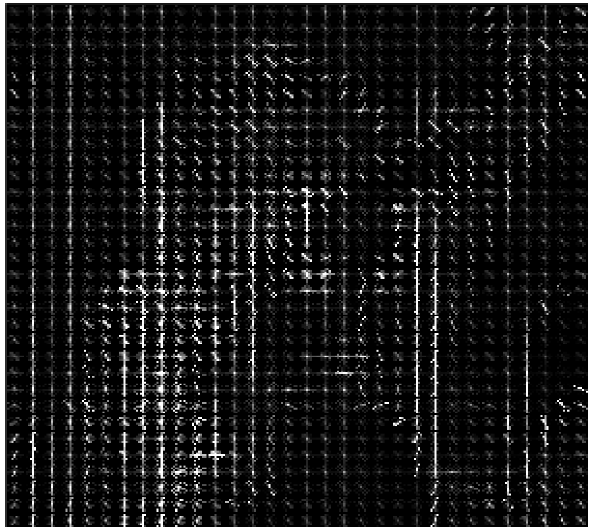
$$f = W_2 \max(0, W_1 x)$$

Flatten lattice into vector

Input image
(2, 2)

(4,)

Histogram of Oriented Gradients



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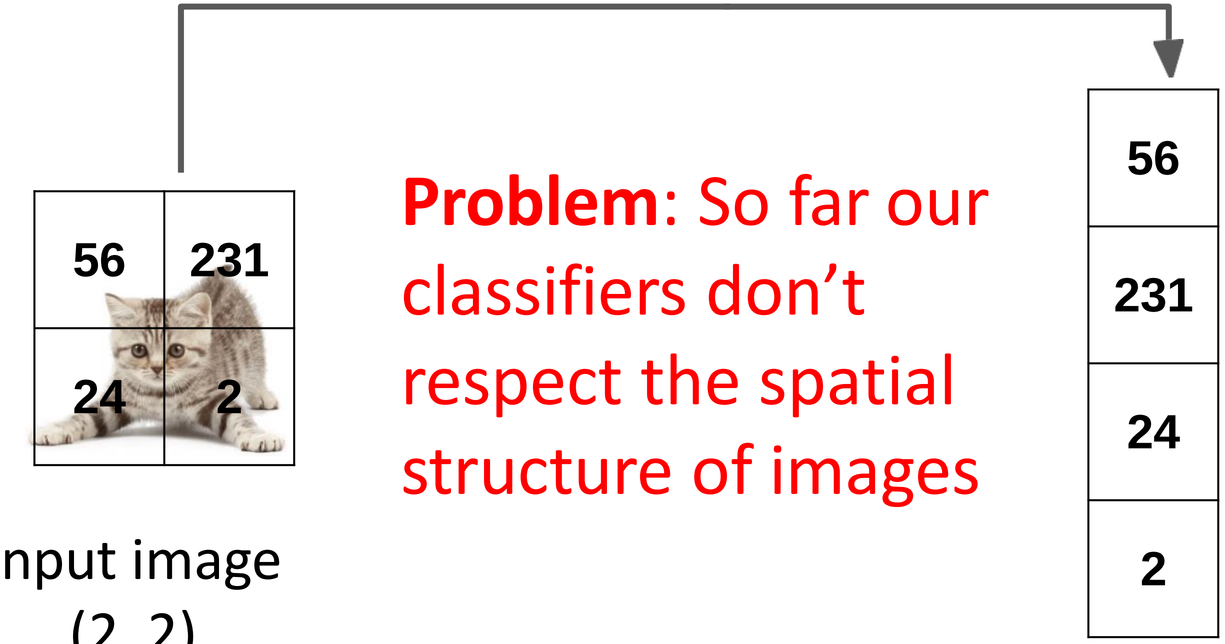


Input image
(2, 2)

(4,)

Problem: So far our classifiers don’t respect the spatial structure of images

Spatial Information

$$f = W_2 \max(0, W_1 x)$$

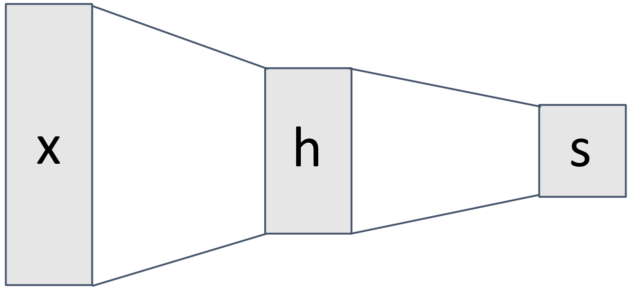
Flatten lattice into vector

Problem: So far our classifiers don’t respect the spatial structure of images

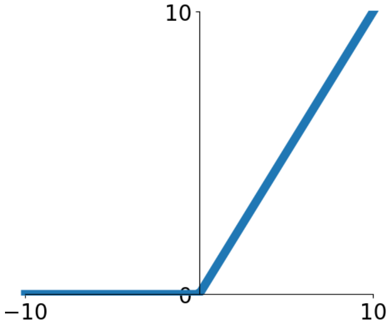
Solution: Define new computational operators

Components of a Fully-Connected Network

Fully-Connected Layers

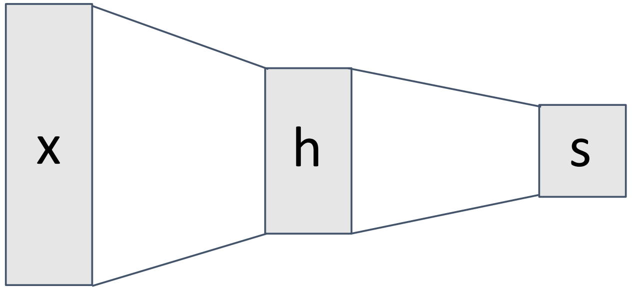


Activation Function



Components of a Convolutional Network

Fully-Connected Layers



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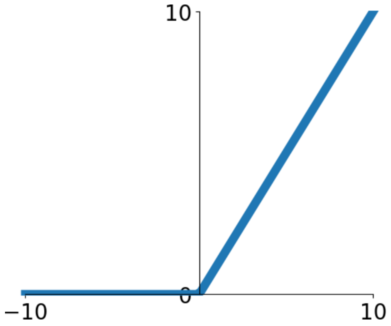
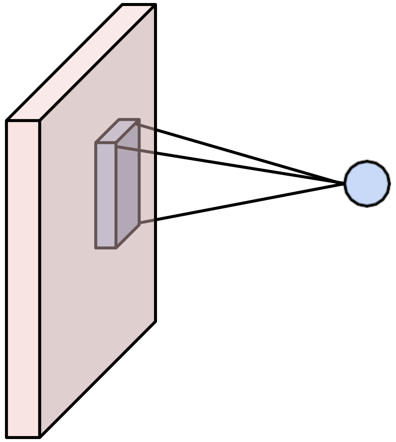
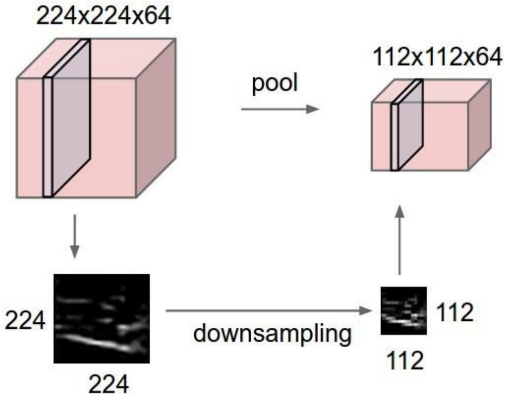


Convolution Layers



Pooling Layers

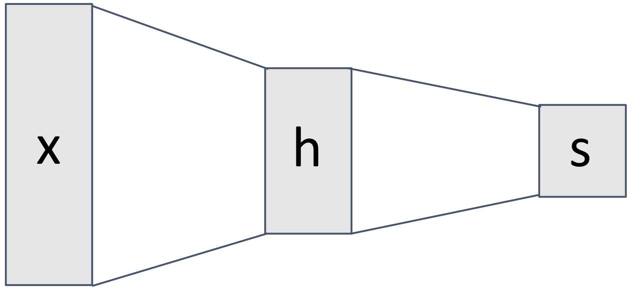


Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Components of a Convolutional Network

Fully-Connected Layers



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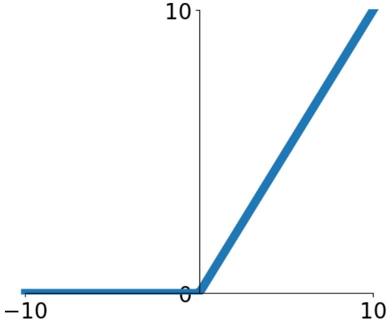
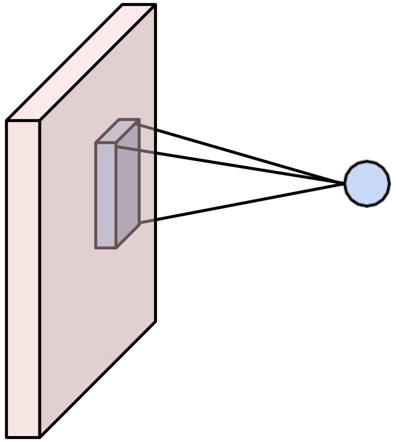
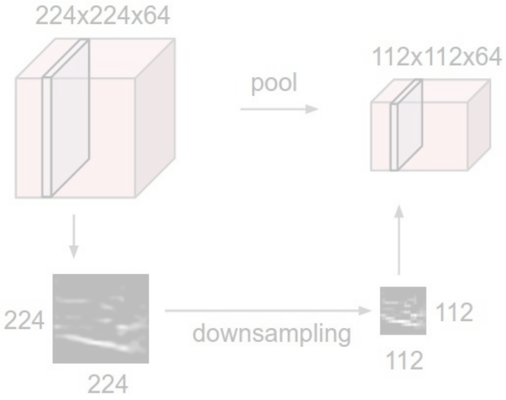


Convolution Layers



Pooling Layers



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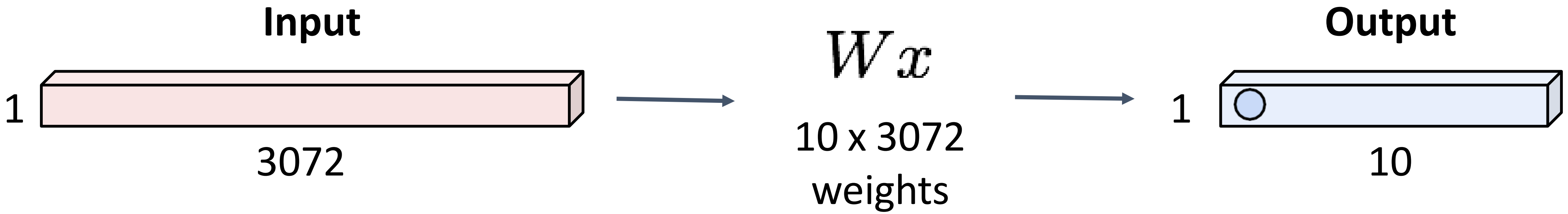
Fully-Connected Layer

32x32x3 image -> flatten to 3072 x 1



Fully-Connected Layer

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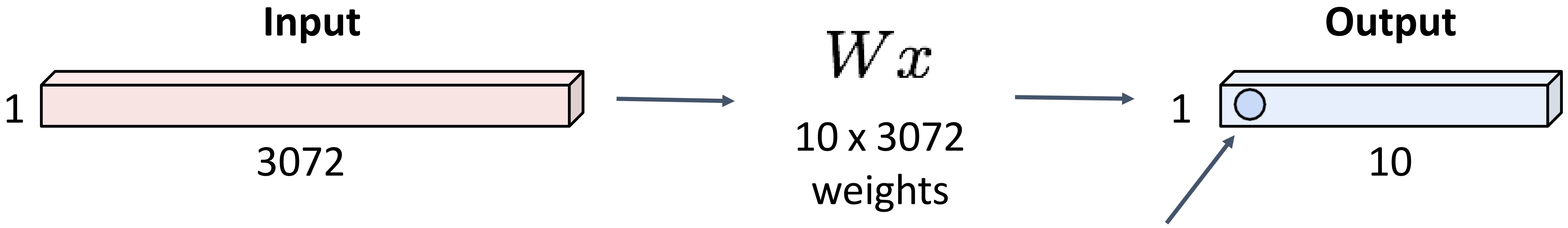
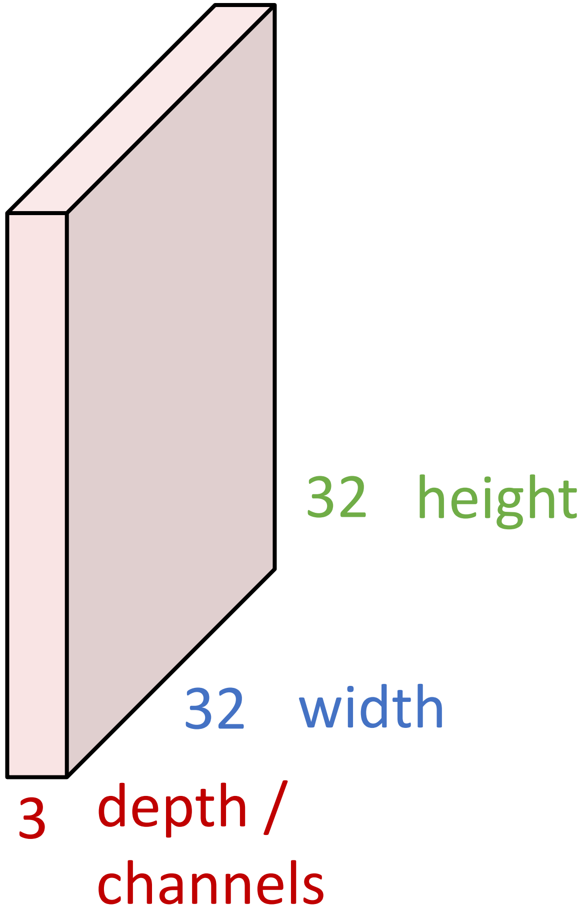


1 number:
the result of taking a dot product between a row of W and the input (a 3072-dimensional dot product)

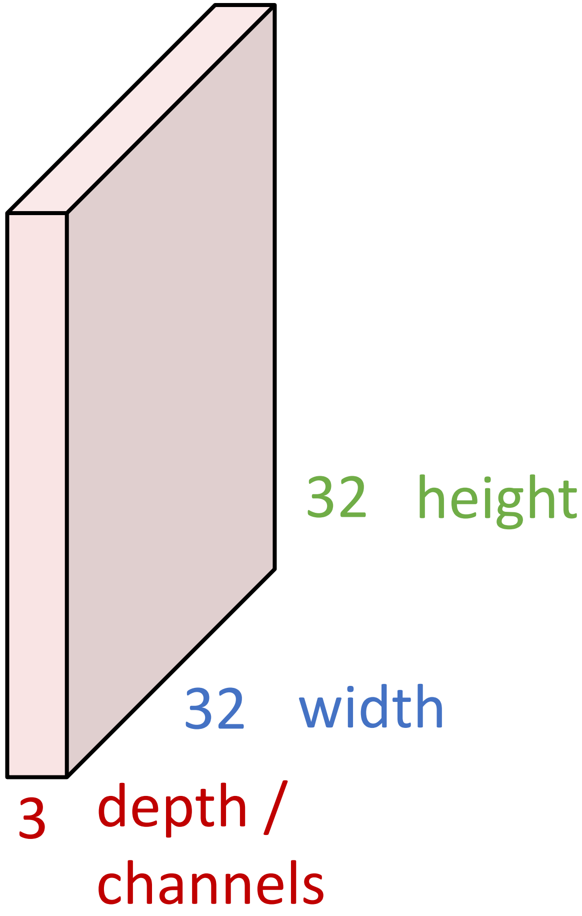
Convolution Layer

3x32x32 image: preserve structure

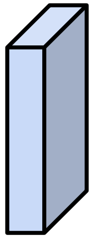


Convolution Layer

3x32x32 image

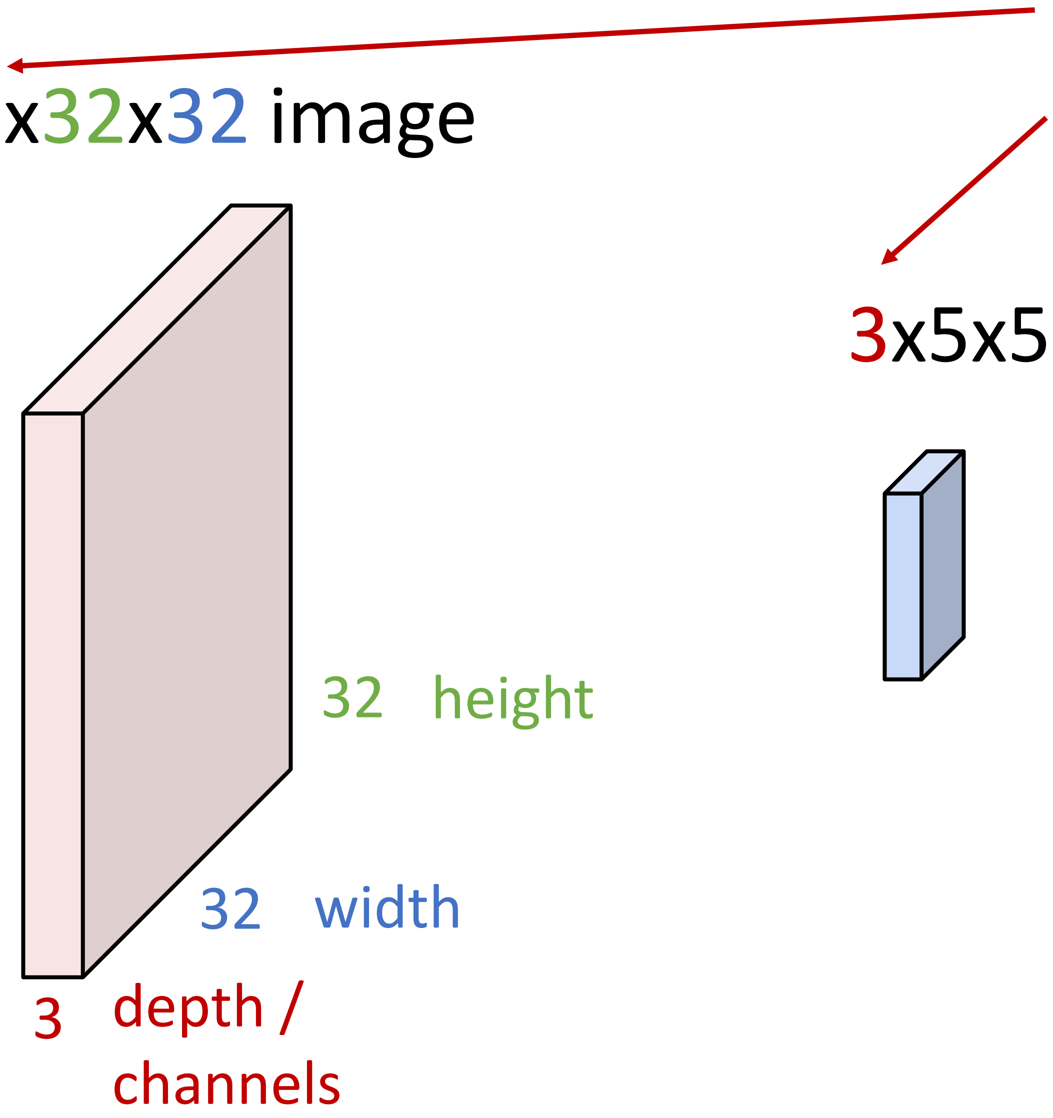


3x5x5 filter



Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

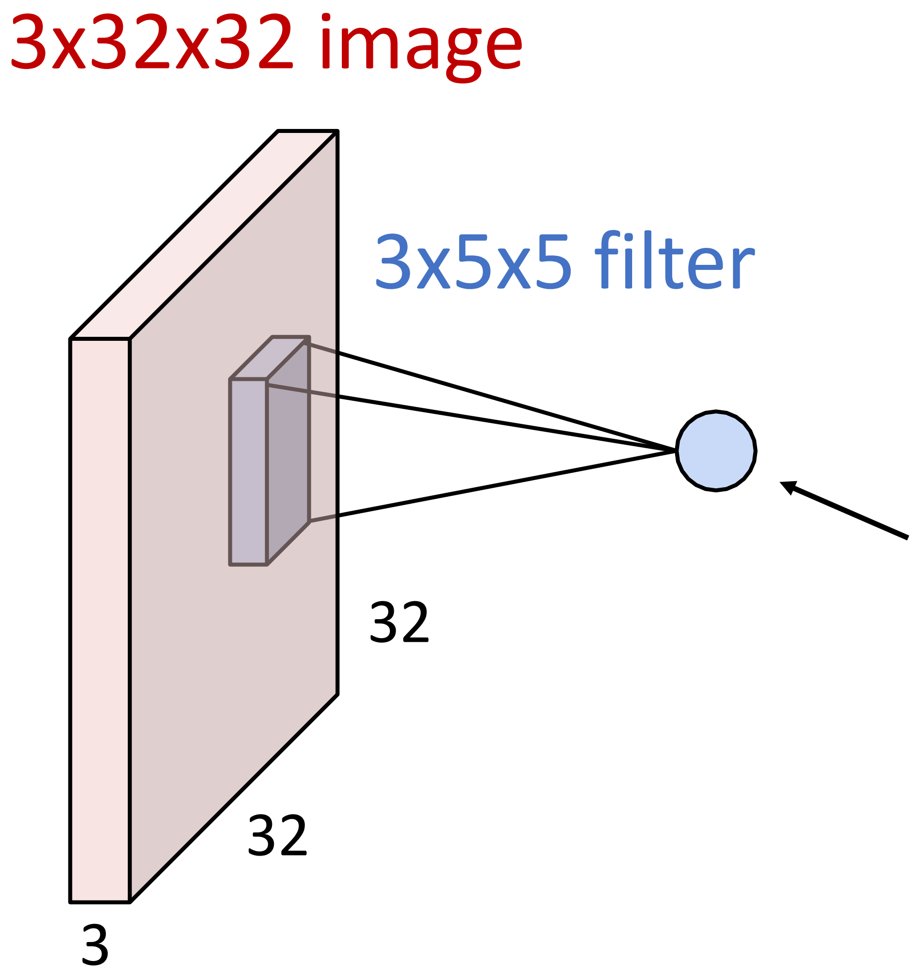
Convolution Layer



Filters always extend the full depth of the input volume

Convolve the filter with the image i.e. “slide over the image spatially, computing dot products”

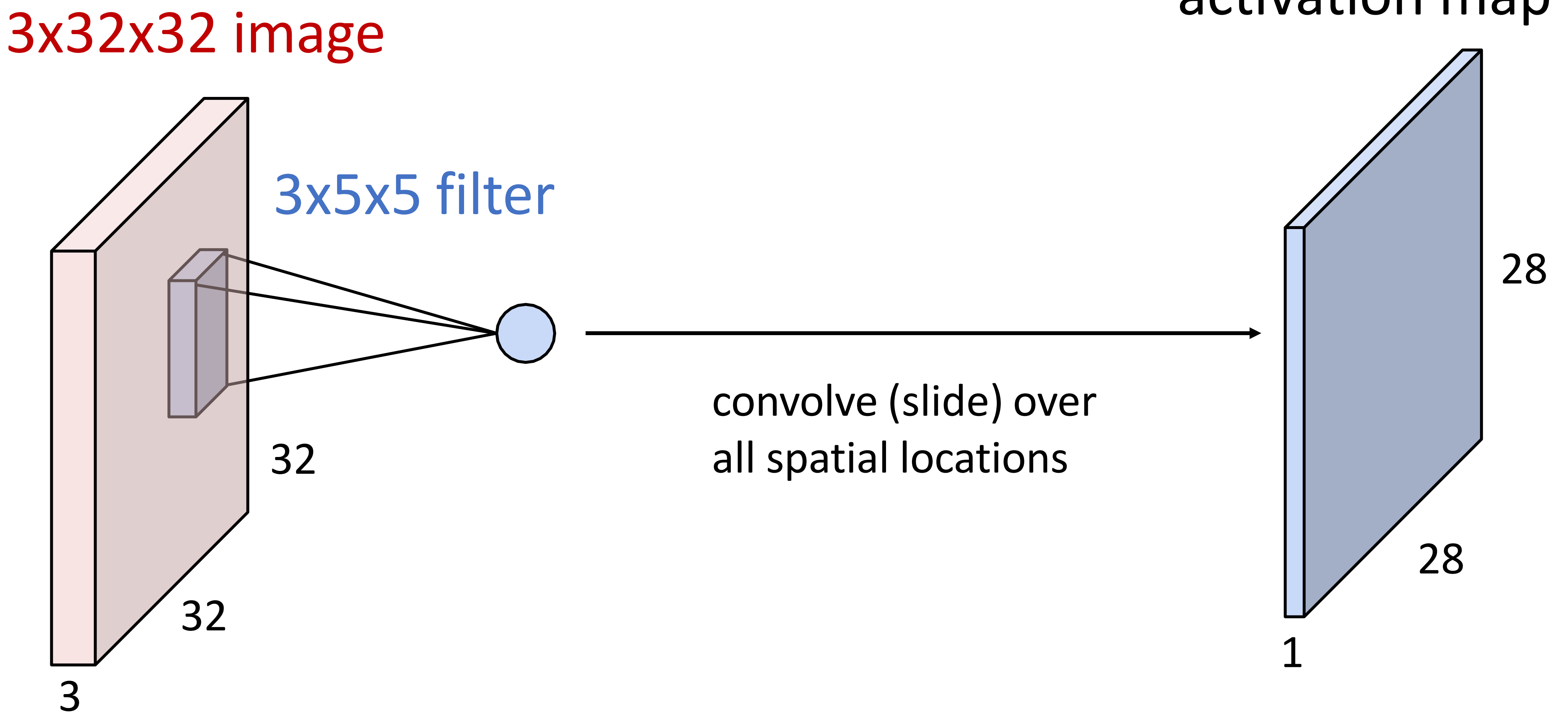
Convolution Layer



1 number:
the result of taking a dot product between the filter and a small 3x5x5 chunk of the image
(i.e. 3*5*5 = 75-dimensional dot product + bias)

$$w^T x + b$$

Convolution Layer



3x32x32 image

3x5x5 filter

32

32

3

convolve (slide) over
all spatial locations

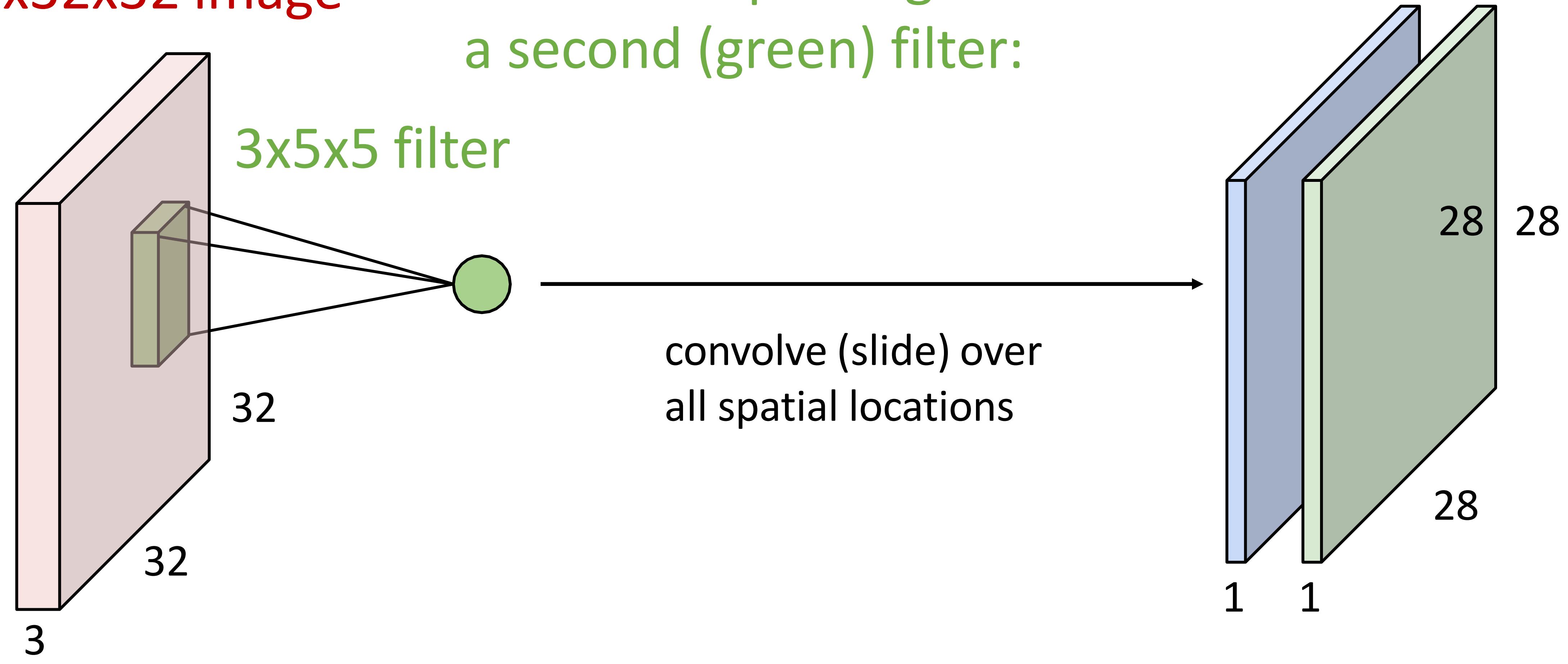
1x28x28
activation map

28

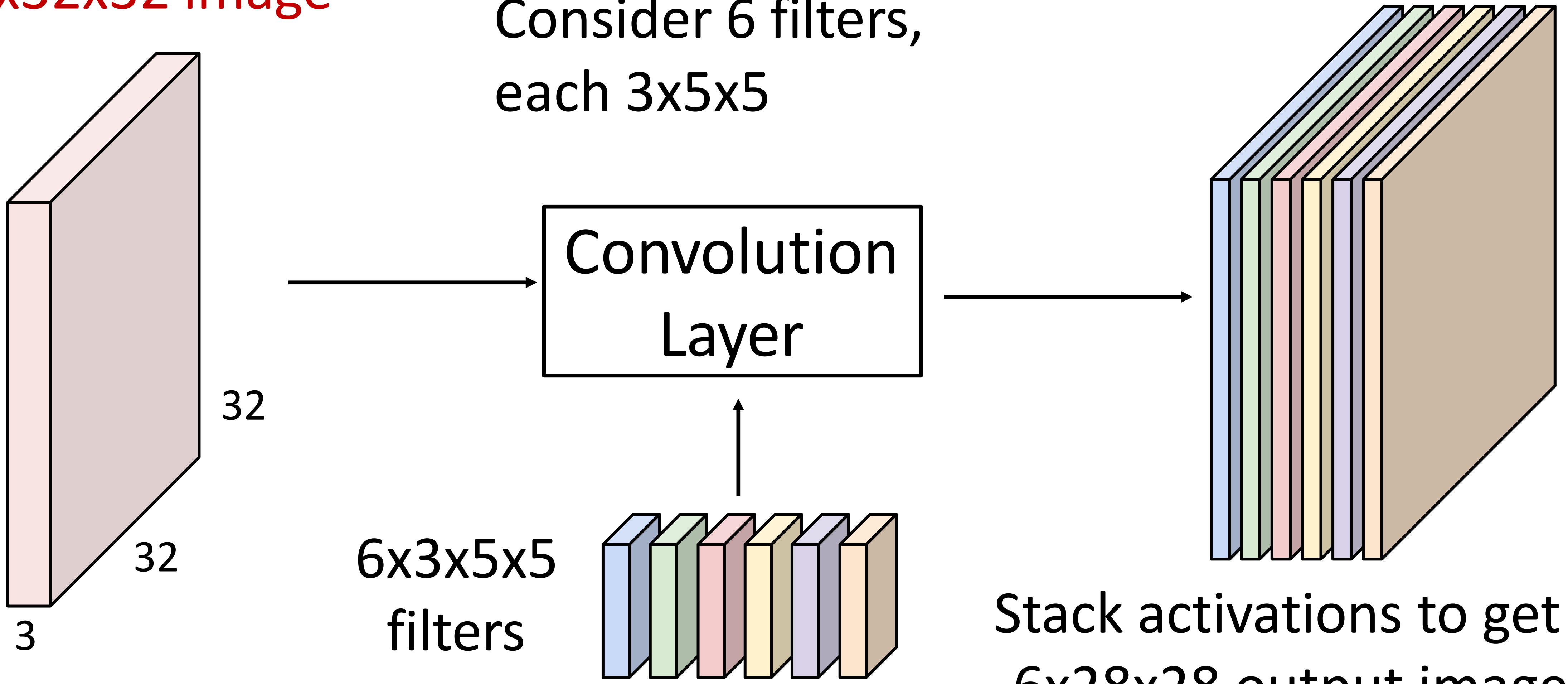
28

1

Convolution Layer



Convolution Layer



3x32x32 image

Consider 6 filters,
each 3x5x5

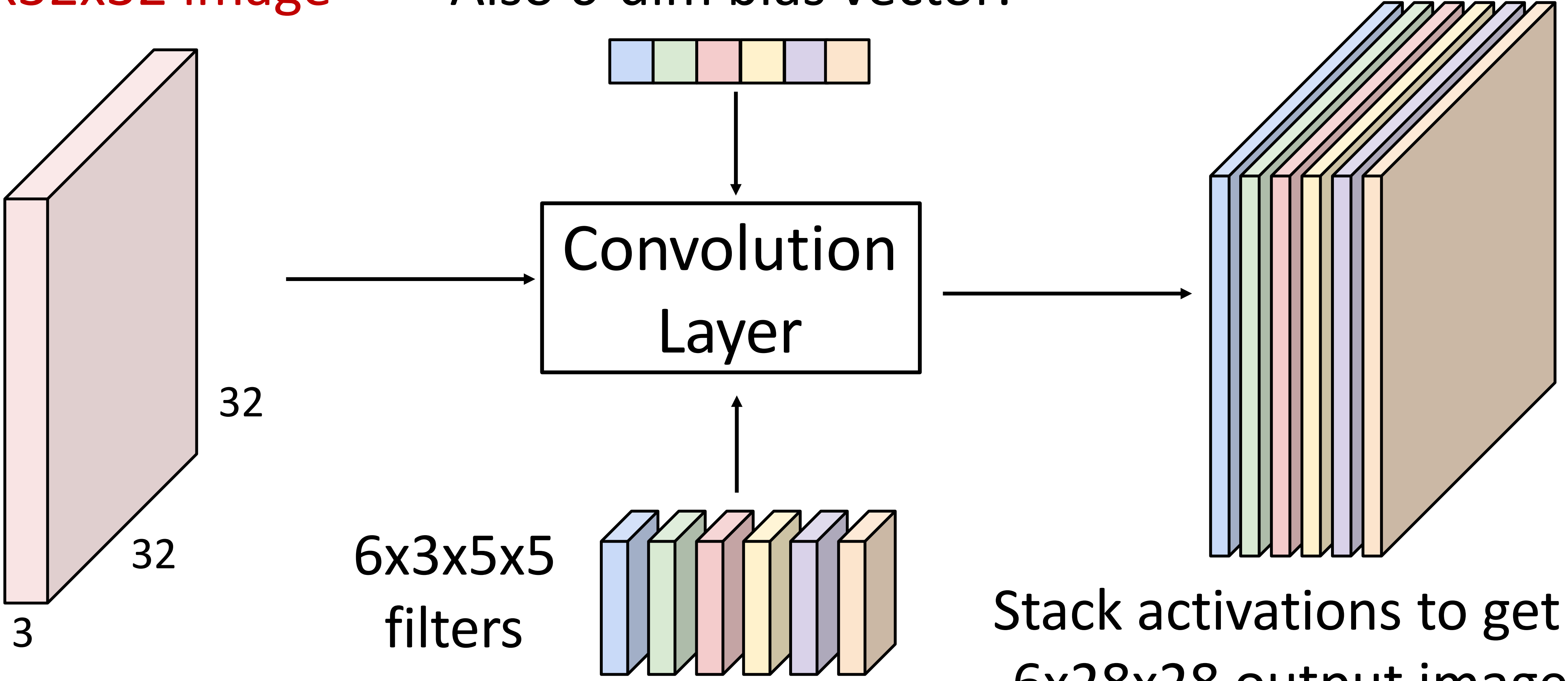
Convolution Layer

6x3x5x5
filters

6 activation maps,
each 1x28x28

Stack activations to get a
6x28x28 output image!

Convolution Layer



3x32x32 image

Also 6-dim bias vector:

Convolution Layer

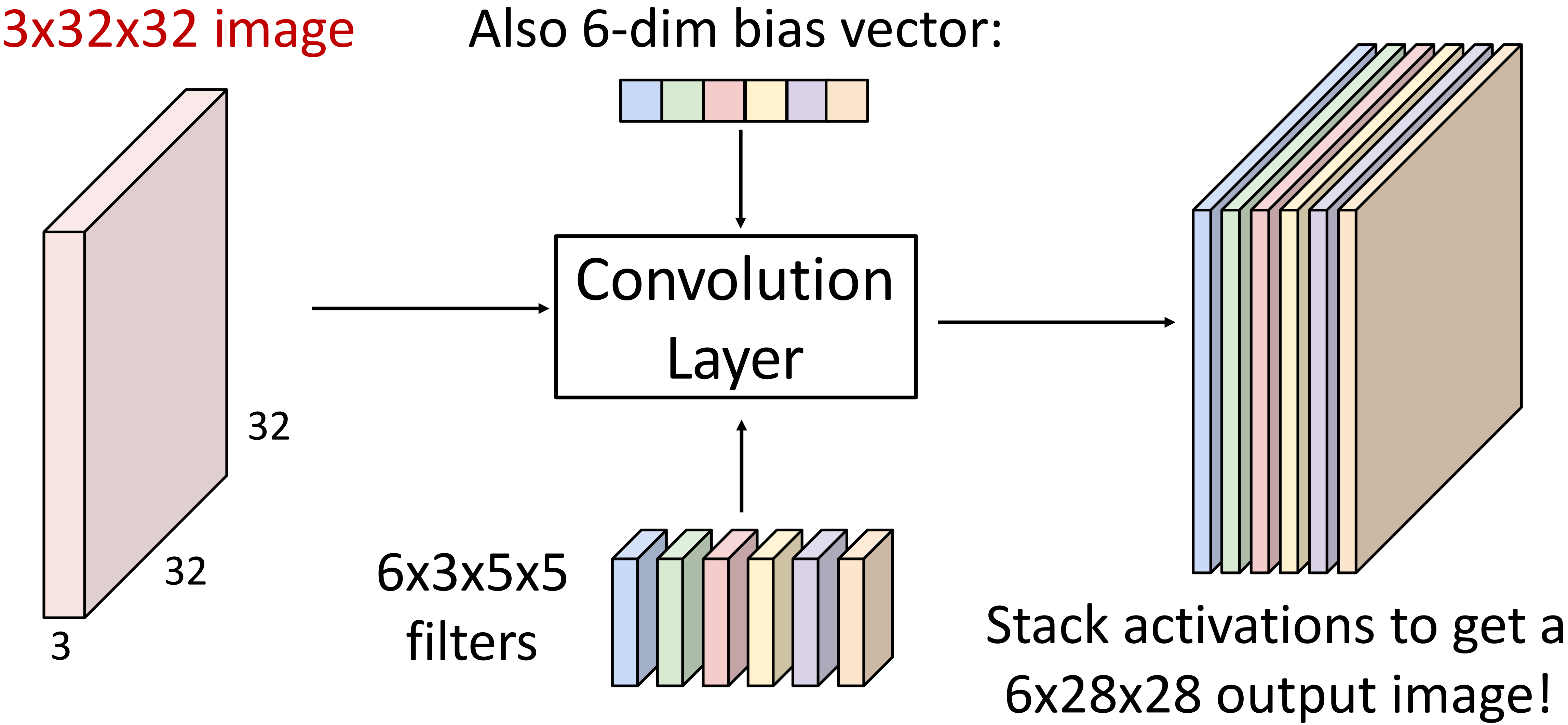
6x3x5x5 filters

6 activation maps, each 1x28x28

Stack activations to get a 6x28x28 output image!

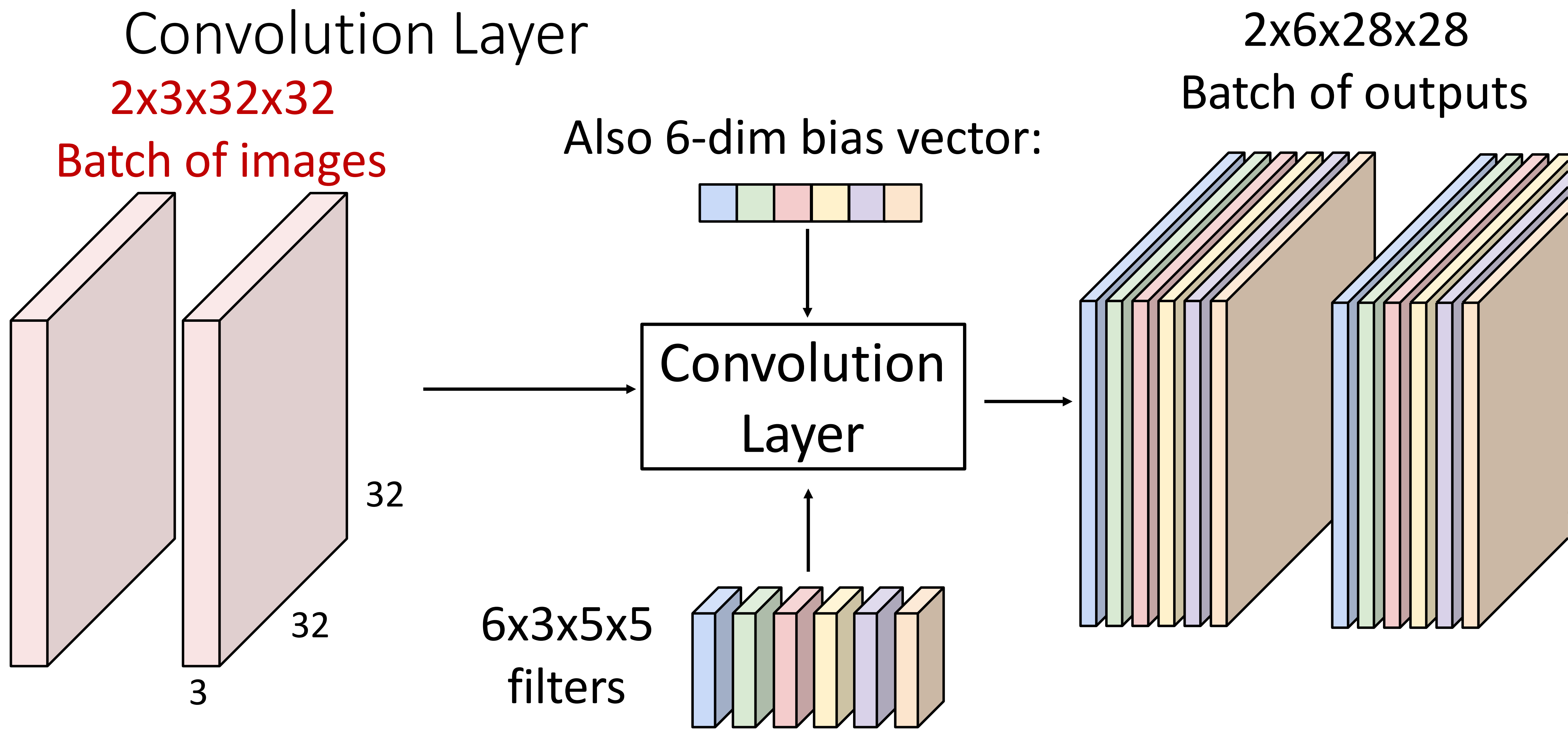
Convolution Layer

28x28 grid, at each point a 6-dim vector

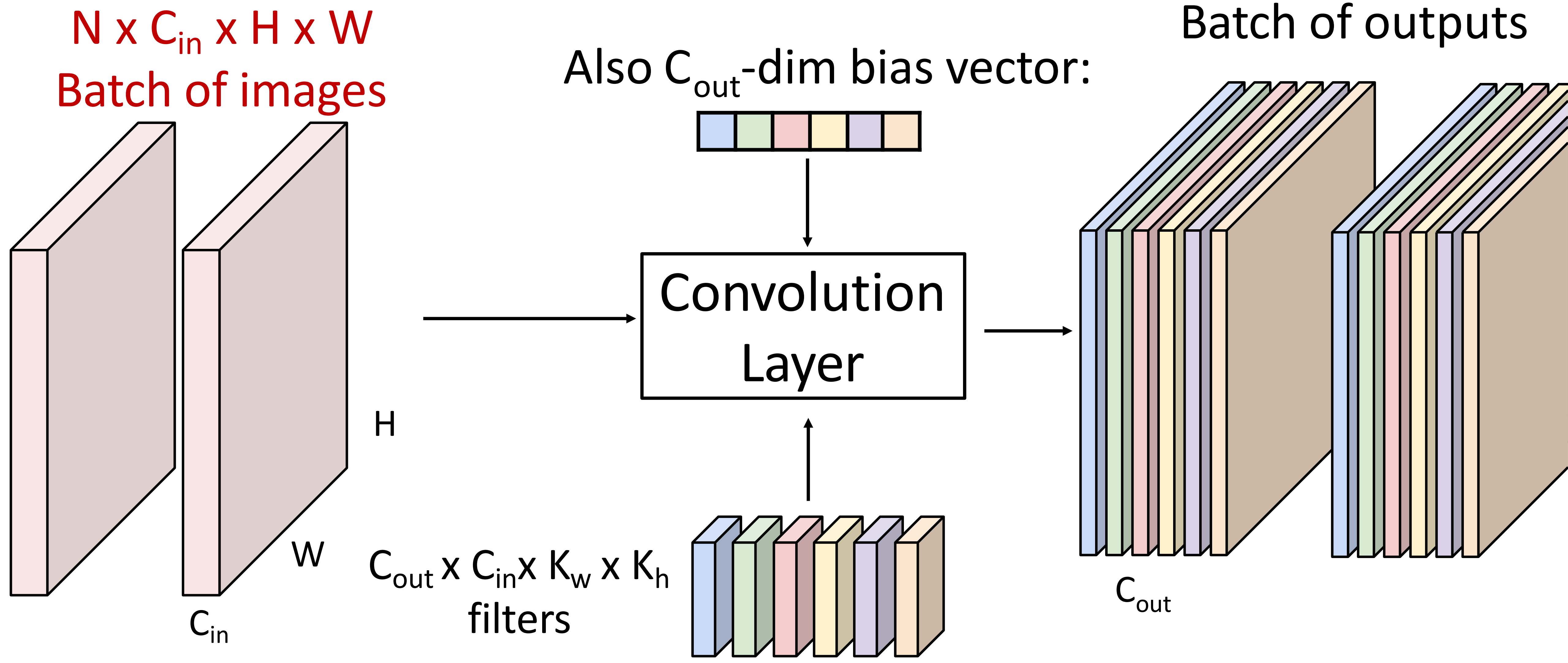


Stack activations to get a 6x28x28 output image!

Convolution Layer



Convolution Layer



$N \times C_{in} \times H \times W$

Batch of images

Also C_{out}-dim bias vector:

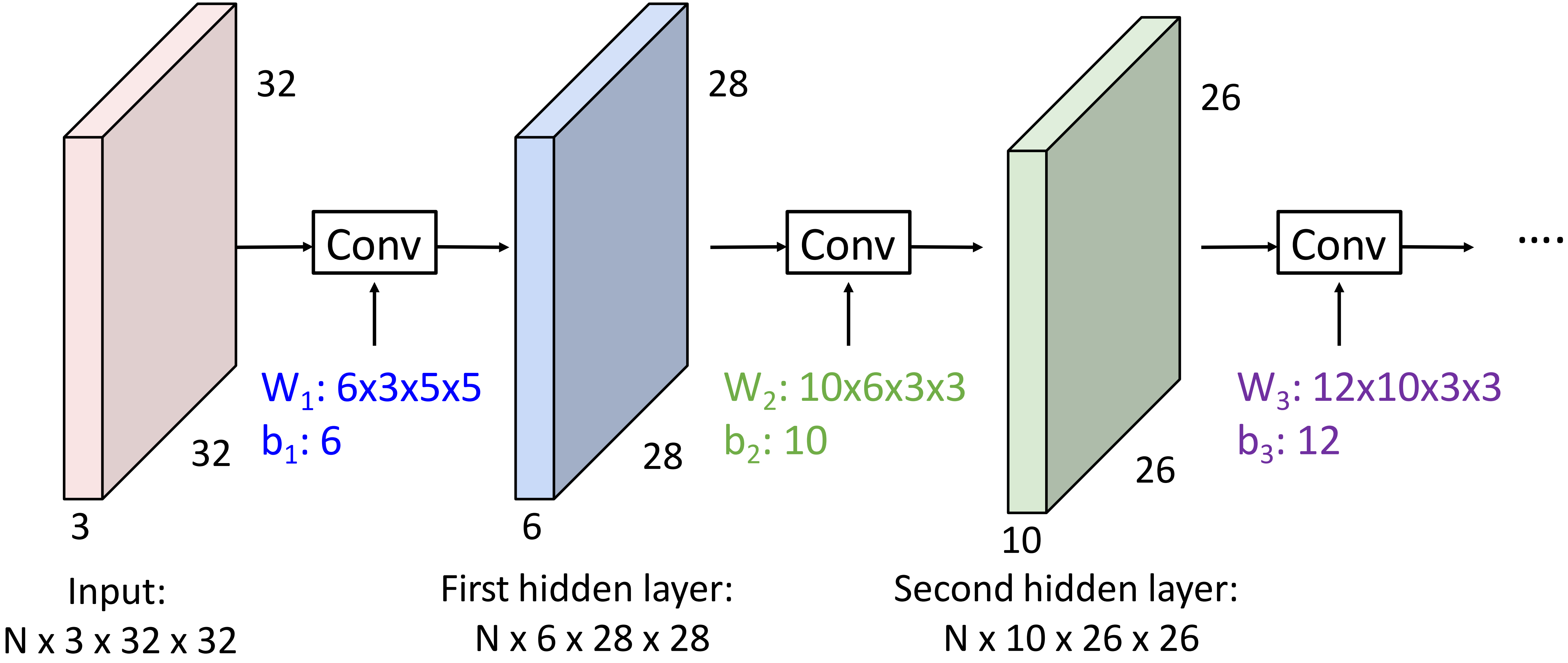
Convolution Layer

$C_{out} \times C_{in} \times K_w \times K_h$ filters

$N \times C_{out} \times H' \times W'$

Batch of outputs

Stacking Convolutions



W_1: 6x3x5x5
b_1: 6

W_2: 10x6x3x3
b_2: 10

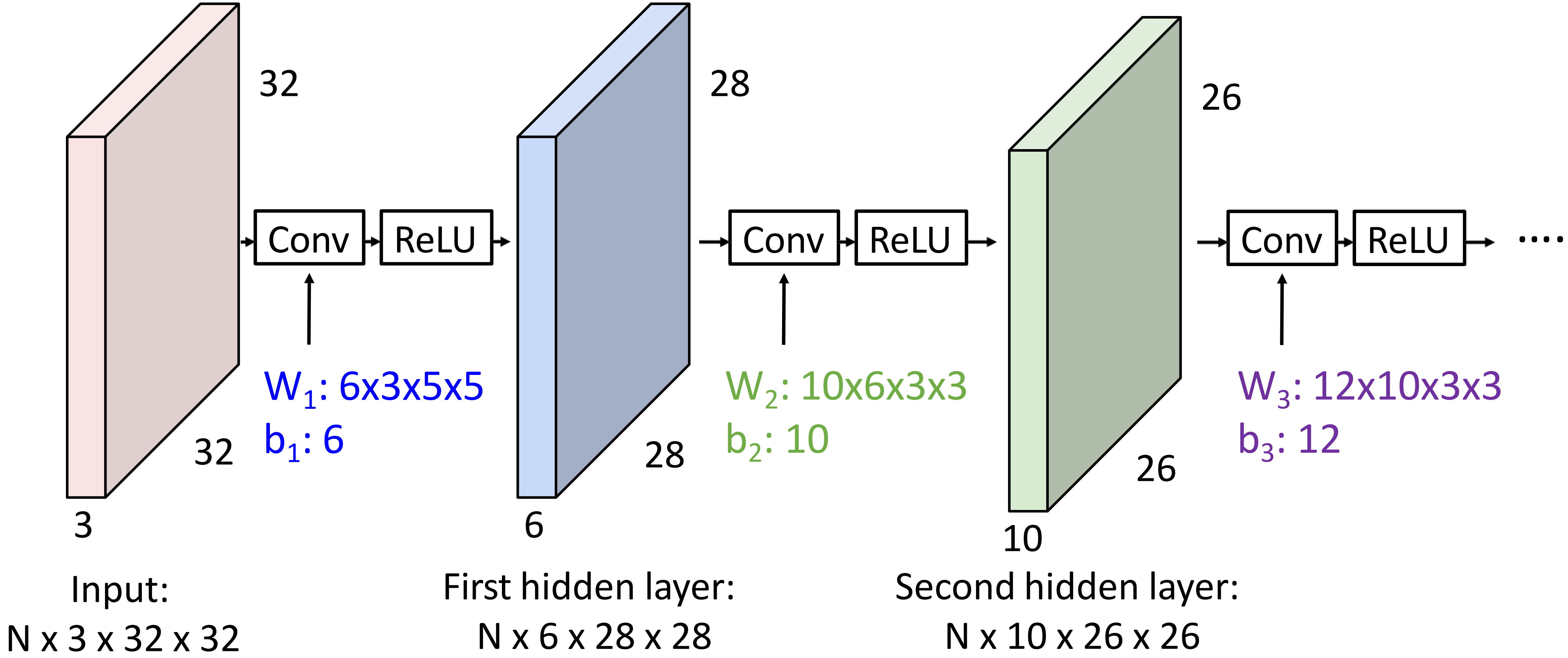
W_3: 12x10x3x3
b_3: 12

Input:
N x 3 x 32 x 32

First hidden layer:
N x 6 x 28 x 28

Second hidden layer:
N x 10 x 26 x 26

Stacking Convolutions: Add Non-linearity



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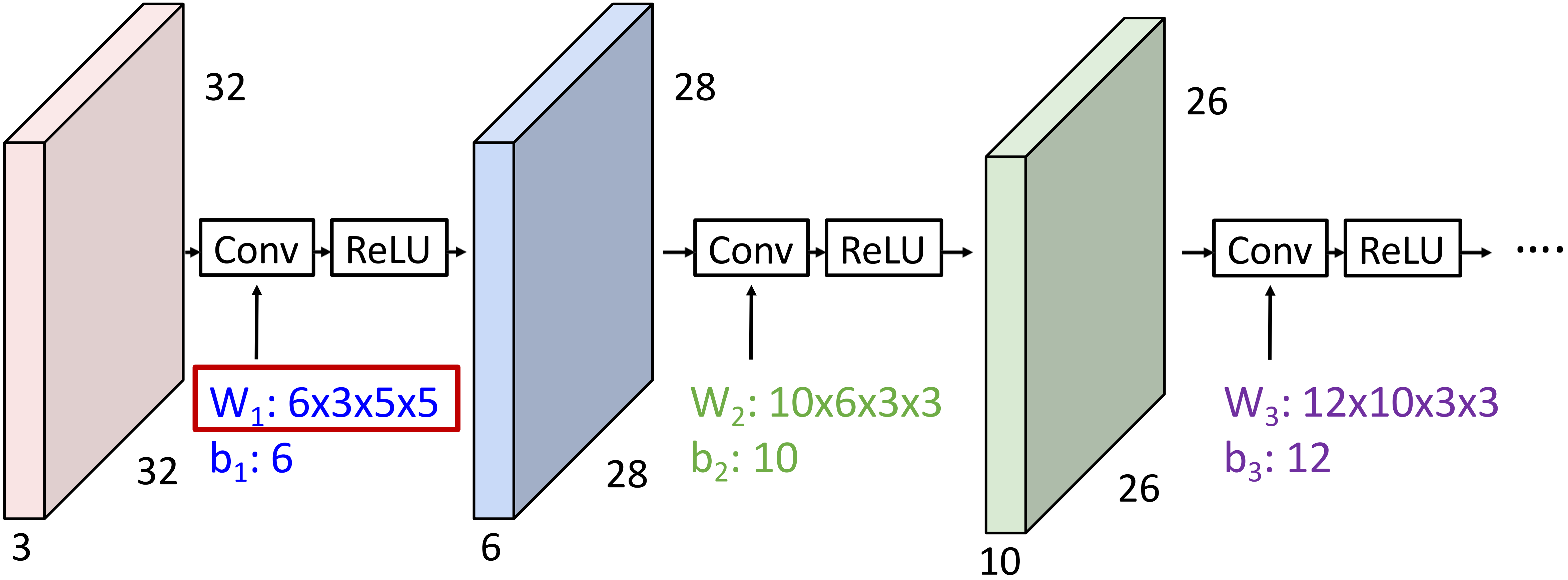
What do convolutional filters learn?



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Input:
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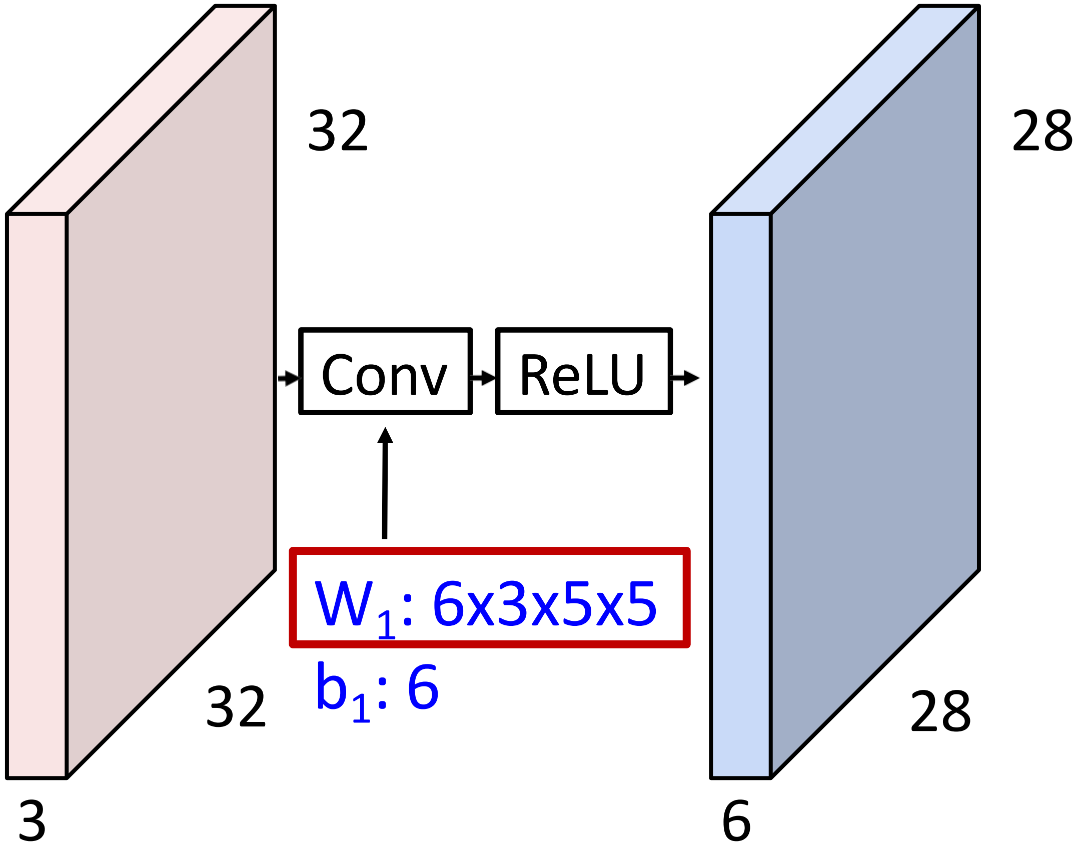


32

32

3

Conv

ReLU

W_1: 6x3x5x5

b_1: 6

28

28

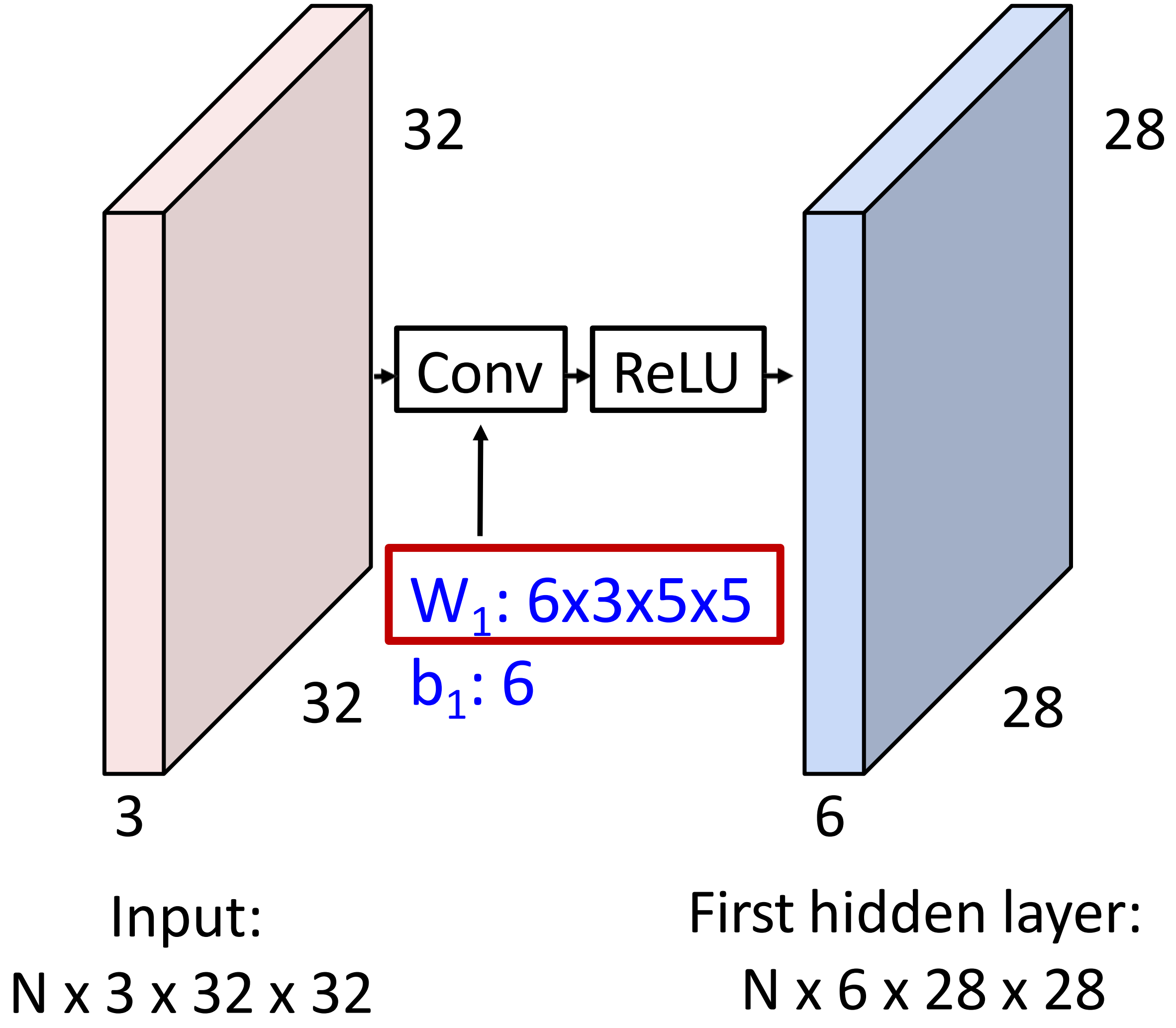
6

Input:
N x 3 x 32 x 32

First hidden layer:
N x 6 x 28 x 28

Linear classifier: One template per class

What do convolutional filters learn?

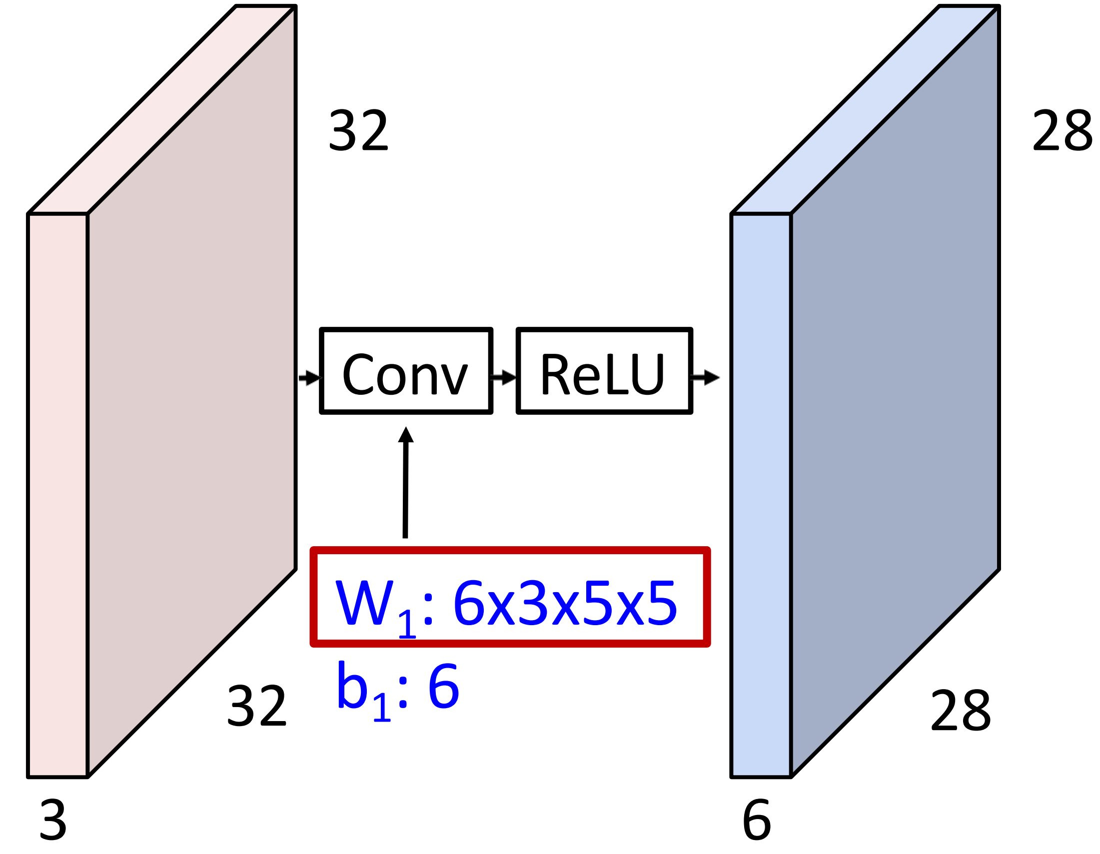


Input:
N x 3 x 32 x 32

First hidden layer:
N x 6 x 28 x 28

MLP: Bank of whole-image templates

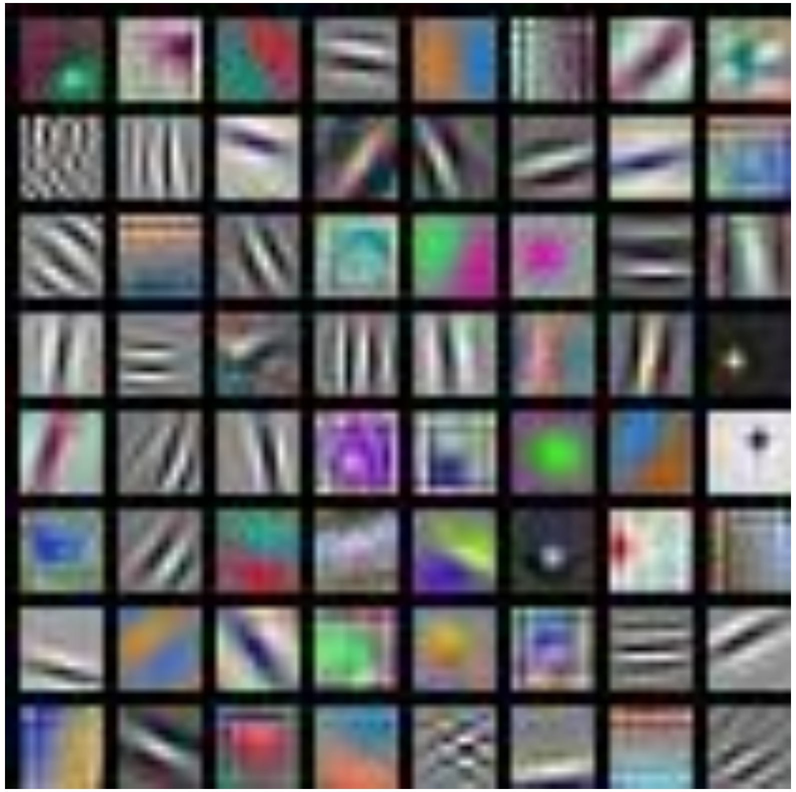
What do convolutional filters learn?



Input:
N x 3 x 32 x 32

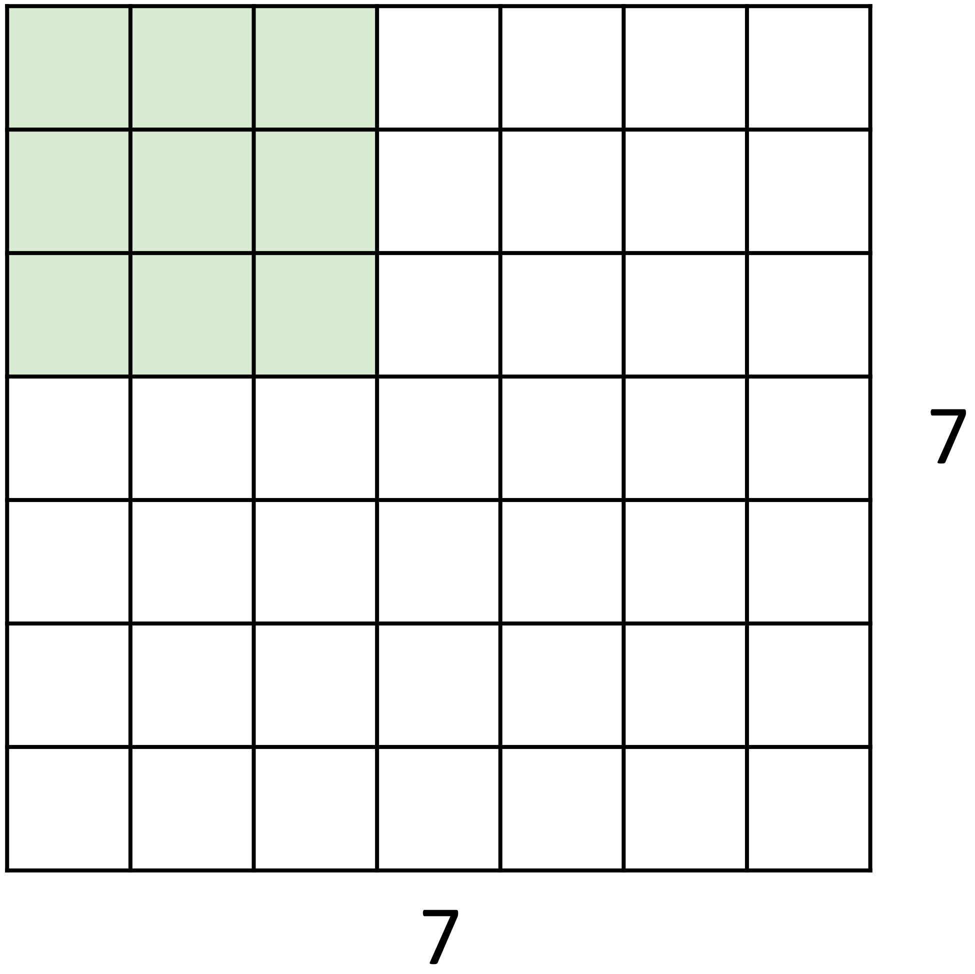
First hidden layer:
N x 6 x 28 x 28

First-layer conv filters: local image templates
(Often learns oriented edges, opposing colors)



AlexNet: 64 filters, each 3x11x11

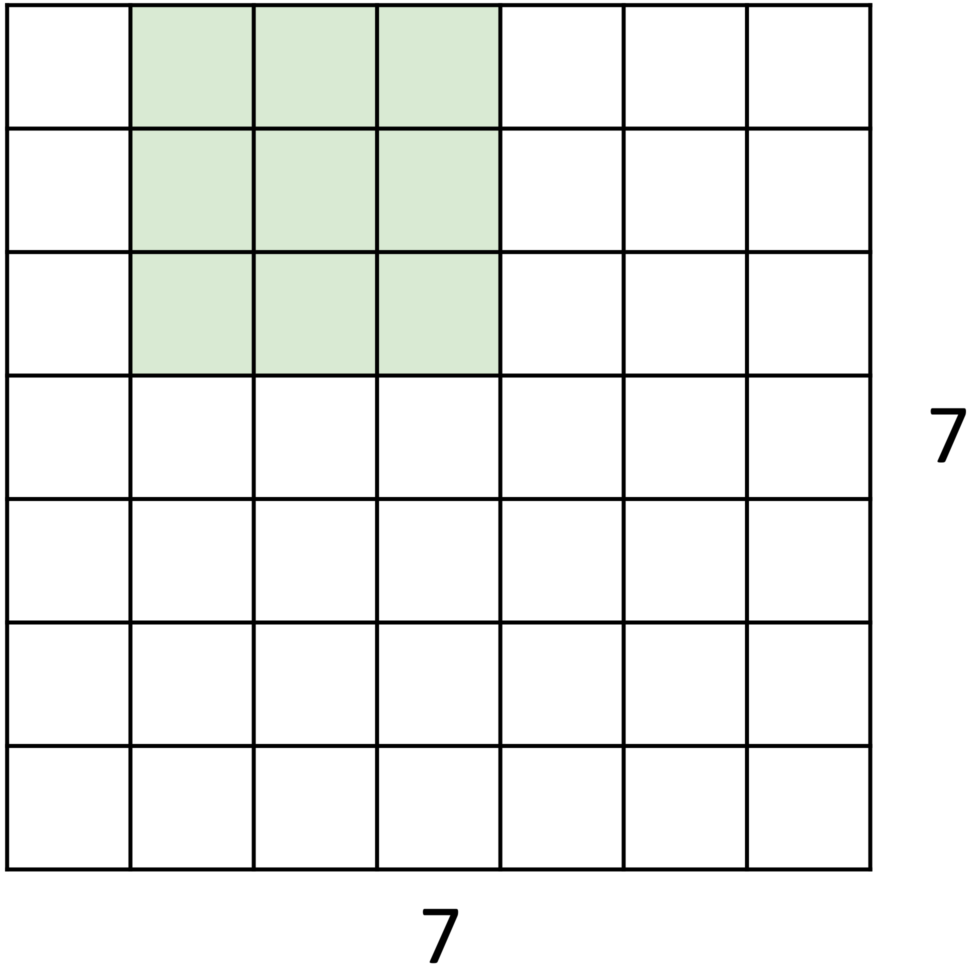
A closer look at spatial dimensions



Input: 7x7

Filter: 3x3

A closer look at spatial dimensions



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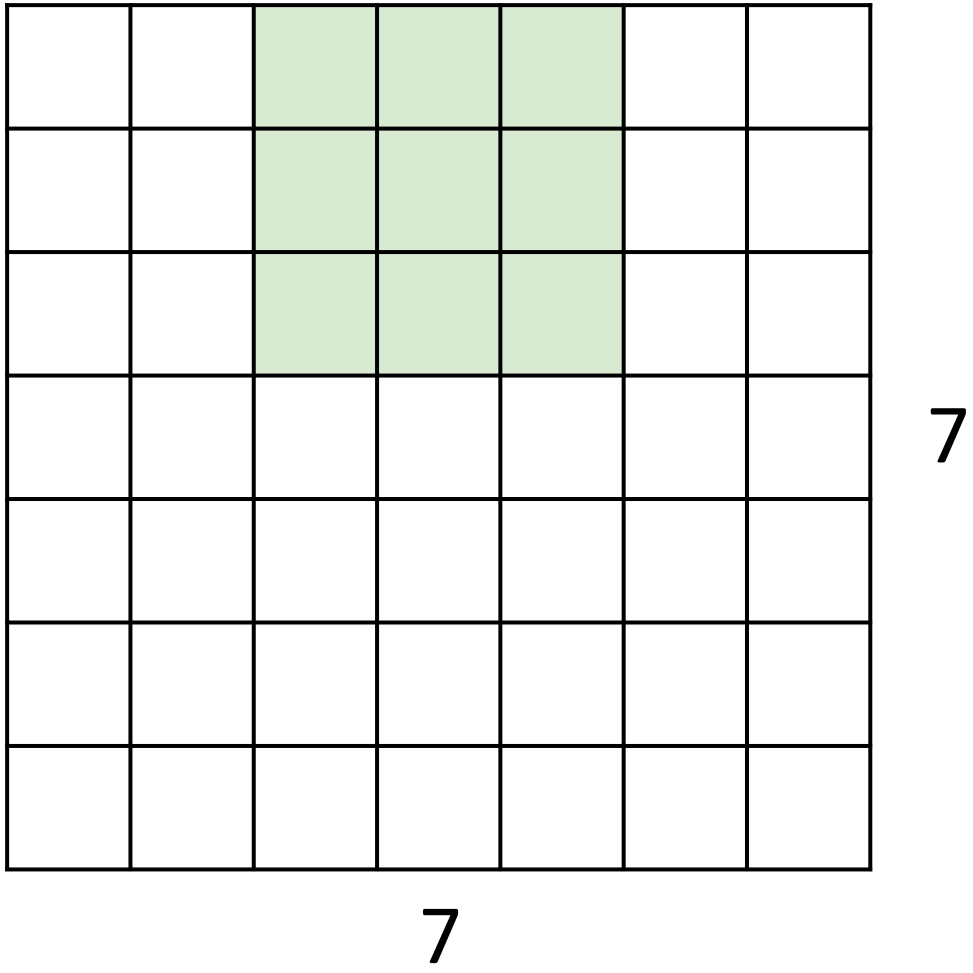

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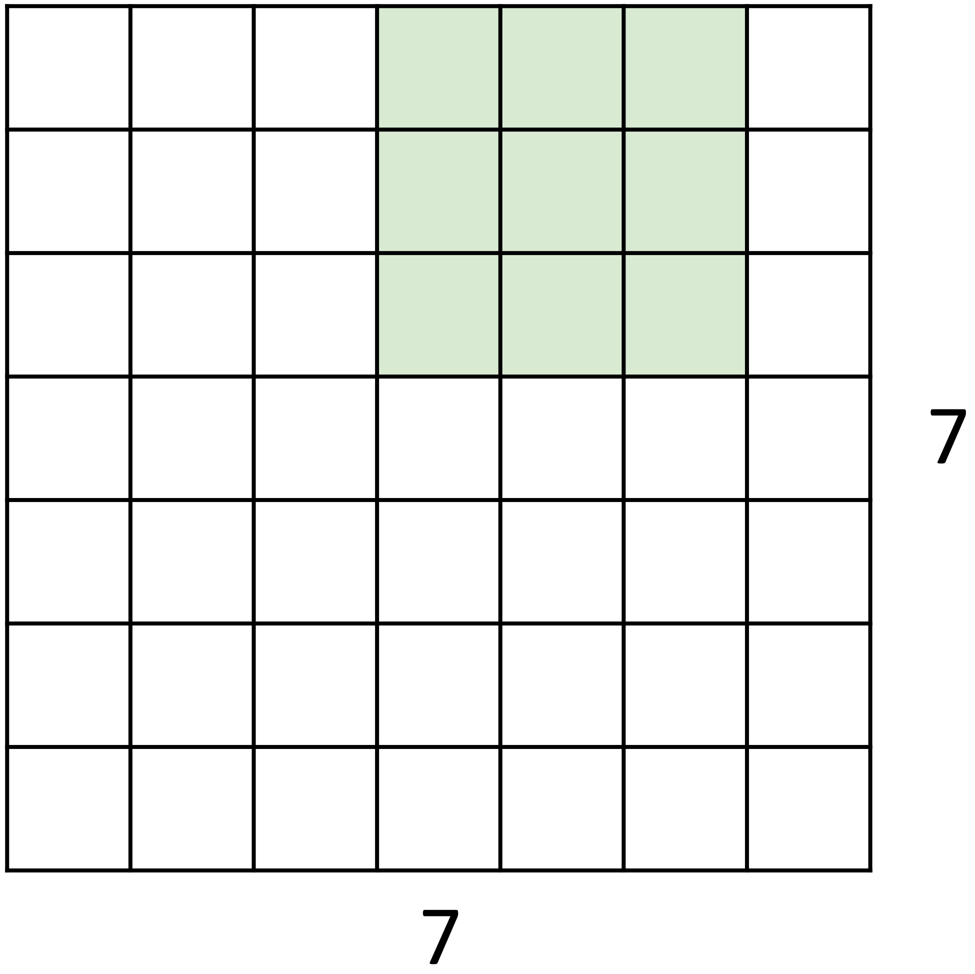

Input: 7x7

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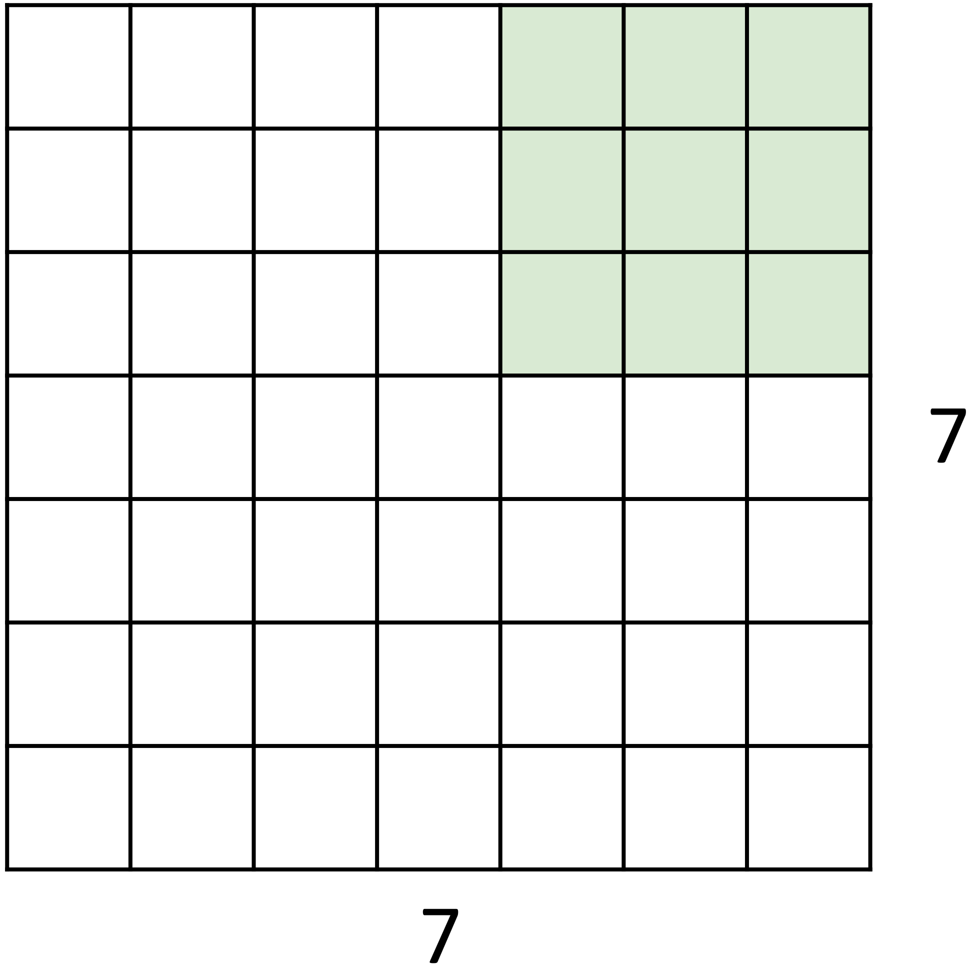
A closer look at spatial dimensions

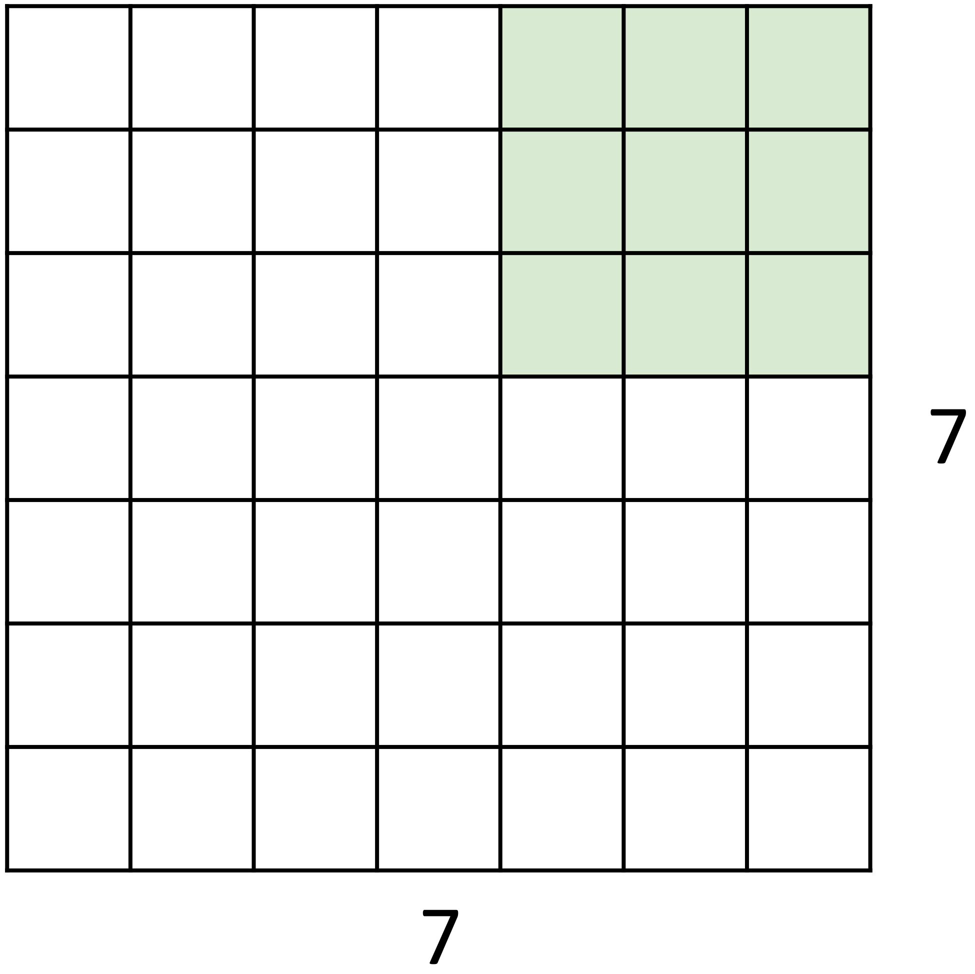


Input: 7x7

Filter: 3x3

Output: 5x5

A closer look at spatial dimensions



Input: 7x7

Filter: 3x3

Output: 5x5

In general:

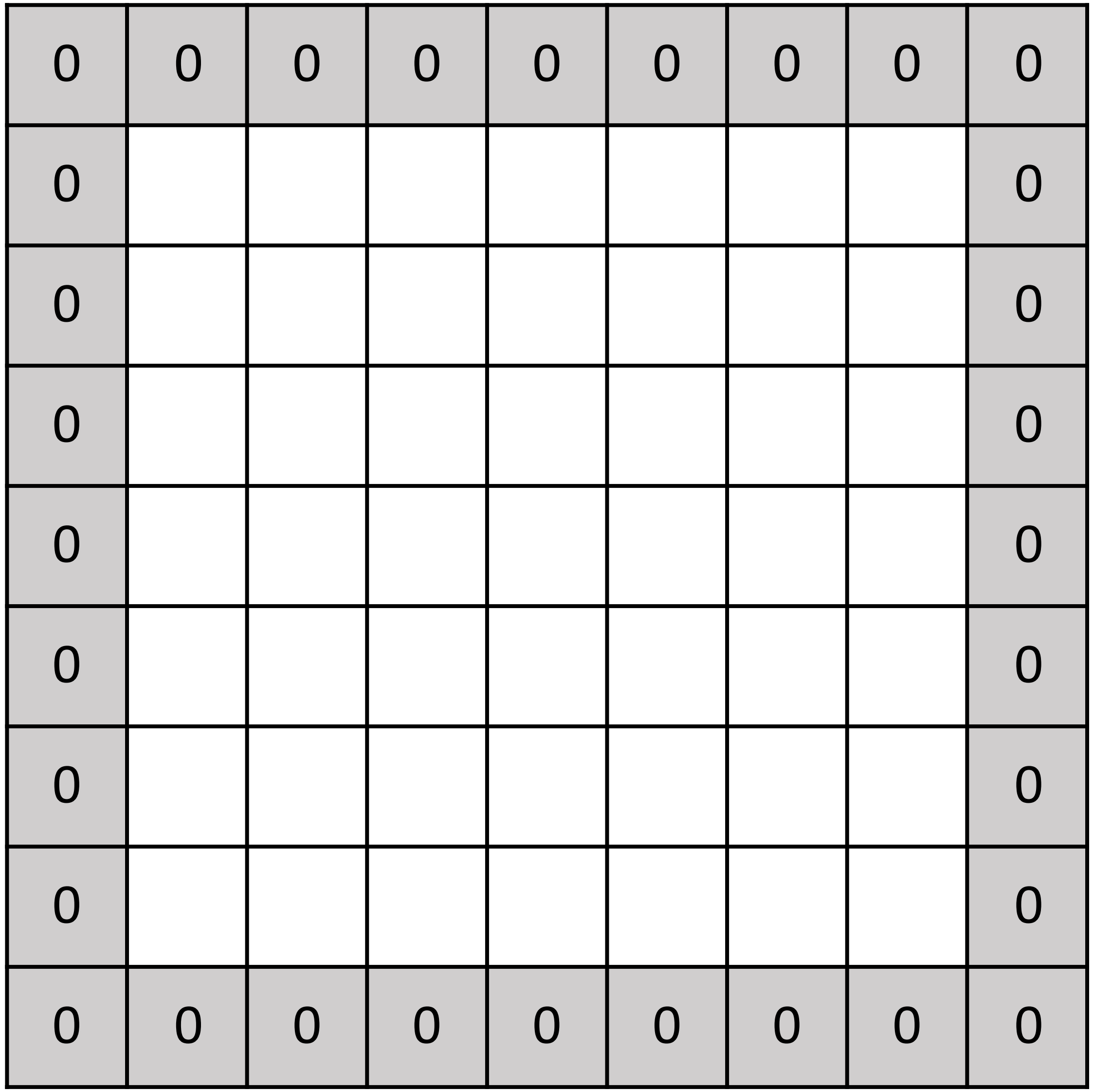
Input: W

Filter: K

Output: W – K + 1

Problem: Feature maps “shrink” with each layer!

A closer look at spatial dimensions



0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

Input: 7x7

Filter: 3x3

Output: 5x5

In general:

Input: W

Filter: K

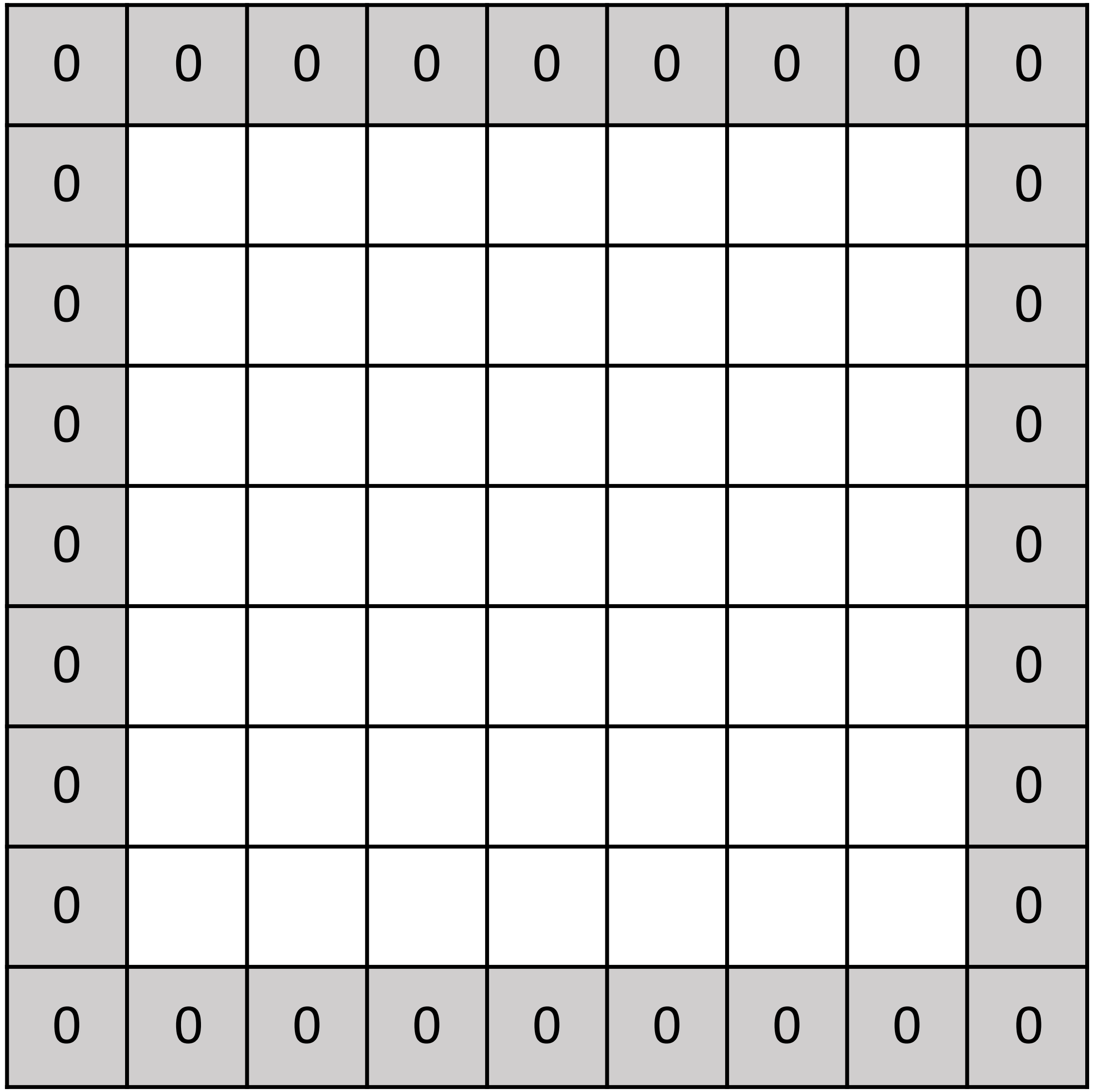
Output: W – K + 1

Problem: Feature maps “shrink” with each layer!

Solution: **padding**

Add zeros around the input

A closer look at spatial dimensions



0	0	0	0	0	0	0	0	0
0								0
0								0
0								0
0								0
0								0
0								0
0								0
0	0	0	0	0	0	0	0	0

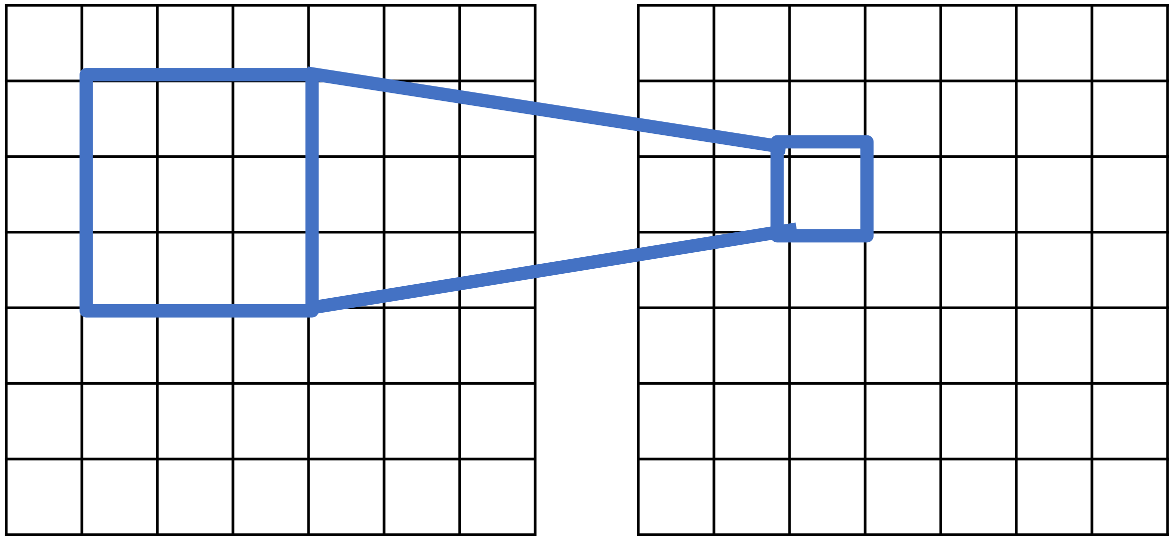
Input: 7x7
Filter: 3x3
Output: 5x5

In general:
Input: W
Filter: K
Padding: P
Output: W – K + 1 + 2P

Common:
Set P = (K – 1) / 2 to make output have same size as input

Receptive Fields

For convolution with kernel size K, each element in the output depends on a K x K **receptive field** in the input

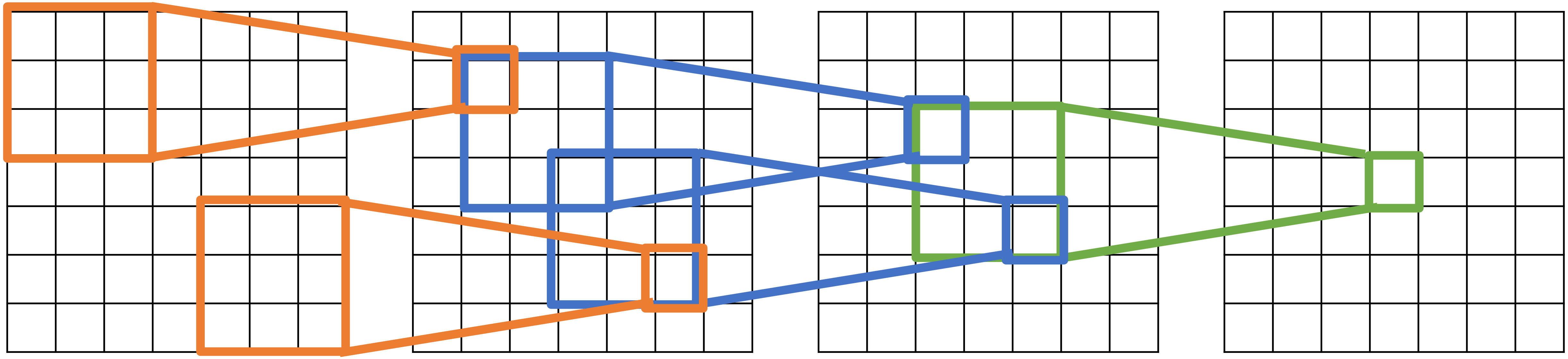


Input

Output

Receptive Fields

Each successive convolution adds K – 1 to the receptive field size
With L layers the receptive field size is 1 + L * (K – 1)

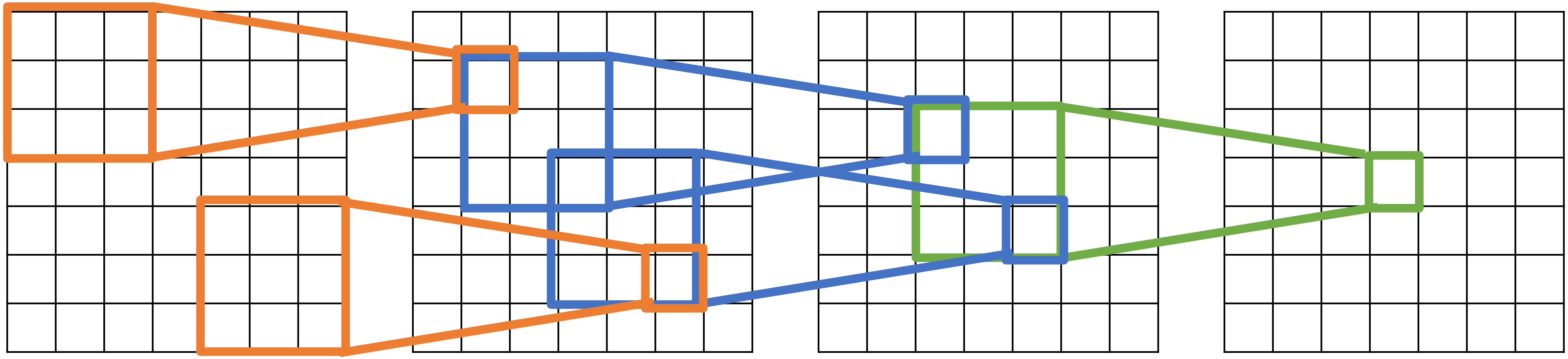


Input

Output

Receptive Fields

Each successive convolution adds K – 1 to the receptive field size
With L layers the receptive field size is 1 + L * (K – 1)



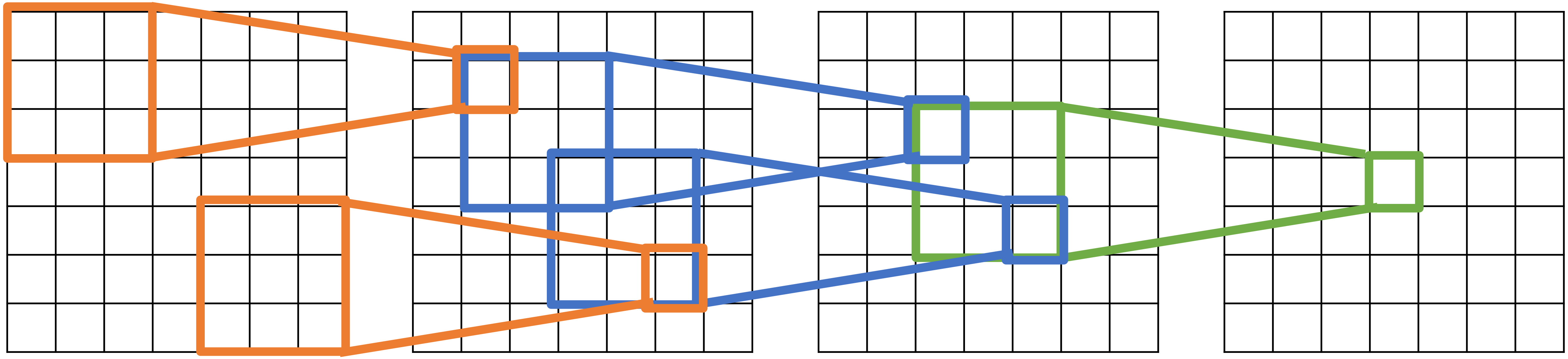
Input

Output

Problem: For large images we need many layers for each output to “see” the whole image image

Receptive Fields

Each successive convolution adds K – 1 to the receptive field size
With L layers the receptive field size is 1 + L * (K – 1)



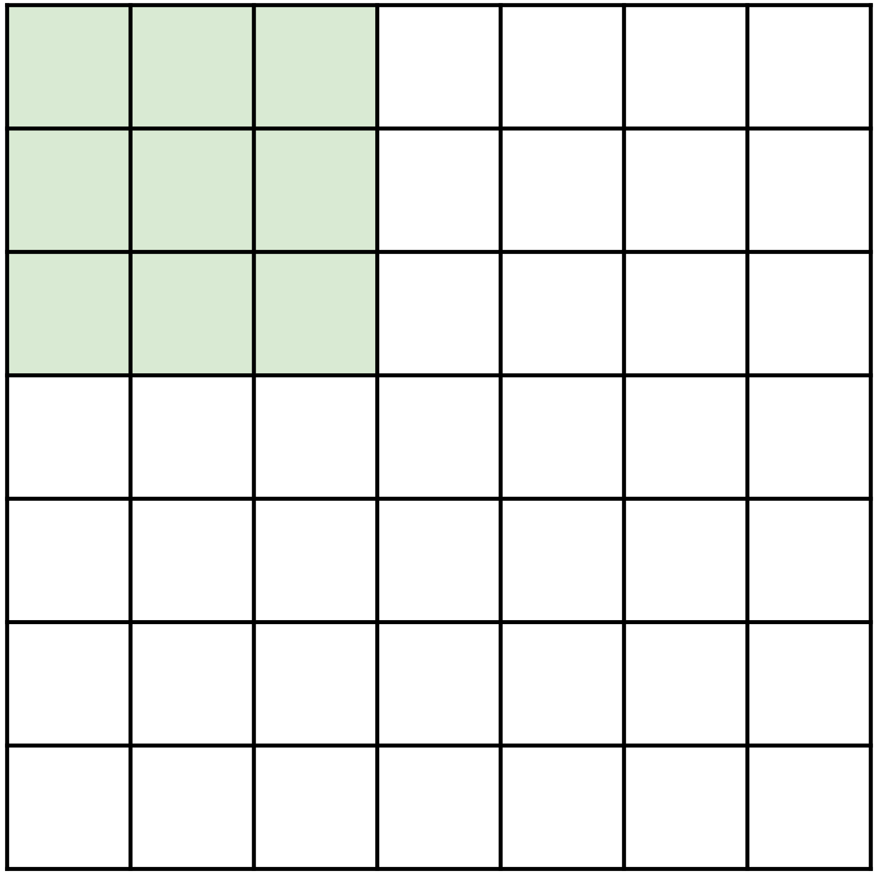
Input

Output

Problem: For large images we need many layers for each output to “see” the whole image image

Solution: Downsample inside the network

Strided Convolution

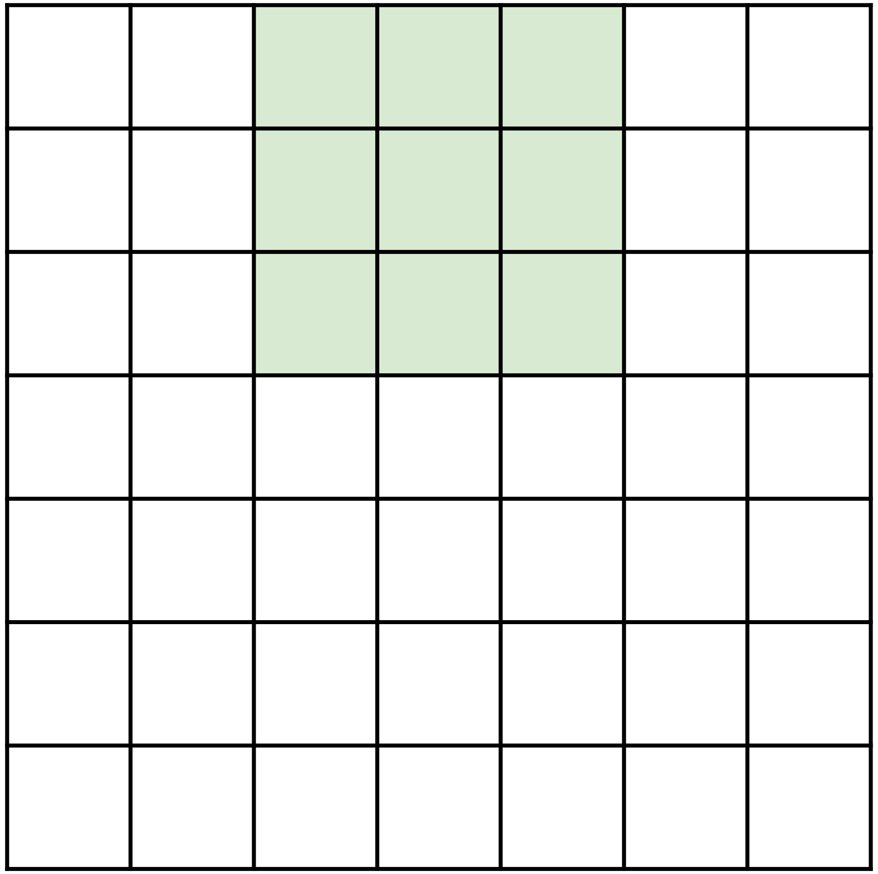


Input: 7x7

Filter: 3x3

Stride: 2

Strided Convolution

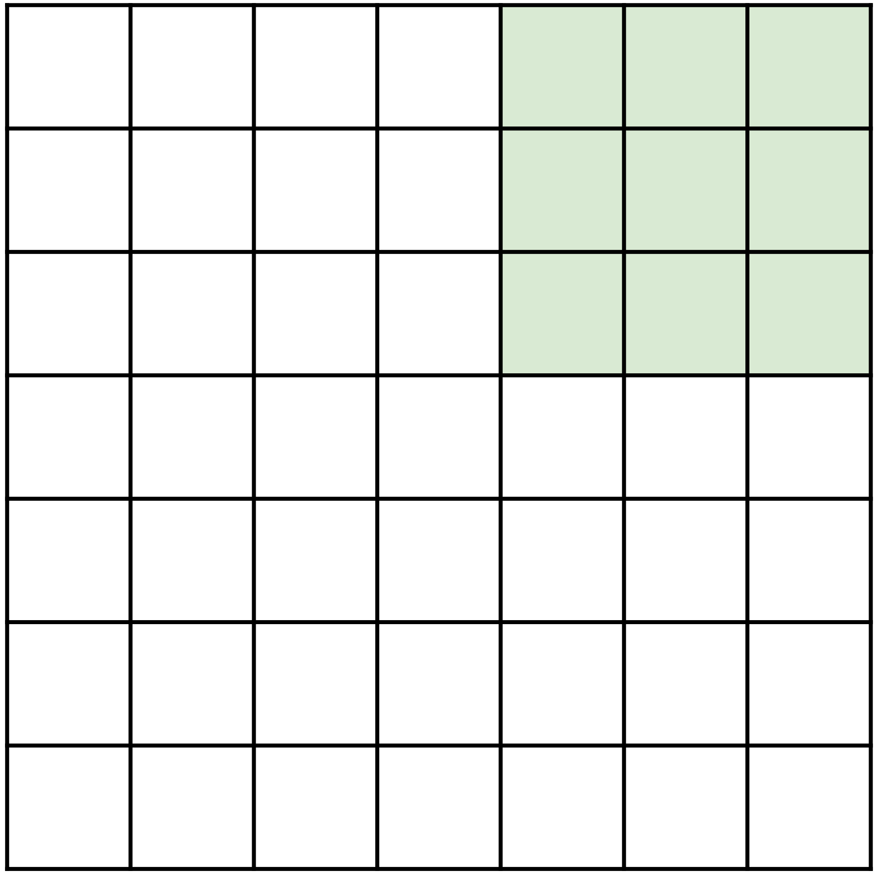


Input: 7x7

Filter: 3x3

Stride: 2

Strided Convolution



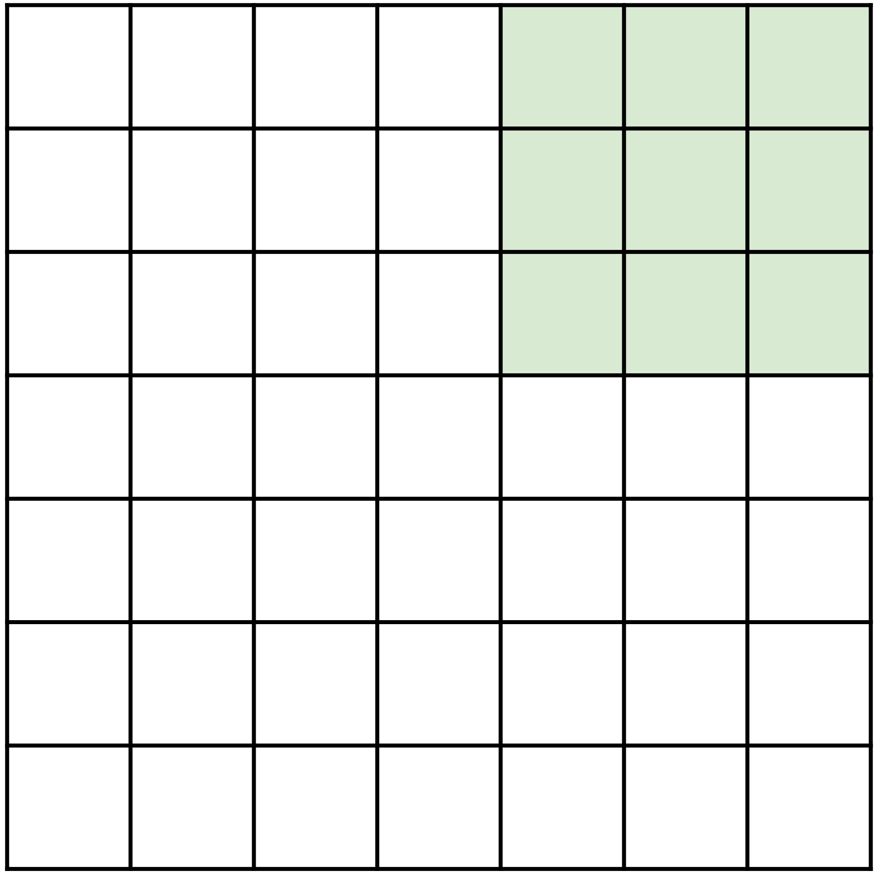
Input: 7x7

Filter: 3x3

Stride: 2

Output: 3x3

Strided Convolution



Input: 7x7

Filter: 3x3

Stride: 2

Output: 3x3

In general:

Input: W

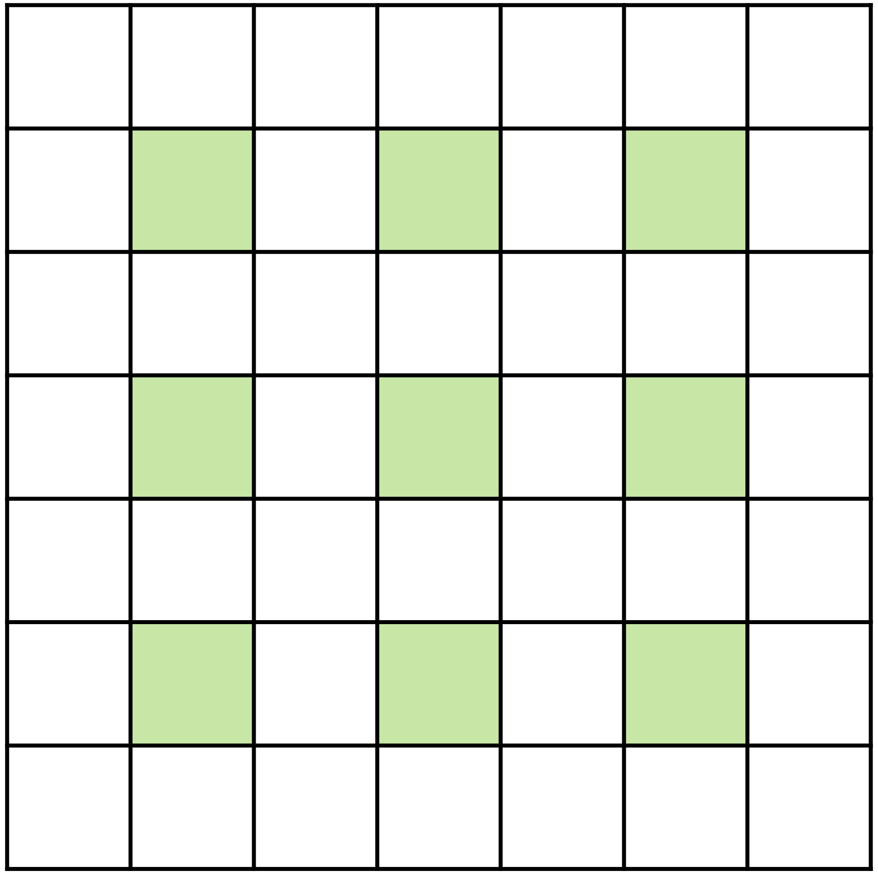
Filter: K

Padding: P

Stride: S

Output: (W – K + 2P) / S + 1

Dilated Convolution



Input: 7x7

Filter: 3x3

Rate: 2

Output: 3x3

In general:

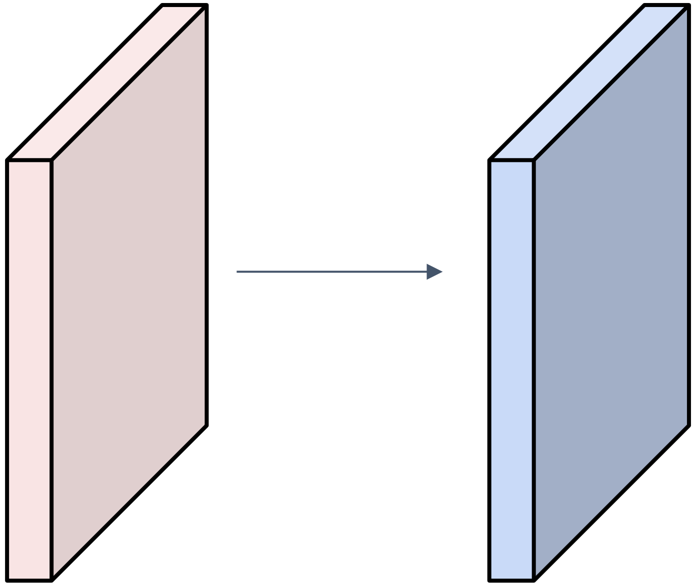
Input: W

Filter: K

Padding: P

Rate: R

Convolution Example

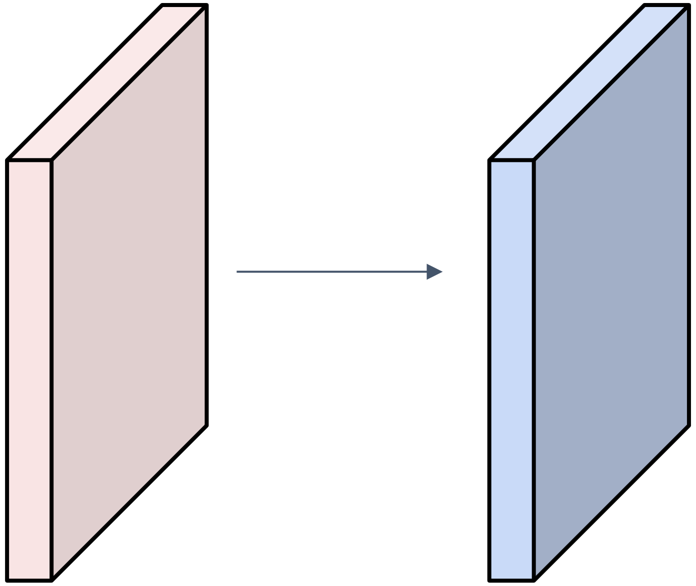


Input volume: 3 x 32 x 32

10 5x5 filters with stride 1, pad 2

Output volume size: ?

Convolution Example



Input volume: 3 x **32** x **32**

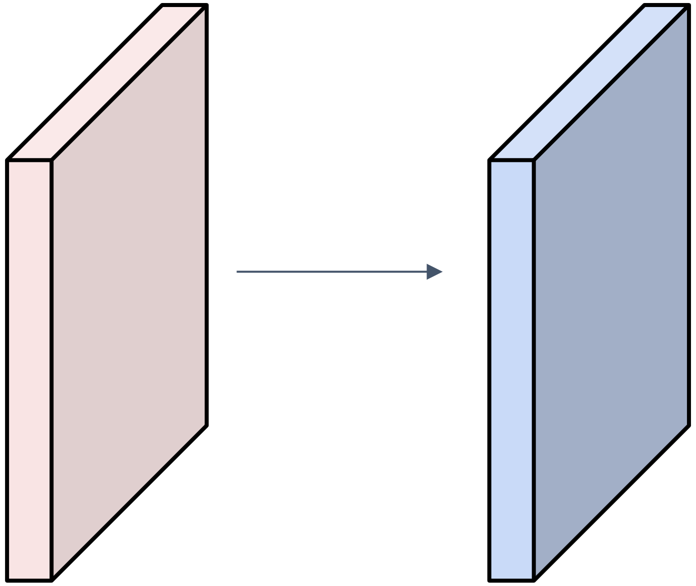
10 **5x5** filters with stride **1**, pad **2**

Output volume size:

(**32**+2***2**-**5**)/**1**+1 = 32 spatially, so

10 x 32 x 32

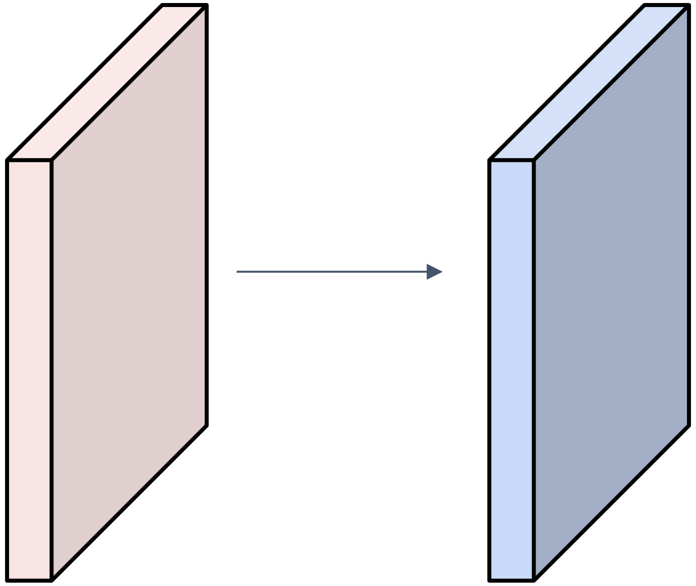
Convolution Example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: 10 x 32 x 32
Number of learnable parameters: ?

Convolution Example



Input volume: 3 x 32 x 32

10 5x5 filters with stride 1, pad 2

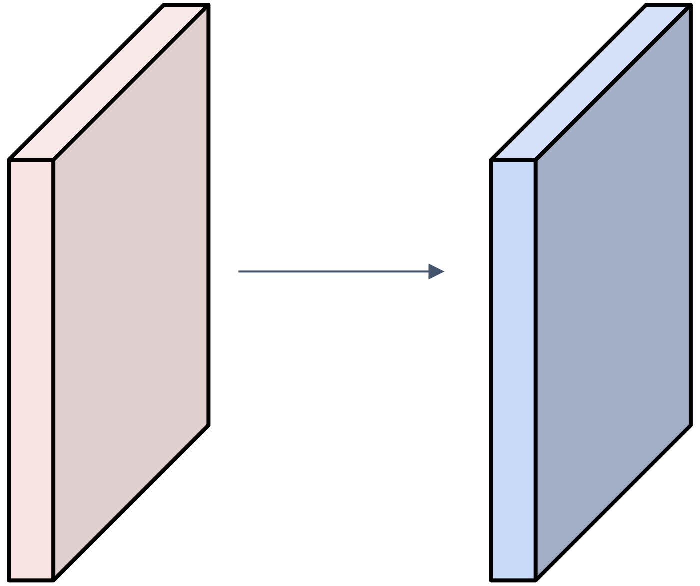
Output volume size: 10 x 32 x 32

Number of learnable parameters: **760**

Parameters per filter: 3*5*5 + 1 (for bias) = 76

10 filters, so total is 10 * 76 = **760**

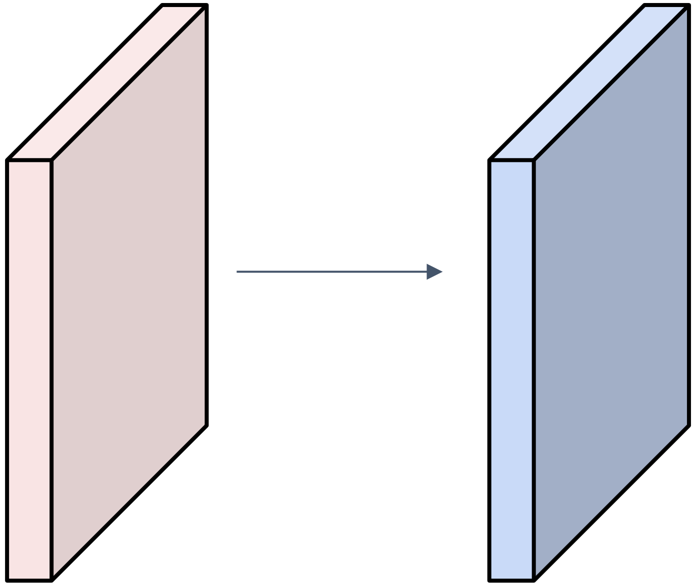
Convolution Example



Input volume: 3 x 32 x 32
10 5x5 filters with stride 1, pad 2

Output volume size: 10 x 32 x 32
Number of learnable parameters: 760
Number of multiply-add operations: ?

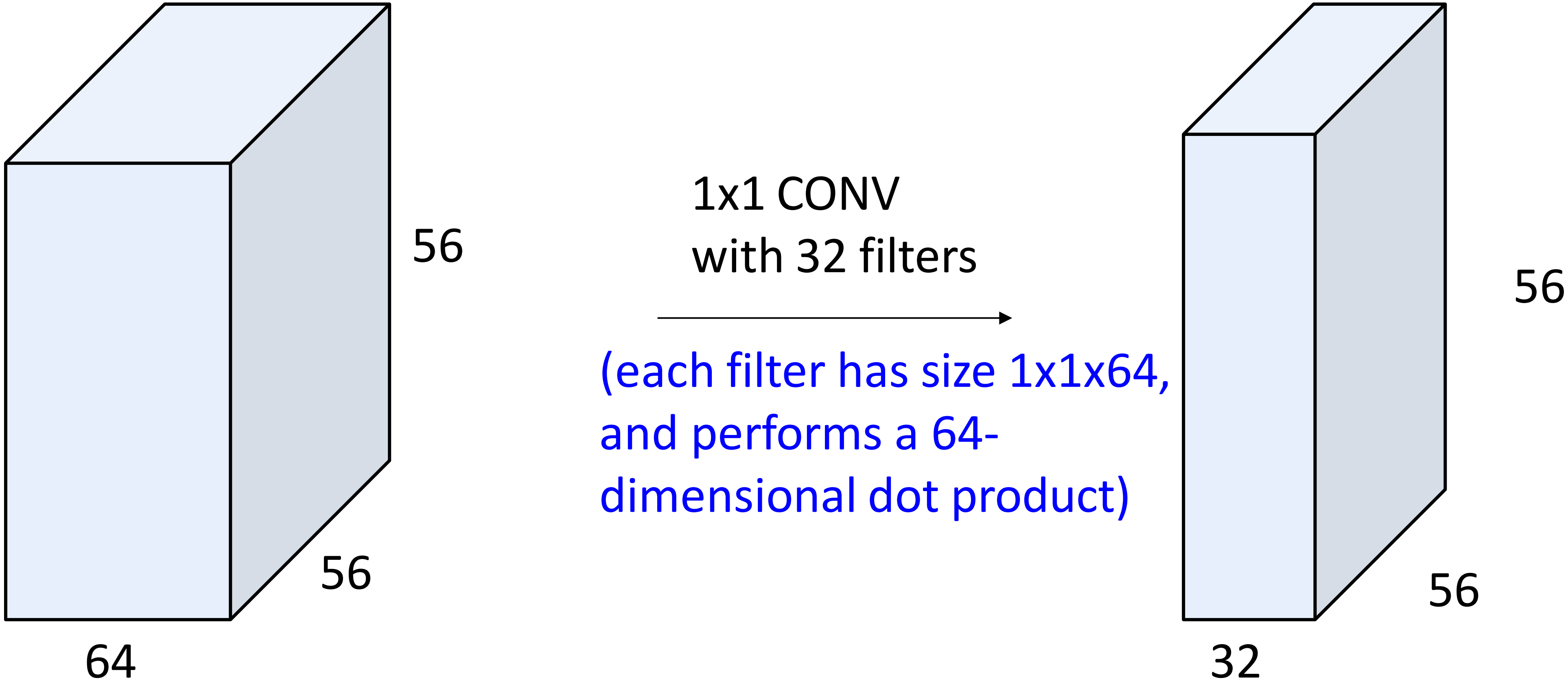
Convolution Example



Input volume: **3** x 32 x 32
10 **5x5** filters with stride 1, pad 2

Output volume size: **10 x 32 x 32**
Number of learnable parameters: 760
Number of multiply-add operations: **768,000**
10*32*32 = 10,240 outputs; each output is the inner product of two **3**x**5x5** tensors (75 elems); total = 75*10240 = **768K**

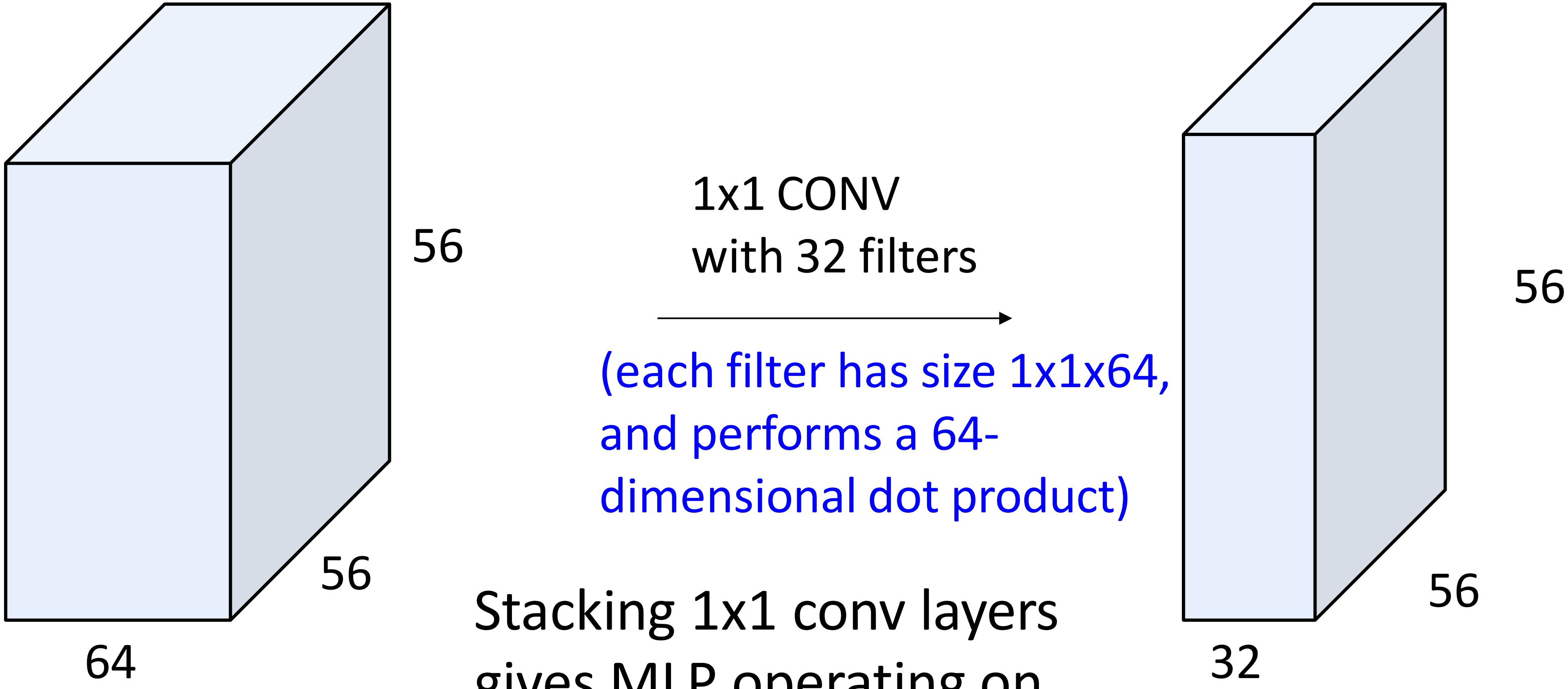
Example: 1x1 Convolution



1x1 CONV
with 32 filters

(each filter has size 1x1x64, and performs a 64-dimensional dot product)

Example: 1x1 Convolution



1x1 CONV
with 32 filters

(each filter has size 1x1x64, and performs a 64-dimensional dot product)

Stacking 1x1 conv layers gives MLP operating on each input position

Lin et al, “Network in Network”, ICLR 2014

Convolution Summary

Input: C_{in} x H x W

Hyperparameters:

- **Kernel size**: K_H x K_W
- **Number filters**: C_{out}
- **Padding**: P
- **Stride**: S

Weight matrix: C_{out} x C_{in} x K_H x K_W

giving C_{out} filters of size C_{in} x K_H x K_W

Bias vector: C_{out}

Output size: C_{out} x H’ x W’ where:

- $H' = (H - K + 2P) / S + 1$
- $W' = (W - K + 2P) / S + 1$

Convolution Summary

Input: C_{in} x H x W

Hyperparameters:

- **Kernel size**: K_H x K_W
- **Number filters**: C_{out}
- **Padding**: P
- **Stride**: S

Weight matrix: C_{out} x C_{in} x K_H x K_W giving C_{out} filters of size C_{in} x K_H x K_W

Bias vector: C_{out}

Output size: C_{out} x H' x W' where:

- $H' = (H - K + 2P) / S + 1$
- $W' = (W - K + 2P) / S + 1$

Common settings:

$K_H = K_W$ (Small square filters)

$P = (K - 1) / 2$ ("Same" padding)

C_{in}, C_{out} = 32, 64, 128, 256 (powers of 2)

K = 3, P = 1, S = 1 (3x3 conv)

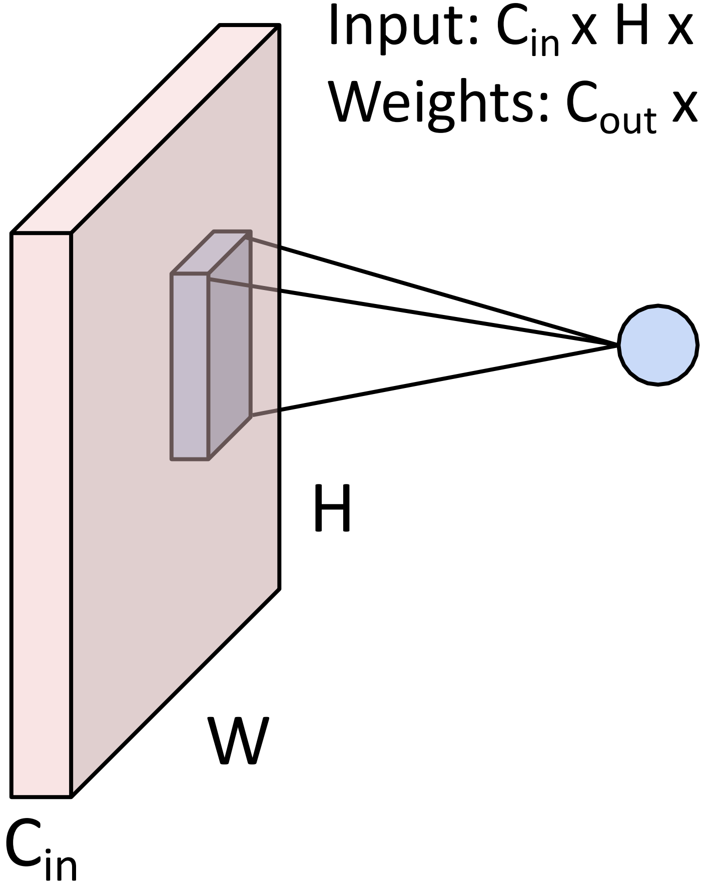
K = 5, P = 2, S = 1 (5x5 conv)

K = 1, P = 0, S = 1 (1x1 conv)

K = 3, P = 1, S = 2 (Downsample by 2)

Other types of convolution

So far: 2D Convolution



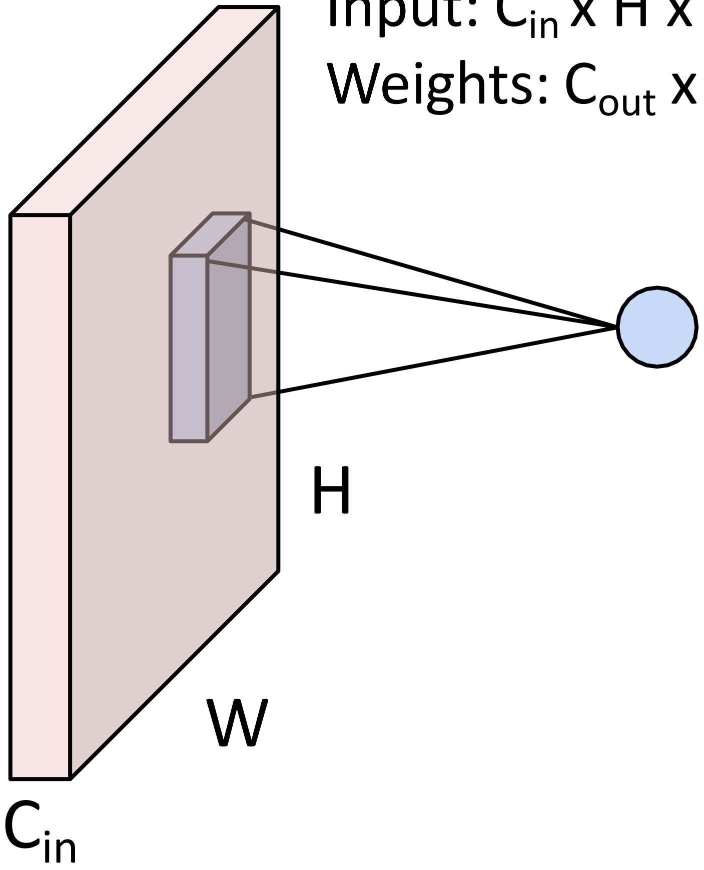
Input: C_{in} x H x W

Weights: C_{out} x C_{in} x K x K

Other types of convolution

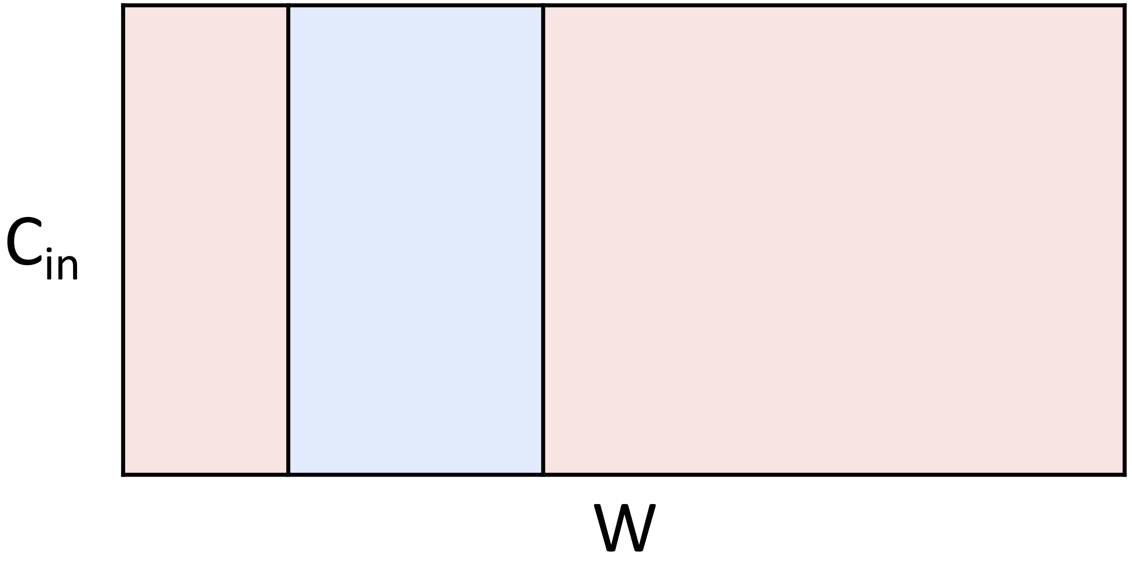
So far: 2D Convolution

Input: C_{in} x H x W
Weights: C_{out} x C_{in} x K x K



1D Convolution

Input: C_{in} x W
Weights: C_{out} x C_{in} x K

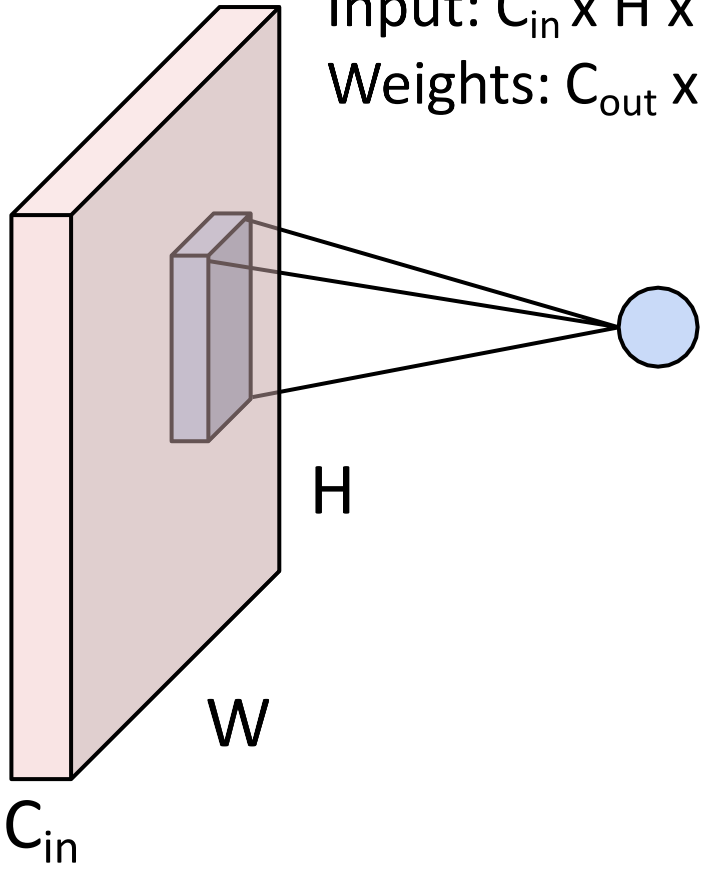


Other types of convolution

So far: 2D Convolution

Input: C_{in} x H x W

Weights: C_{out} x C_{in} x K x K

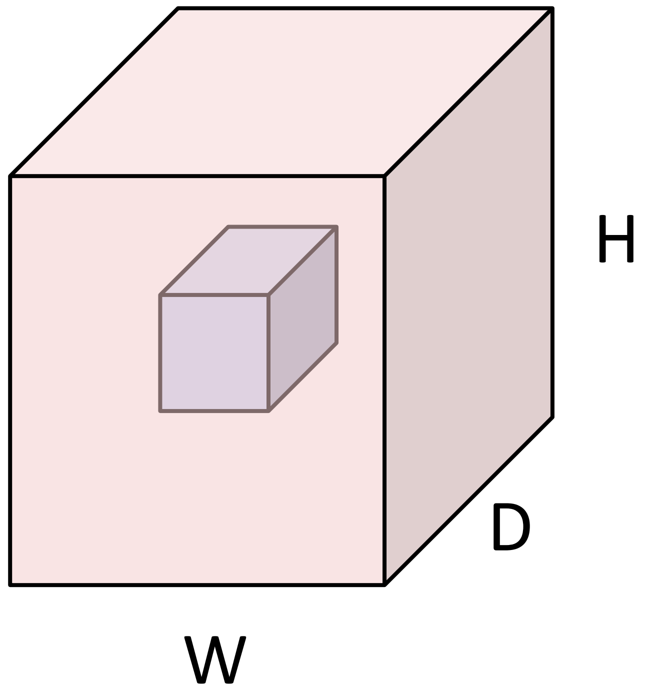


3D Convolution

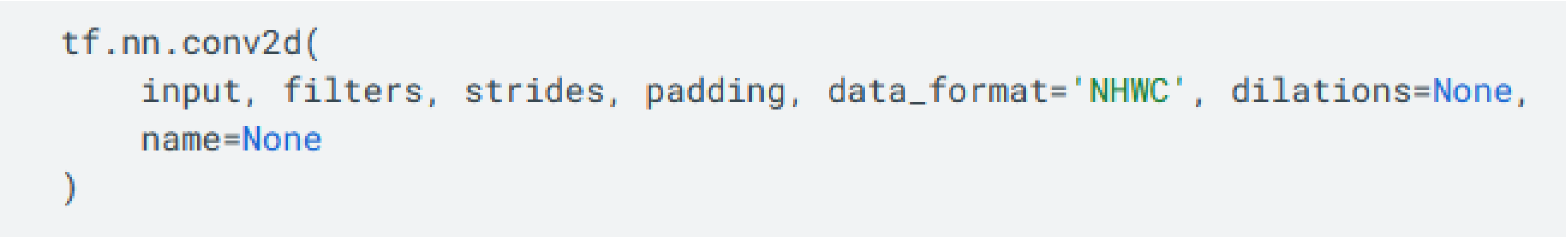
Input: C_{in} x H x W x D

Weights: C_{out} x C_{in} x K x K x K

C_{in}-dim vector at each point in the volume

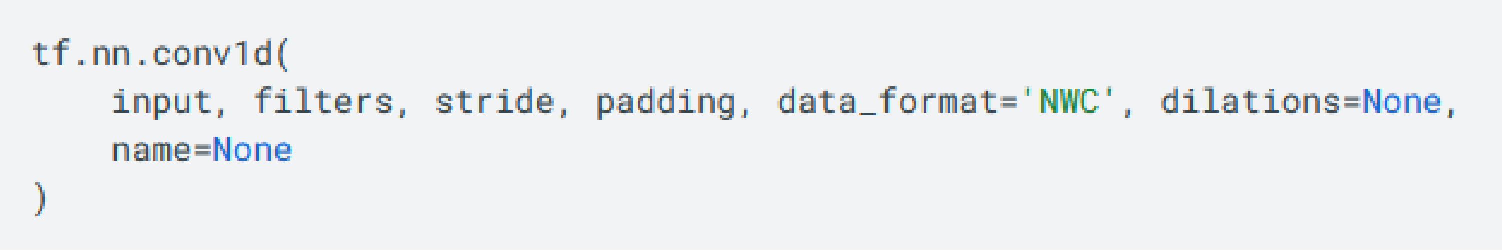


Tensorflow Convolution Layer

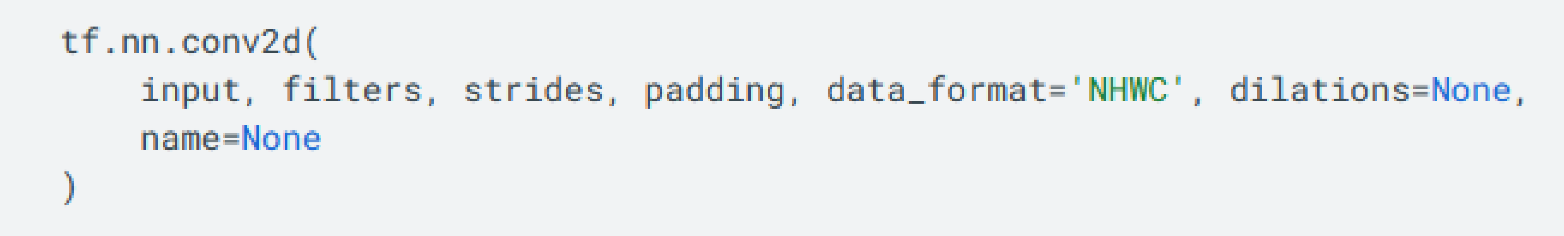


```
tf.nn.conv2d(
    input, filters, strides, padding, data_format='NHWC', dilations=None,
    name=None
)
```

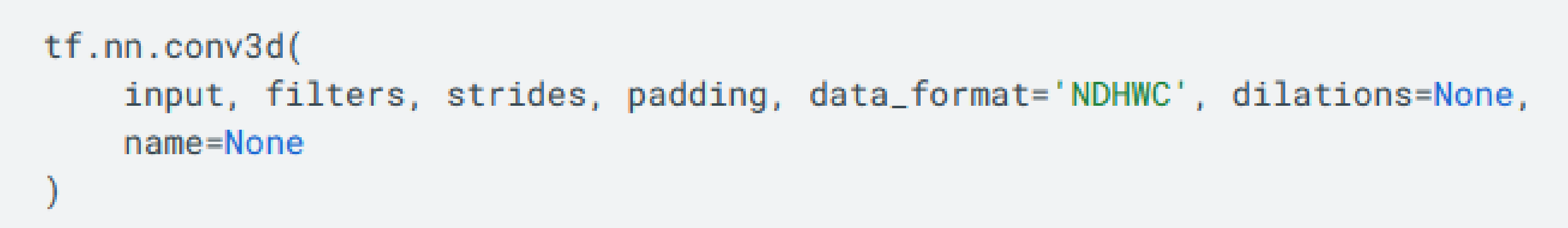
Tensorflow Convolution Layer



```
tf.nn.conv1d(
    input, filters, stride, padding, data_format='NWC', dilations=None,
    name=None
)
```



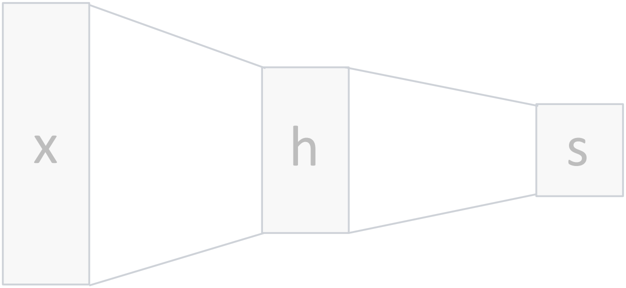
```
tf.nn.conv2d(
    input, filters, strides, padding, data_format='NHWC', dilations=None,
    name=None
)
```



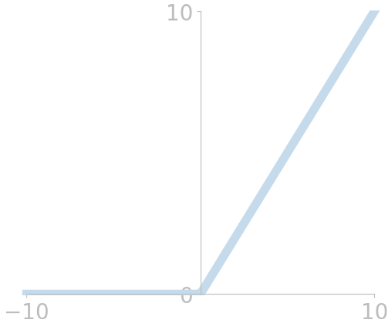
```
tf.nn.conv3d(
    input, filters, strides, padding, data_format='NDHWC', dilations=None,
    name=None
)
```

Components of a Convolutional Network

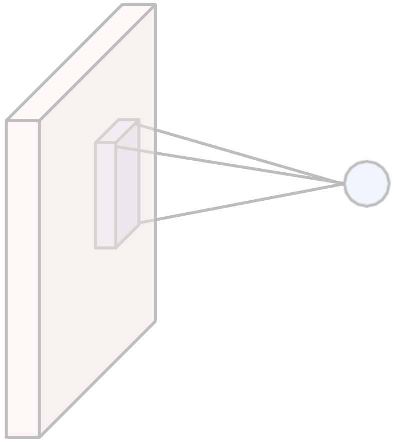
Fully-Connected Layers



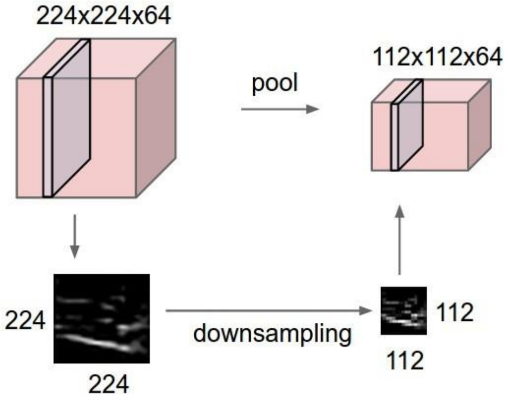
Activation Function



Convolution Layers



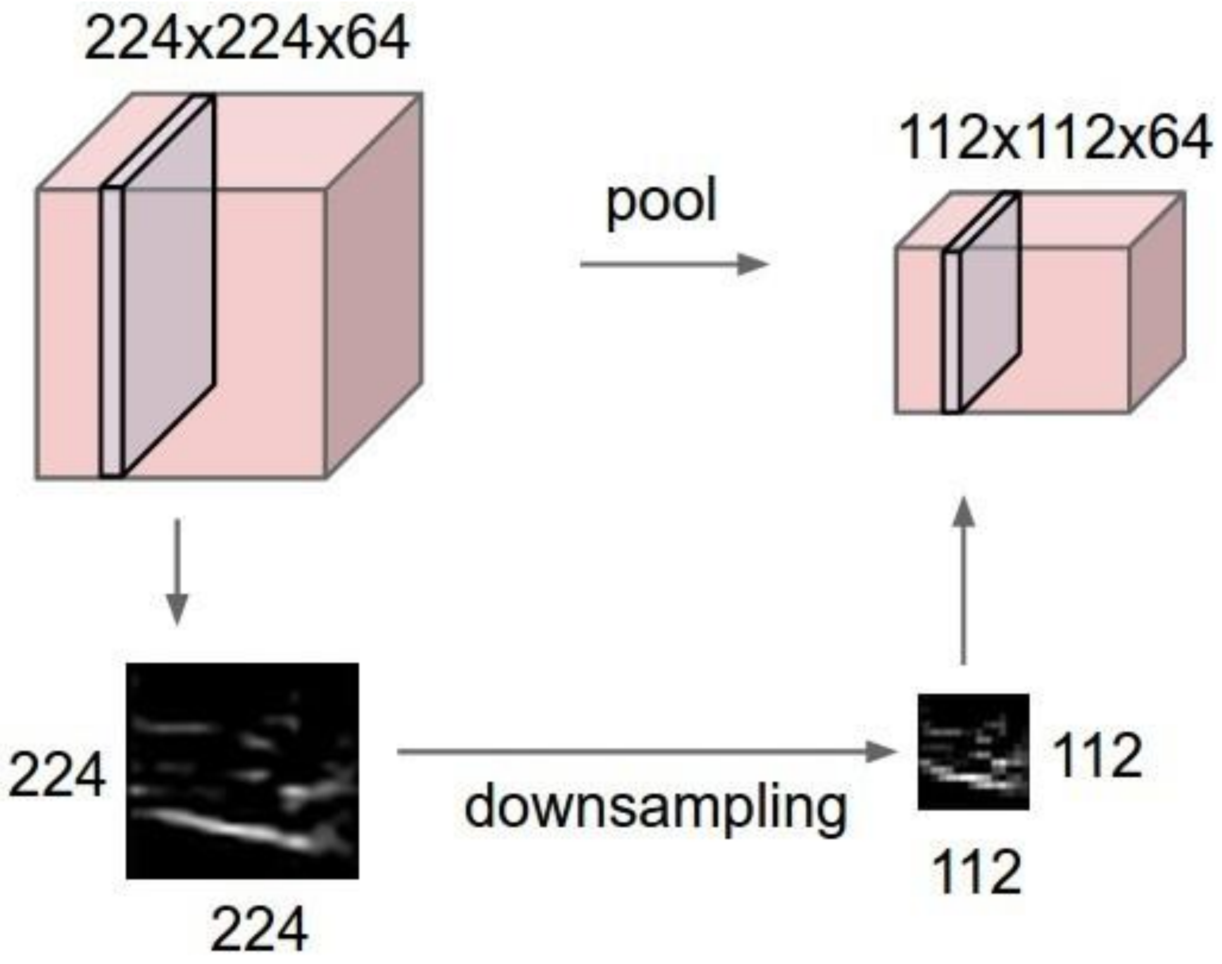
Pooling Layers



Normalization

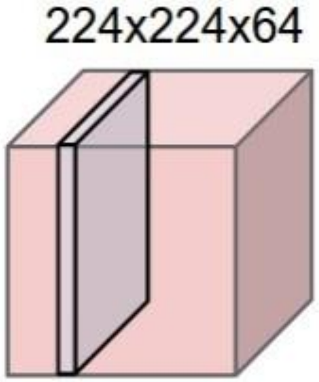
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

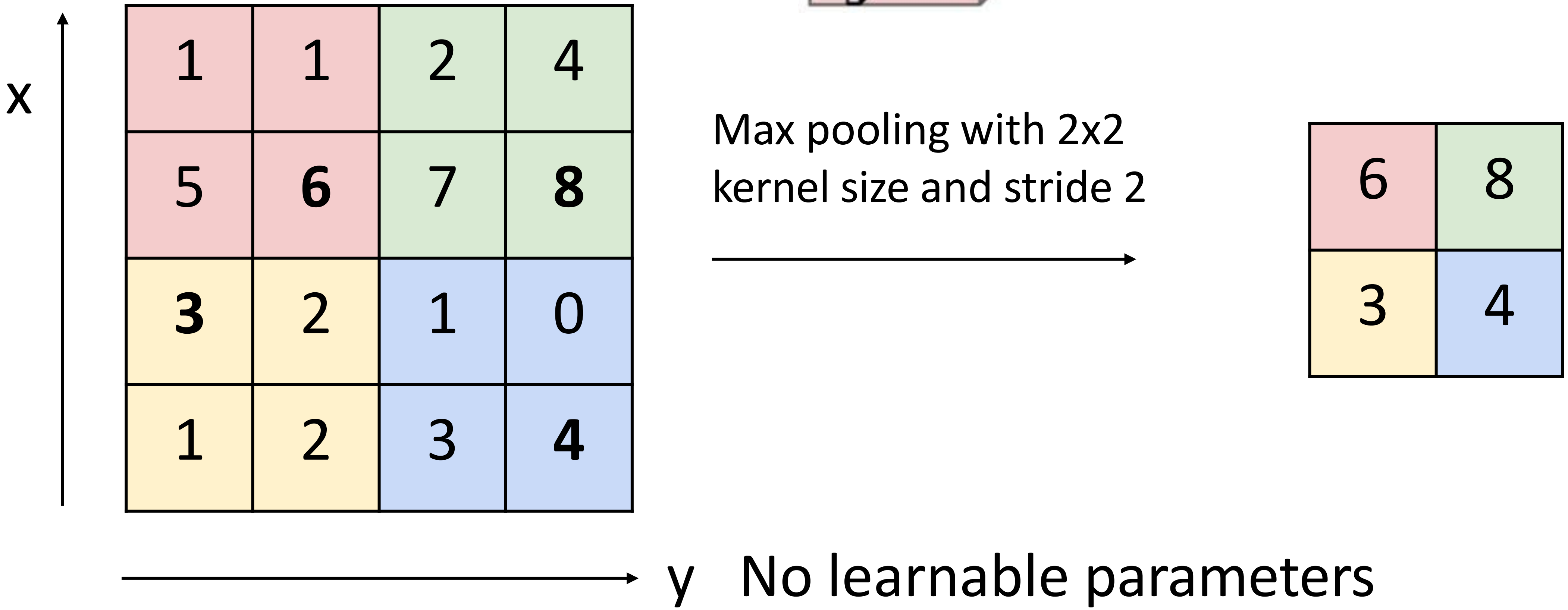
Pooling Layers: Another way to downsample



Hyperparameters:
Kernel Size
Stride
Pooling function

Max Pooling





Single depth slice

1	1	2	4
5	**6**	7	**8**
3	2	1	0
1	2	3	**4**

Max pooling with 2x2
kernel size and stride 2

6	8
3	4

No learnable parameters

Pooling Summary

Input: C x H x W

Hyperparameters:

- Kernel size: K
- Stride: S
- Pooling function (max, avg)

Output: C x H' x W' where

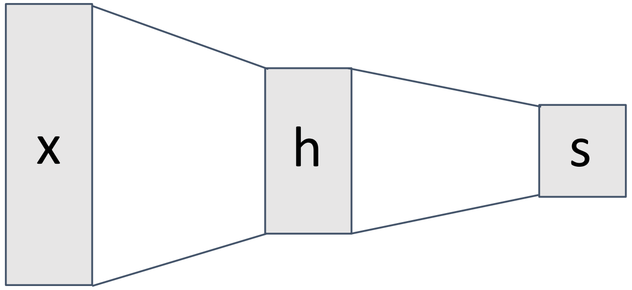
- H' = (H – K) / S + 1
- W' = (W – K) / S + 1

Learnable parameters: None

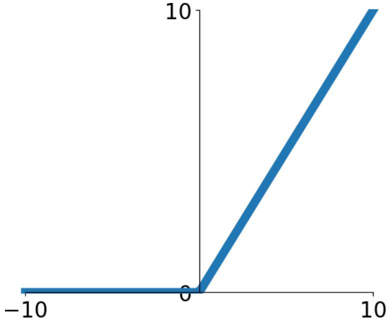
Common settings:
max, K = 2, S = 2
max, K = 3, S = 2 (AlexNet)

Components of a Convolutional Network

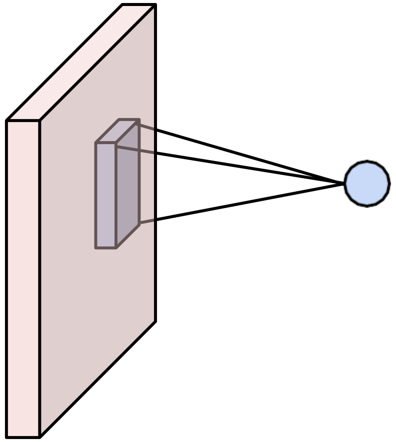
Fully-Connected Layers



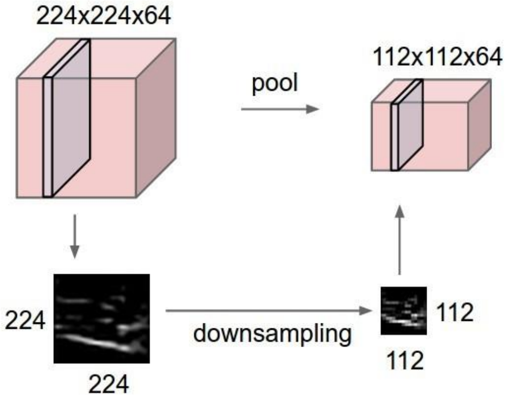
Activation Function



Convolution Layers



Pooling Layers



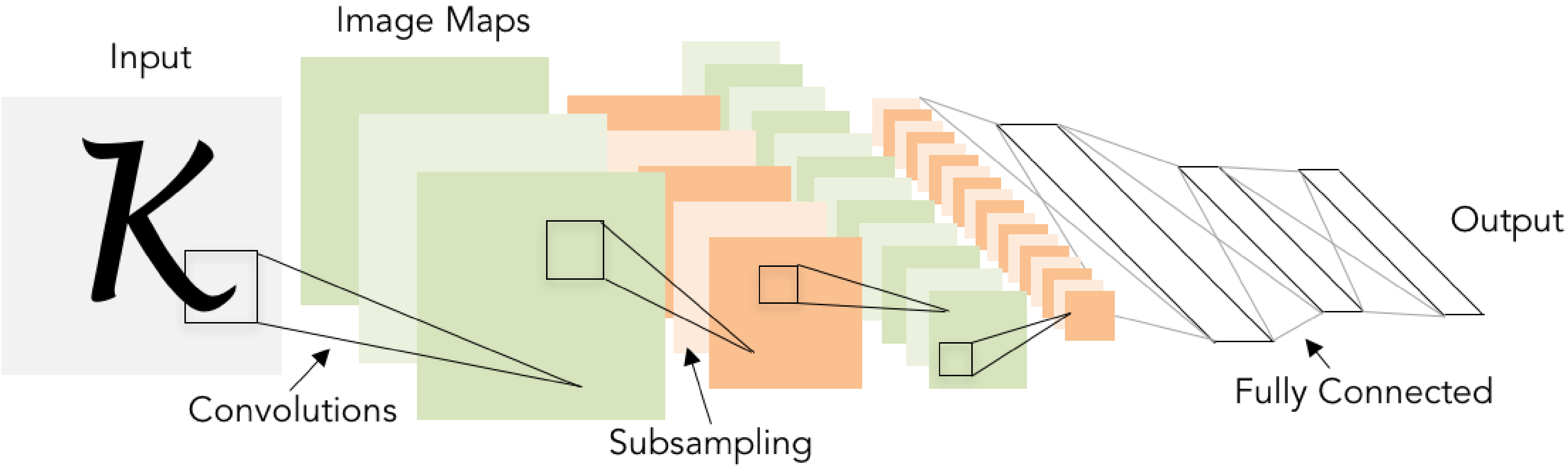
Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Convolutional Networks

Classic architecture: [Conv, ReLU, Pool] x N, flatten, [FC, ReLU] x N, FC

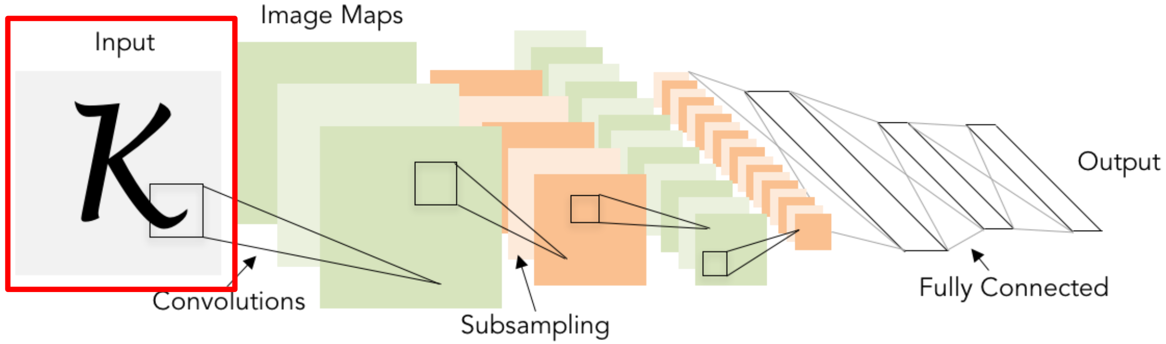
Example: LeNet-5



Lecun et al, “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

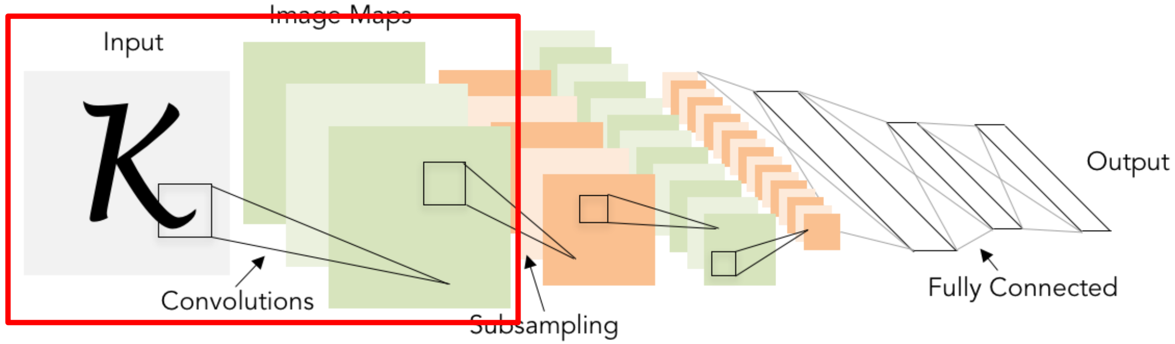
Layer	Output Size	Weight Size
Input	1 x 28 x 28	



Lecun et al, “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

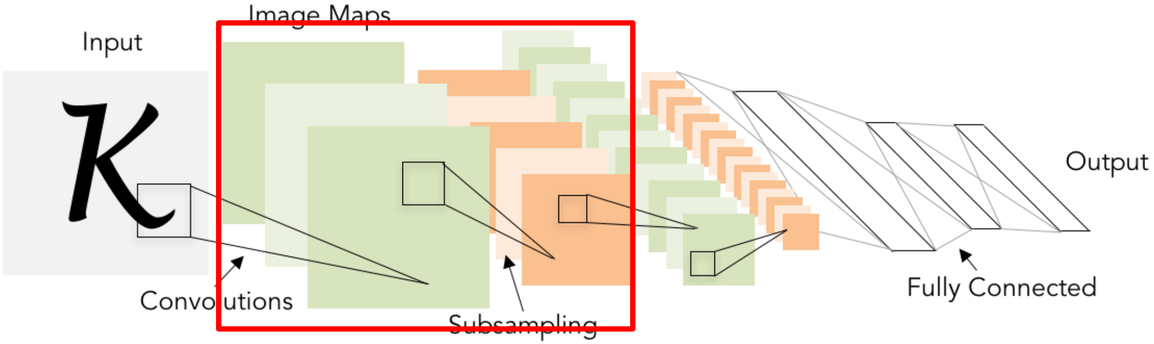
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	



Lecun et al, "Gradient-based learning applied to document recognition", 1998

Example: LeNet-5

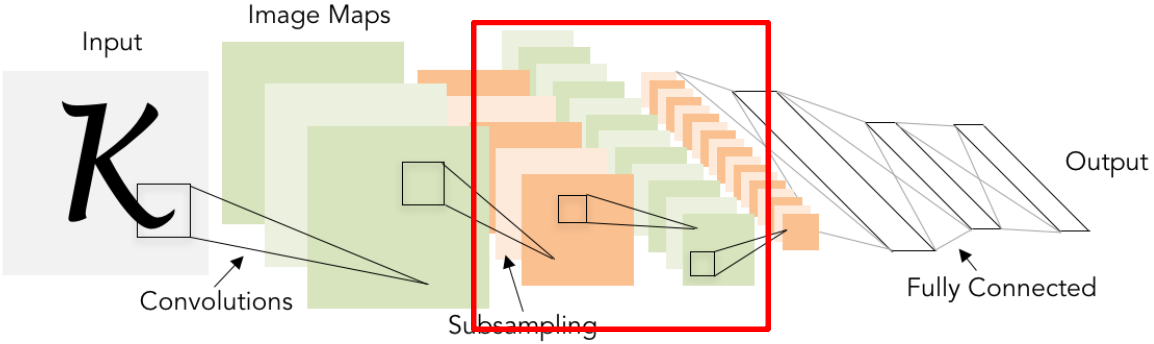
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	



Lecun et al, “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

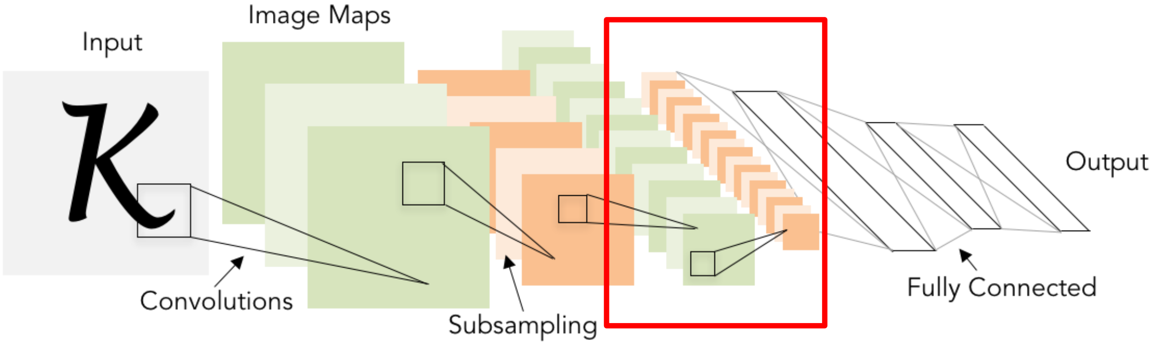
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	



Lecun et al, “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

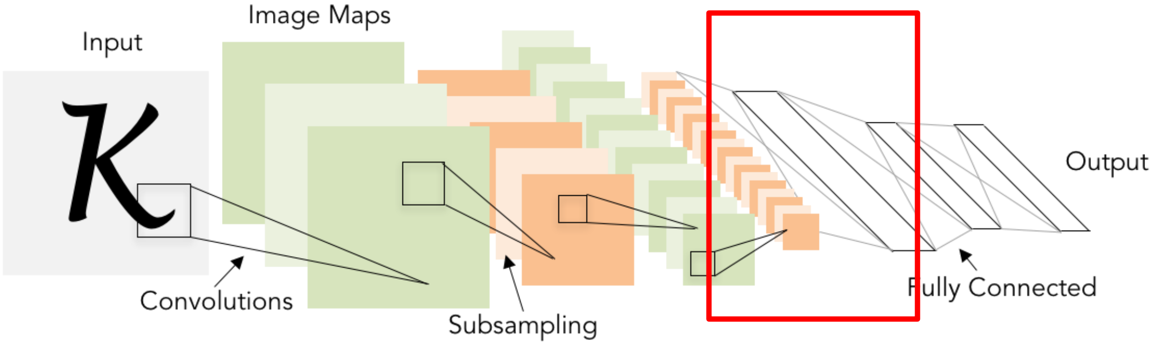
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	



Lecun et al, “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

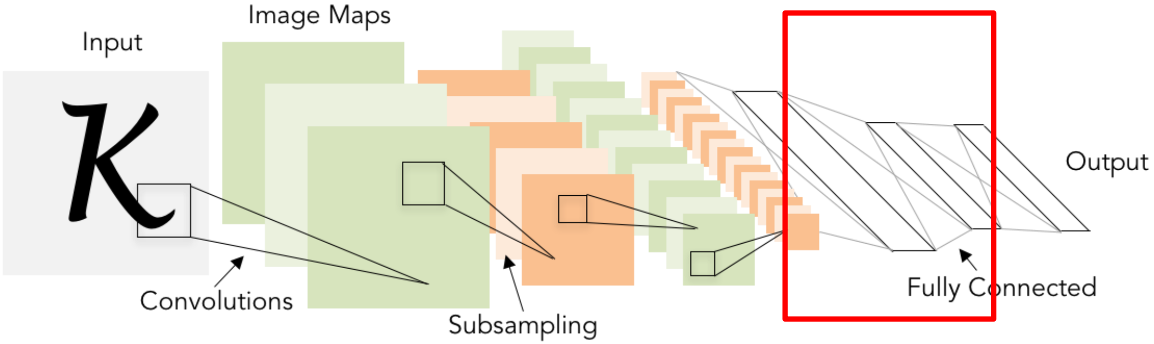
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	



Lecun et al, “Gradient-based learning applied to document recognition”, 1998

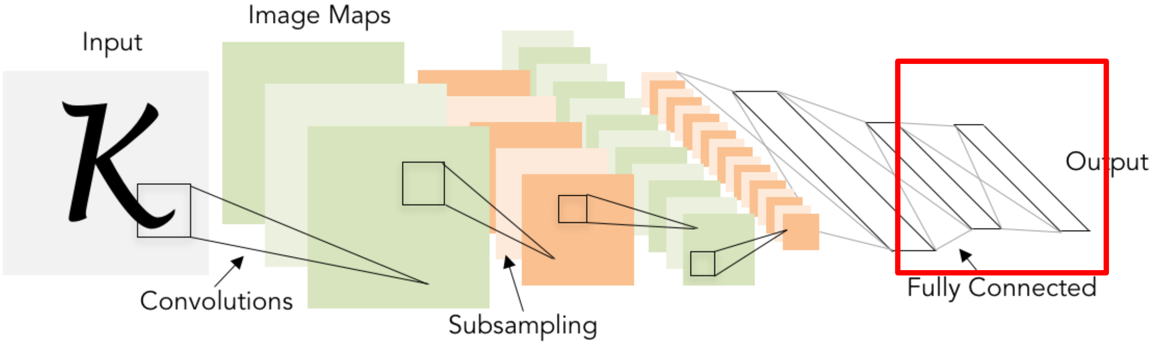
Example: LeNet-5

Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	
Linear (2450 -> 500)	500	2450 x 500
ReLU	500	



Lecun et al, “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

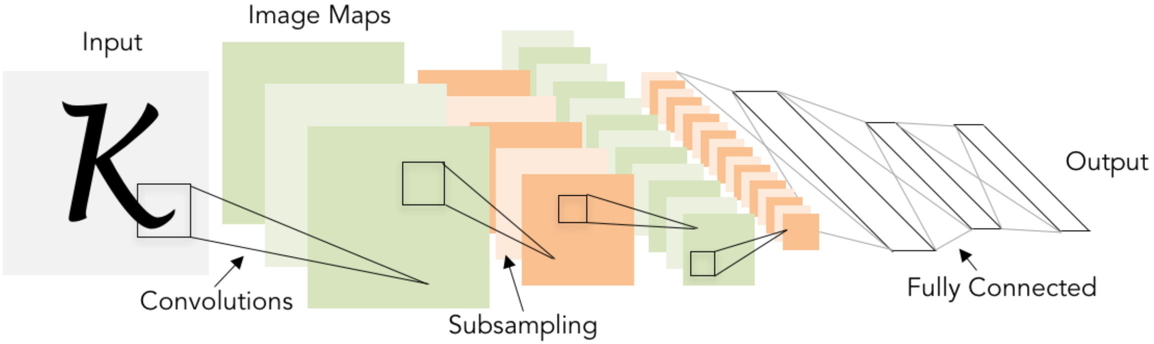


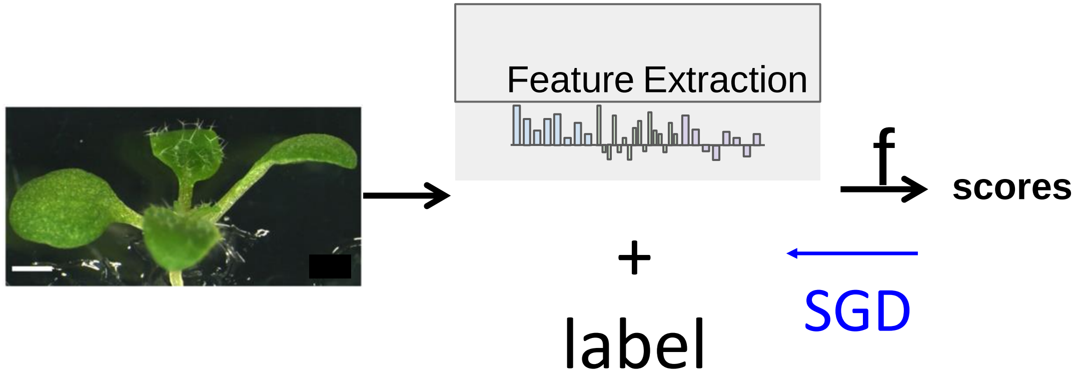
Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	
Linear (2450 -> 500)	500	2450 x 500
ReLU	500	
Linear (500 -> 10)	10	500 x 10

Lecun et al, “Gradient-based learning applied to document recognition”, 1998

Example: LeNet-5

Layer	Output Size	Weight Size
Input	1 x 28 x 28	
Conv (C_{out}=20, K=5, P=2, S=1)	20 x 28 x 28	20 x 1 x 5 x 5
ReLU	20 x 28 x 28	
MaxPool(K=2, S=2)	20 x 14 x 14	
Conv (C_{out}=50, K=5, P=2, S=1)	50 x 14 x 14	50 x 20 x 5 x 5
ReLU	50 x 14 x 14	
MaxPool(K=2, S=2)	50 x 7 x 7	
Flatten	2450	
Linear (2450 -> 500)	500	2450 x 500
ReLU	500	
Linear (500 -> 10)	10	500 x 10

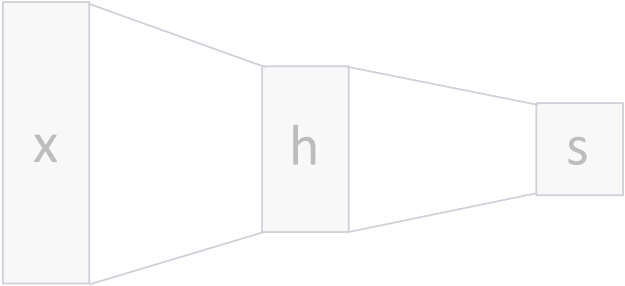




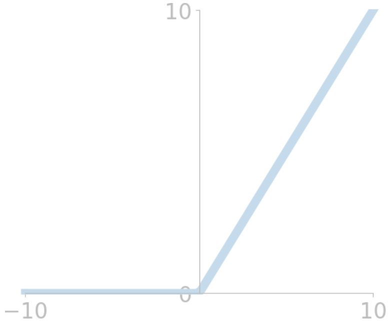
Lecun et al, “Gradient-based learning applied to document recognition”, 1998

Components of a Convolutional Network

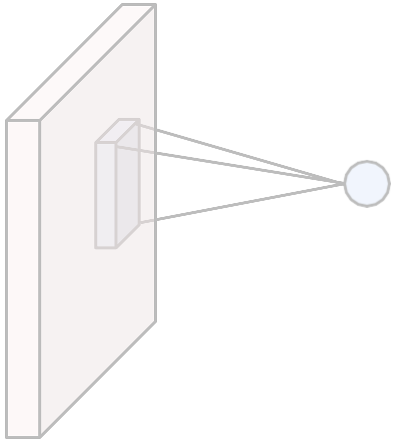
Fully-Connected Layers



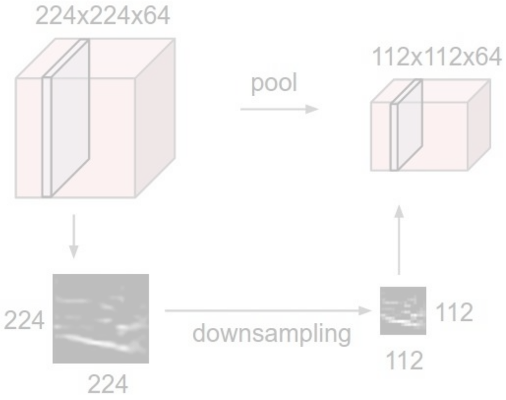
Activation Function



Convolution Layers



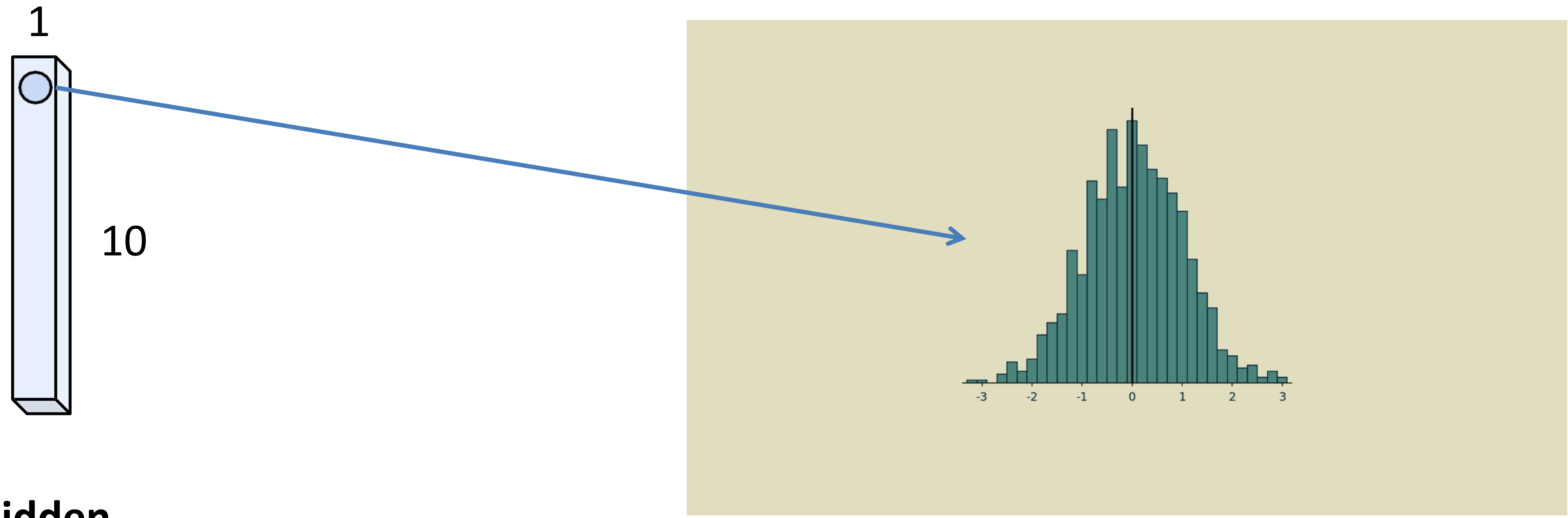
Pooling Layers



Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Node Activation as a Distribution

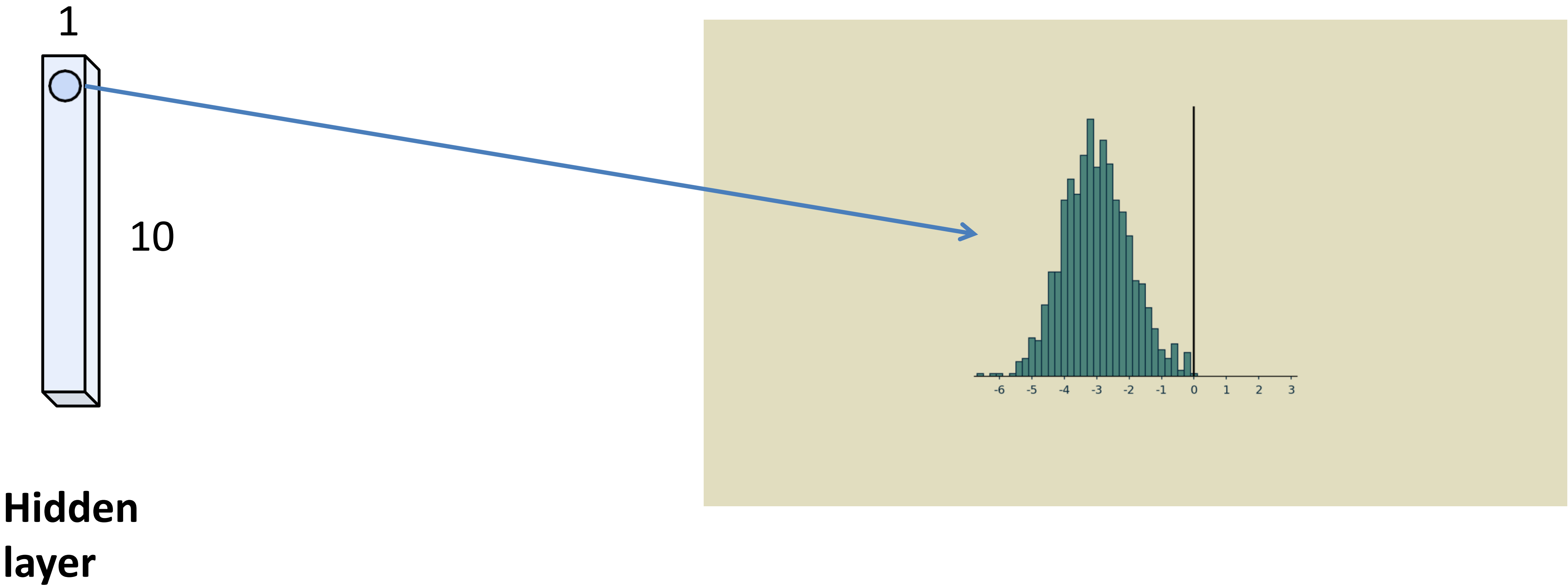


1

10

Hidden layer

Node Activation as a Distribution

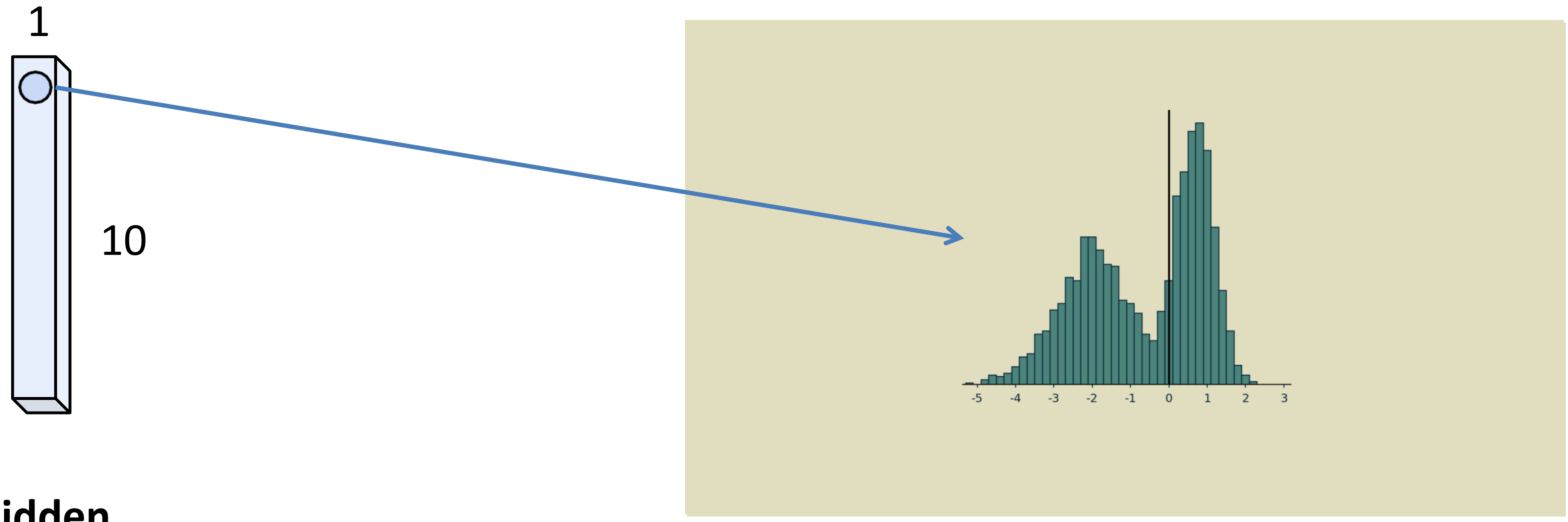


1

10

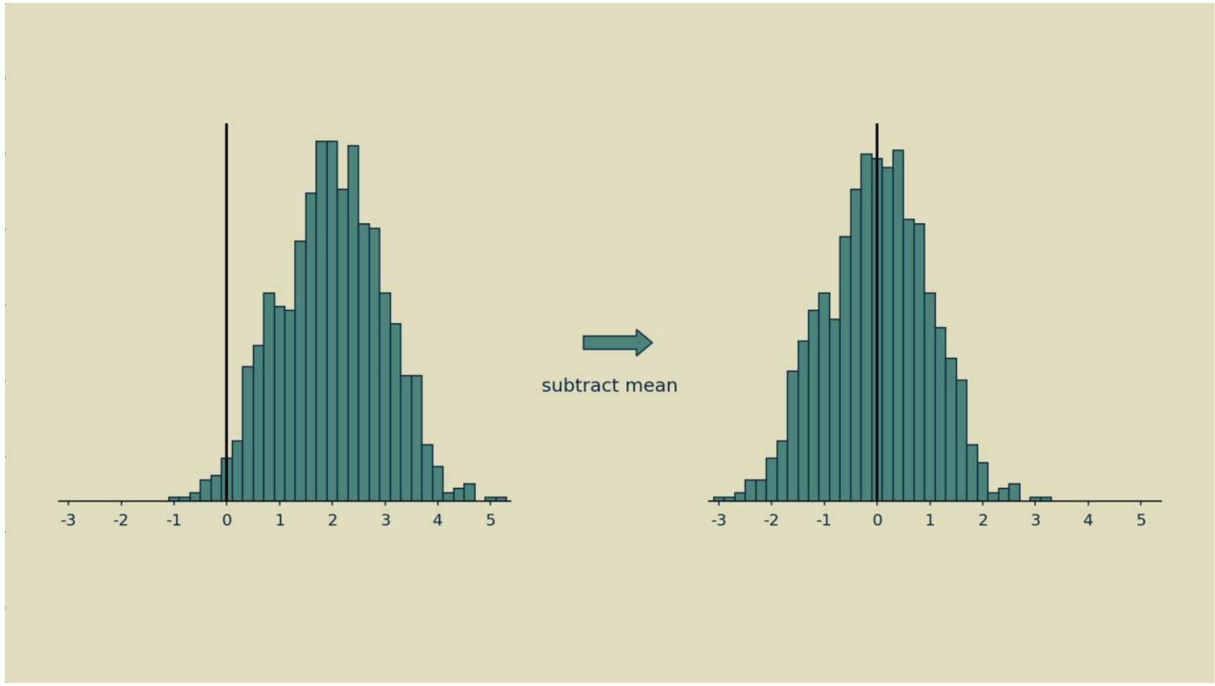
Hidden layer

Node Activation as a Distribution



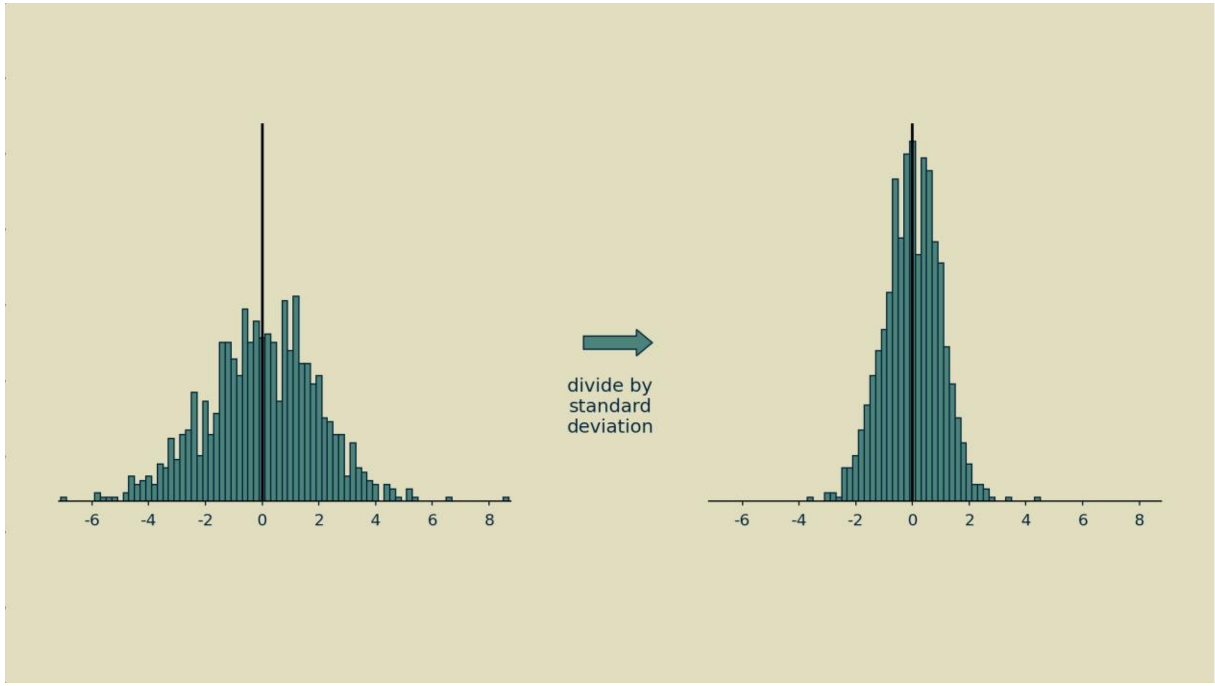
Hidden layer

Normalization of Activation Distribution



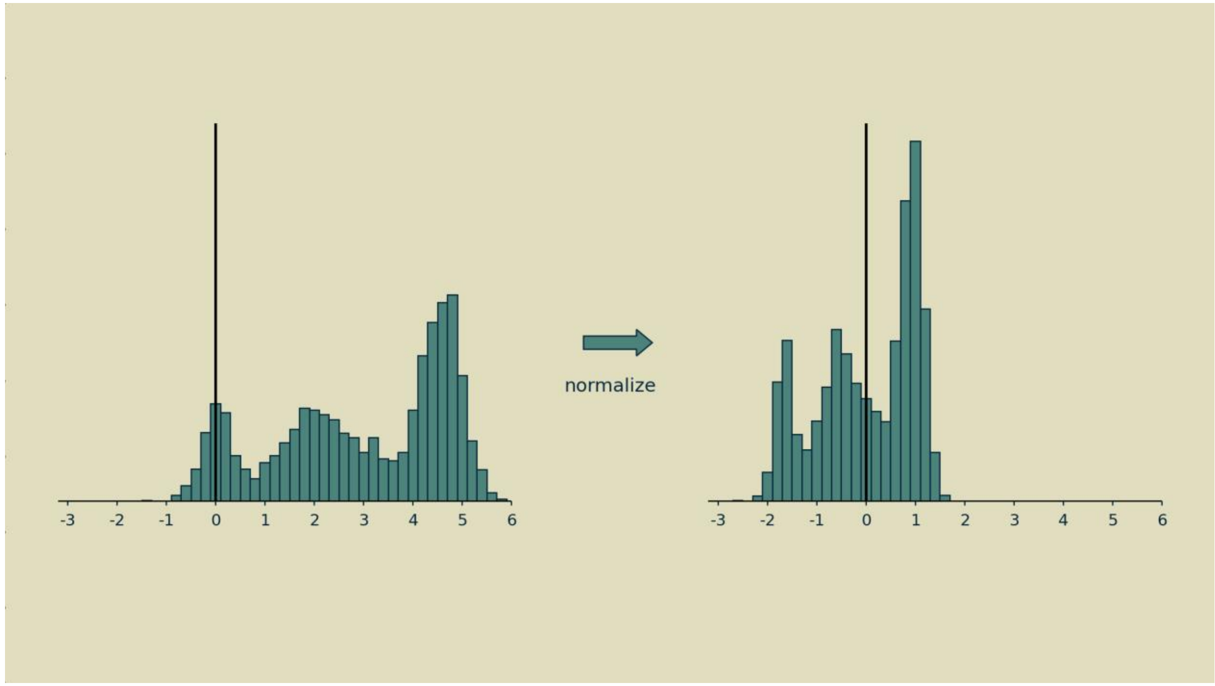
Normalization here → subtract the mean

Normalization of Activation Distribution



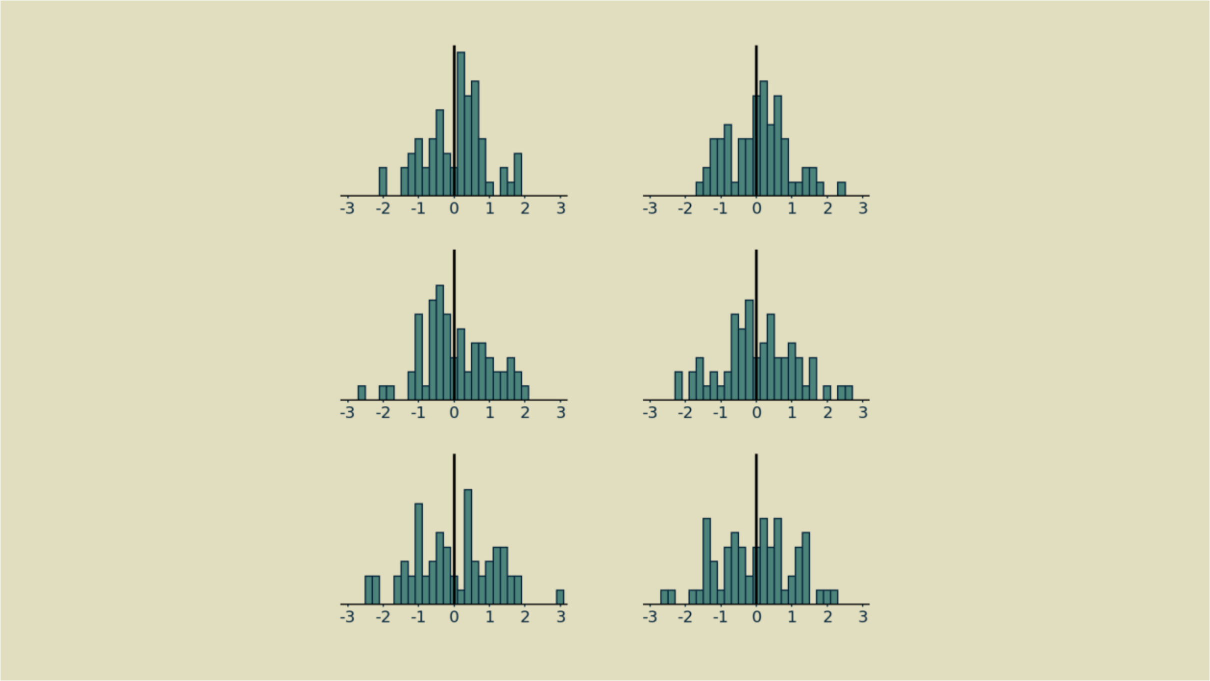
Normalization here → divide by standard deviation

Normalization of Activation Distribution



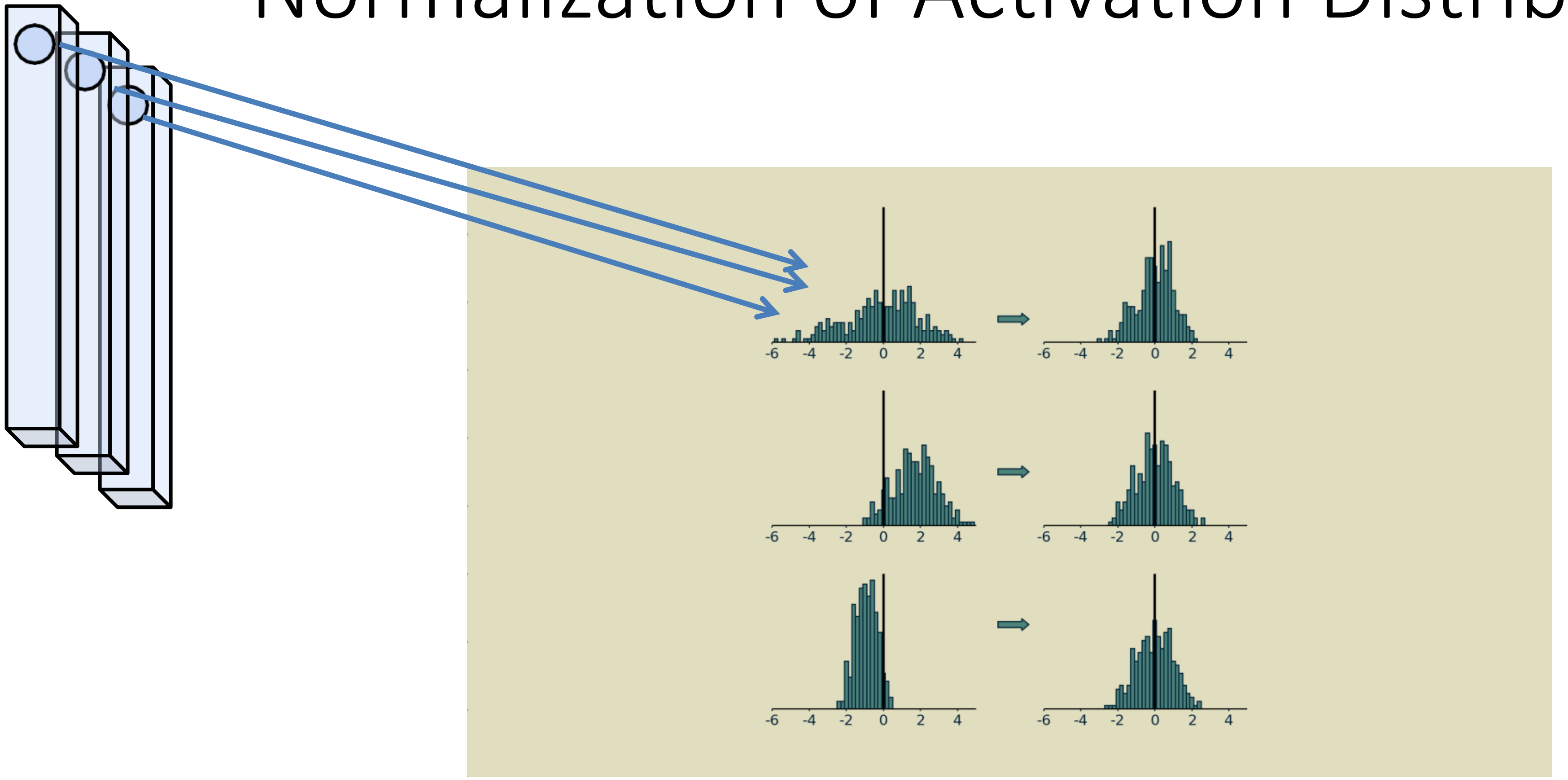
Distribution is centered around the origin and has unit variance

Batch Size



Trade-off between too few observations resulting in noisy estimations for means and variance and loss of meaning due to network learning

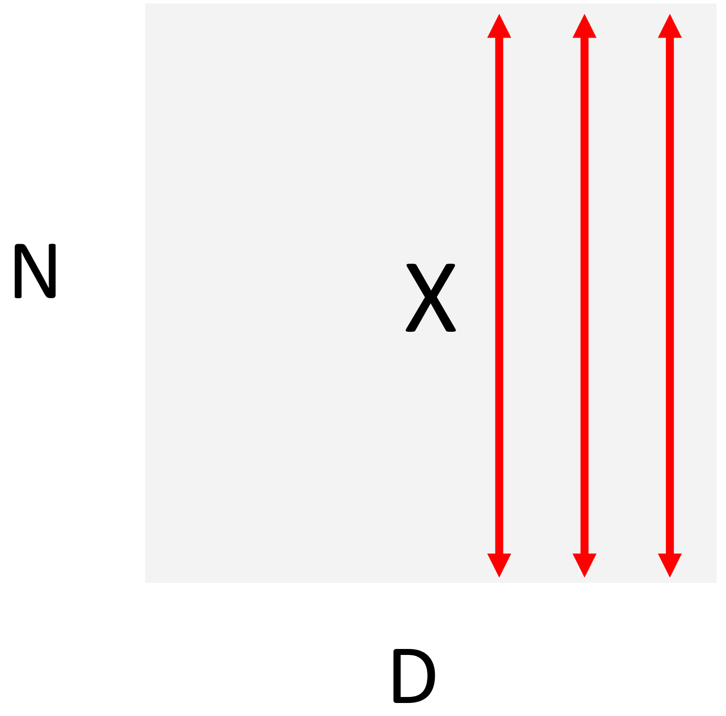
Normalization of Activation Distribution



The size of a single batch used for computing SGD in parallel is considered good practice

Batch Normalization

Input: $x : N \times D$



$$\mu_j = \frac{1}{N} \sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^{N} (x_{i,j} - \mu_j)^2$$

Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

Ioffe and Szegedy, "Batch normalization: Accelerating deep network training by reducing internal covariate shift", ICML 2015

Batch Normalization

Input: $x : N \times D$

Learnable scale and shift parameters:

$\gamma, \beta : D$

$$\mu_j = \frac{1}{N} \sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^{N} (x_{i,j} - \mu_j)^2$$

Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization: Test time

Estimates depend on minibatch; can't do this at test-time

Input: $x : N \times D$

Learnable scale and shift parameters:

$\gamma, \beta : D$

$$\mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N}\sum_{i=1}^{N}(x_{i,j} - \mu_j)^2$$

Per-channel std, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization: Test time

(Running) average of values seen during training

Input: $x : N \times D$

Learnable scale and shift parameters:

$$\gamma, \beta : D$$

$$\mu_j = \frac{1}{N} \sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^{N} (x_{i,j} - \mu_j)^2$$

Per-channel std, shape is D

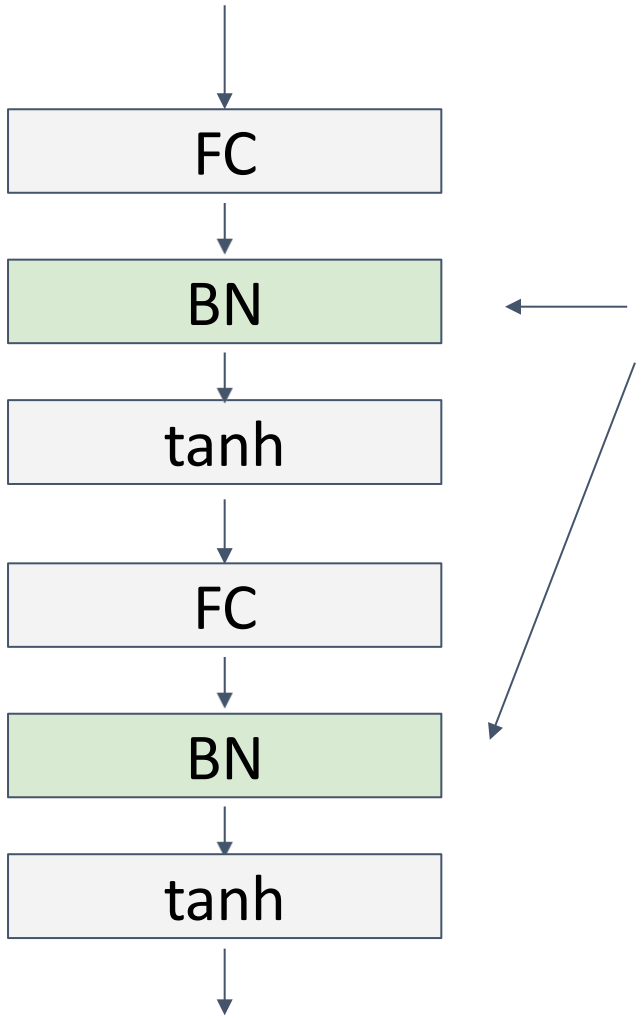
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization



Usually inserted after Fully Connected or Convolutional layers, and before nonlinearity.

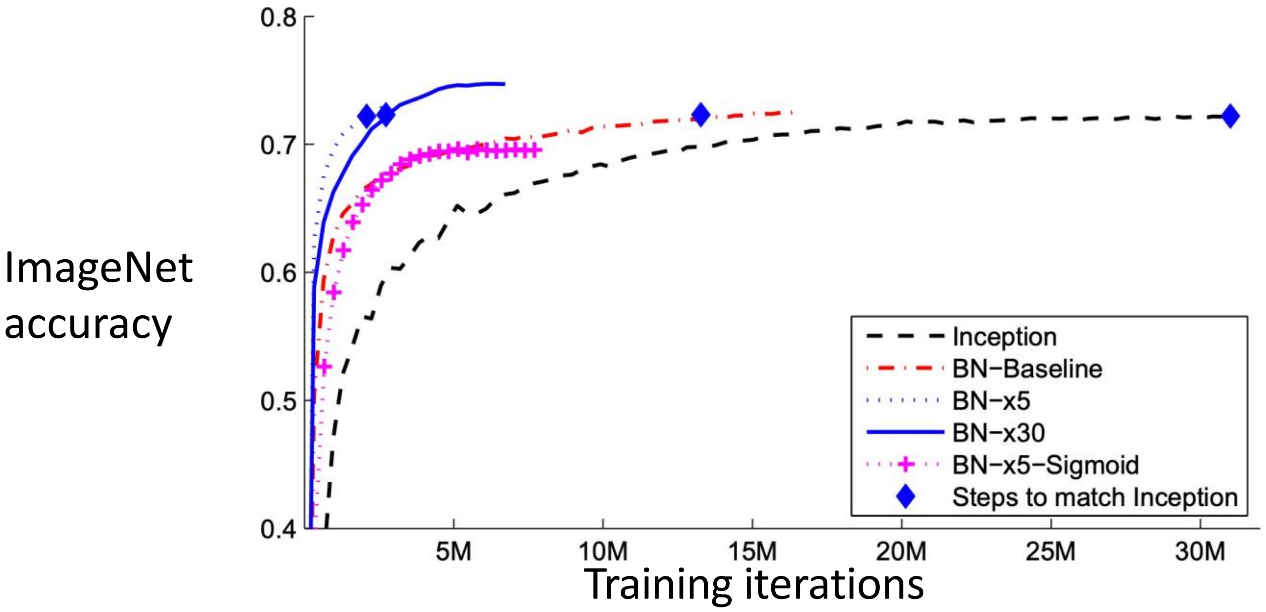
$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

Ioffe and Szegedy, "Batch normalization: Accelerating deep network training by reducing internal covariate shift", ICML 2015

Batch Normalization

FC
BN
tanh
FC
BN
tanh

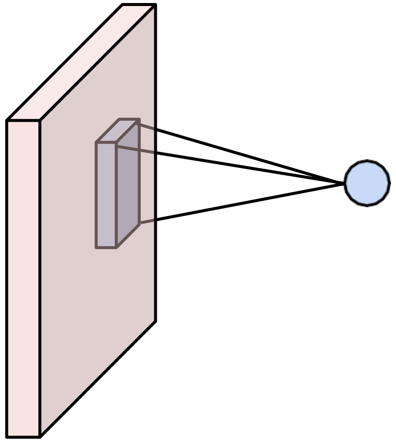
- Makes deep networks **much** easier to train
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as regularization during training
- Zero overhead at test-time



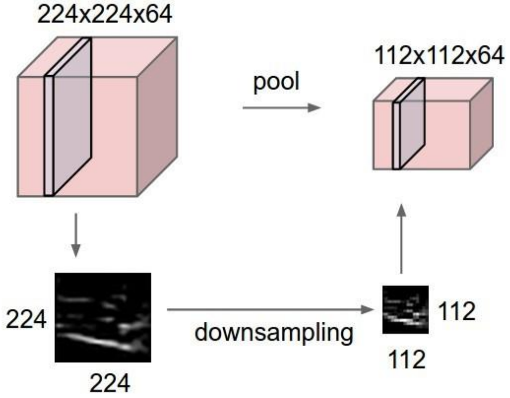
Ioffe and Szegedy, "Batch normalization: Accelerating deep network training by reducing internal covariate shift", ICML 2015

Components of a Convolutional Network

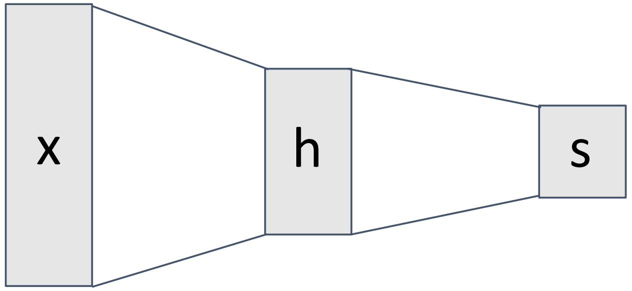
Convolution Layers



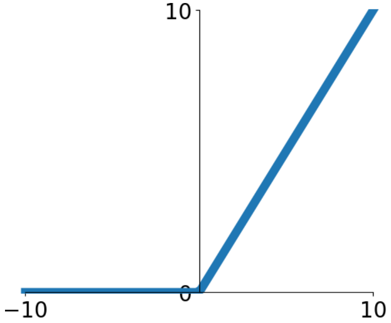
Pooling Layers



Fully-Connected Layers



Activation Function

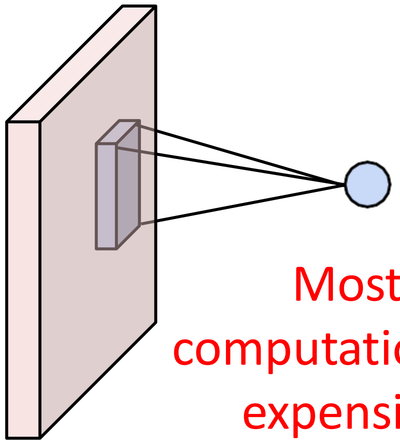


Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

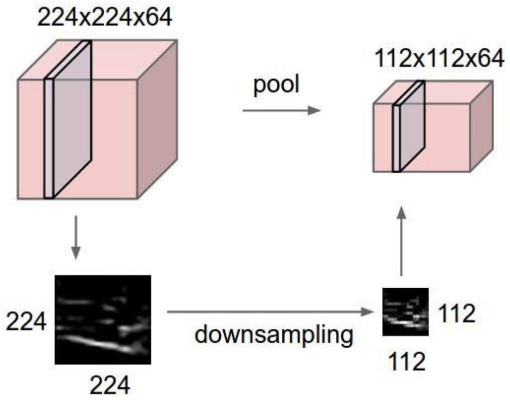
Components of a Convolutional Network

Convolution Layers

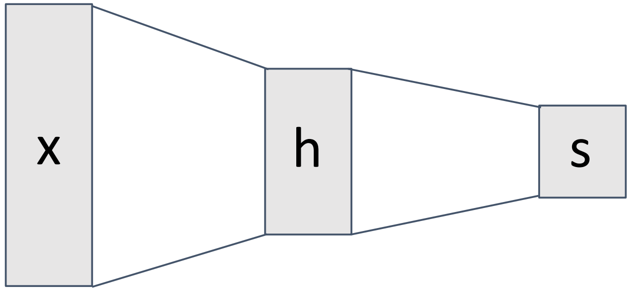


Most computationally expensive

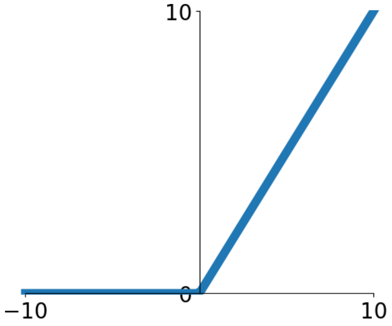
Pooling Layers



Fully-Connected Layers



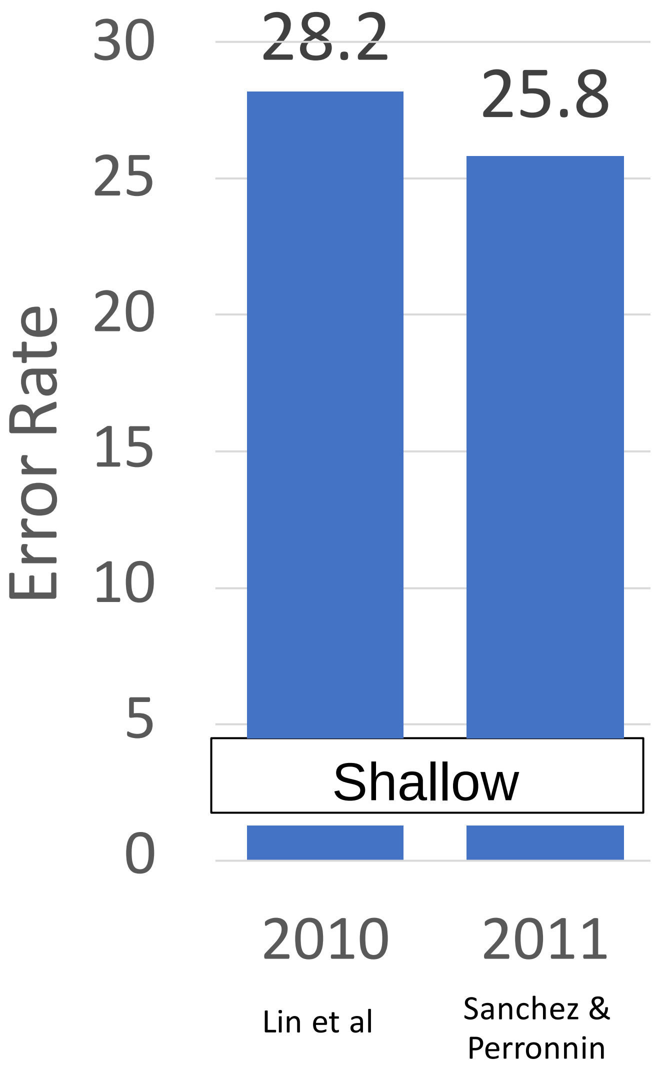
Activation Function



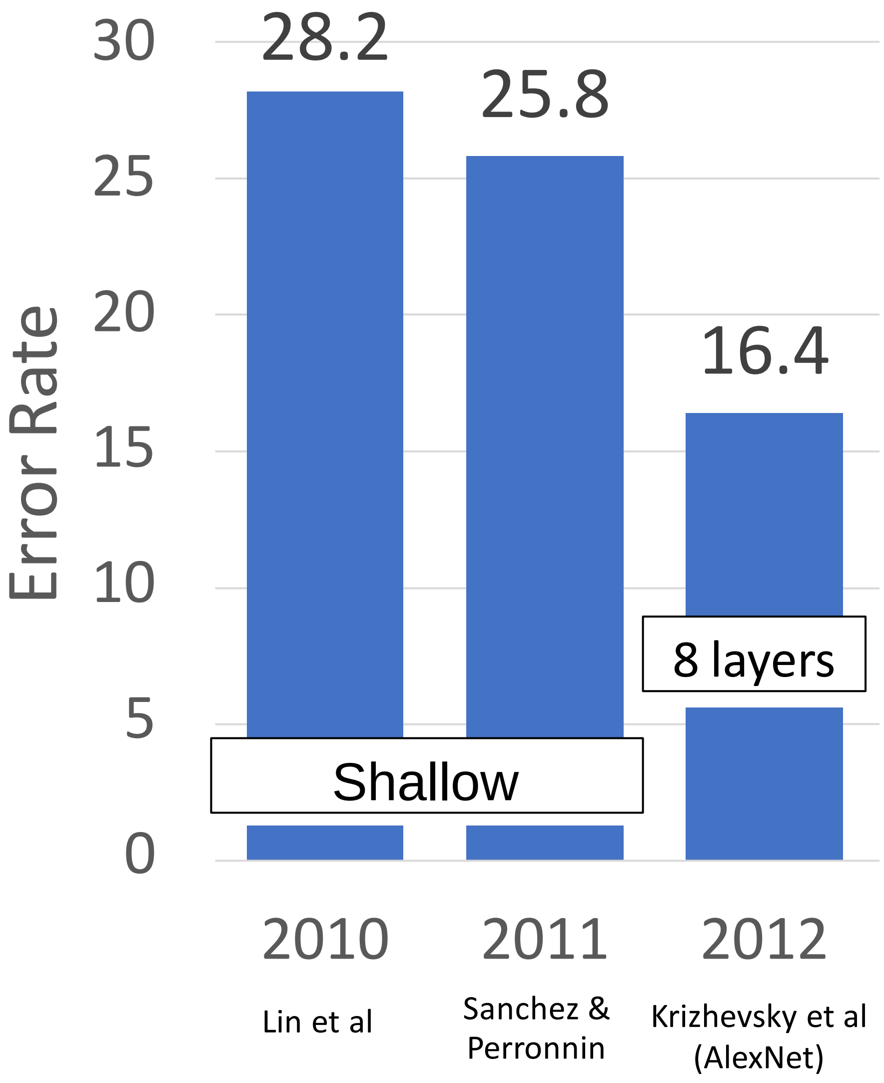
Normalization

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

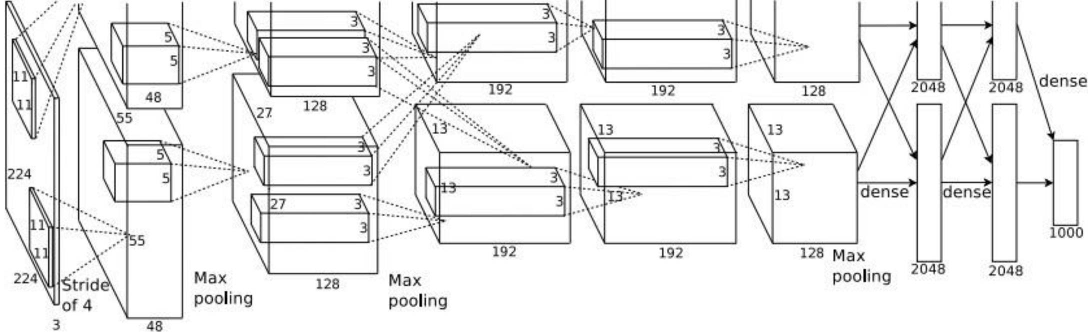
ImageNet Classification Challenge



ImageNet Classification Challenge

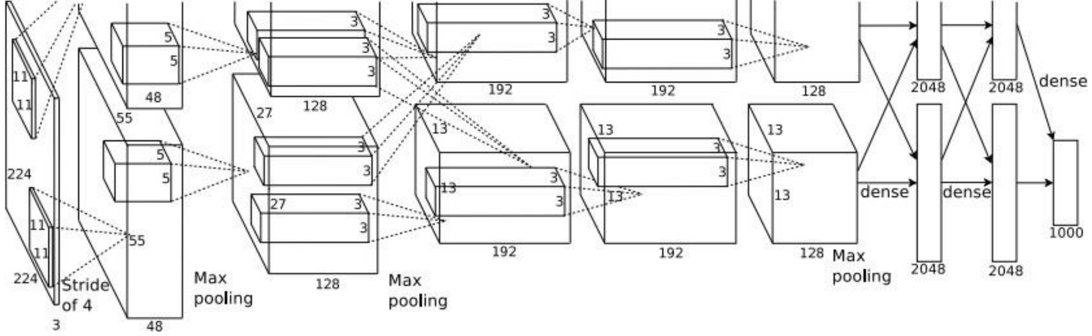


AlexNet



227 x 227 inputs
5 Convolutional layers
Max pooling
3 fully-connected layers
ReLU nonlinearities

AlexNet



227 x 227 inputs
5 Convolutional layers
Max pooling
3 fully-connected layers
ReLU nonlinearities

Used “Local response normalization”; Not used anymore

Trained on two GTX 580 GPUs – only 3GB of memory each – model split over two GPUs

AlexNet

	Input size		**Layer**				**Output size**	
Layer	C	H / W	filters	kernel	stride	pad	C	H / W
conv1	3	227	64	11	4	2		

AlexNet

	Input size		Layer				Output size	
Layer	C	H / W	filters	kernel	stride	pad	C	H / W
conv1	3	227	64	11	4	2	64	56

Recall: W' = (W – K + 2P) / S + 1

= (227 – 11 + 2*2) / 4 + 1

= 220/4 + 1 = 56

AlexNet

	Input size		**Layer**				**Output size**		
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)
conv1	3	227	64	11	4	2	64	56	

AlexNet

	Input size		Layer				Output size		
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)
conv1	3	227	64	11	4	2	64	56	784

Number of output elements = C * H' * W'

= 64*56*56 = 200,704

Bytes per element = 4 (for 32-bit floating point)

KB = (number of elements) * (bytes per elem) / 1024

= 200704 * 4 / 1024

= **784**

AlexNet

	Input size		**Layer**				**Output size**			
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)	params (k)
conv1	3	227	64	11	4	2	64	56	784	

AlexNet

	Input size		Layer				Output size			
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)	params (k)
conv1	3	227	64	11	4	2	64	56	784	23

Weight shape = C_{out} x C_{in} x K x K

= 64 x 3 x 11 x 11

Bias shape = C_{out} = 64

Number of weights = 64*3*11*11 + 64

= **23,296**

AlexNet

	Input size		**Layer**				**Output size**				
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)	params (k)	flop (M)
conv1	3	227	64	11	4	2	64	56	784	23	

AlexNet

	Input size		**Layer**				**Output size**				
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)	params (k)	flop (M)
conv1	3	227	64	11	4	2	64	56	784	23	73

Number of floating point operations (multiply+add)

= (number of output elements) * (ops per output elem)

= $(C_{out} \times H' \times W') * (C_{in} \times K \times K)$

= (64 * 56 * 56) * (3 * 11 * 11)

= 200,704 * 363

= **72,855,552**

AlexNet

	Input size		Layer				Output size				
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)	params (k)	flop (M)
conv1	3	227	64	11	4	2	64	56	784	23	73
pool1	64	56		3	2	0					

AlexNet

	Input size		**Layer**				**Output size**				
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)	params (k)	flop (M)
conv1	3	227	64	11	4	2	64	56	784	23	73
pool1	64	56		3	2	0	64	27			

For pooling layer:

#output channels = #input channels = 64

$$\begin{aligned} W' &= \text{floor}((W - K) / S + 1) \\ &= \text{floor}(53 / 2 + 1) = \text{floor}(27.5) = \mathbf{27} \end{aligned}$$

AlexNet

	Input size		Layer				Output size				
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)	params (k)	flop (M)
conv1	3	227	64	11	4	2	64	56	784	23	73
pool1	64	56		3	2	0	64	27	182		

#output elems = C_{out} x H’ x W’

Bytes per elem = 4

$$KB = C_{out} * H' * W' * 4 / 1024$$
$$= 64 * 27 * 27 * 4 / 1024$$
$$= \mathbf{182.25}$$

AlexNet

	Input size		Layer				Output size				
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)	params (k)	flop (M)
conv1	3	227	64	11	4	2	64	56	784	23	73
pool1	64	56		3	2	0	64	27	182	0	

Pooling layers have no learnable parameters!

AlexNet

	Input size		Layer				Output size				
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)	params (k)	flop (M)
conv1	3	227	64	11	4	2	64	56	784	23	73
pool1	64	56		3	2	0	64	27	182	0	0

Floating-point ops for pooling layer

= (number of output positions) * (flops per output position)

= $(C_{out} * H' * W') * (K * K)$

= $(64 * 27 * 27) * (3 * 3)$

= 419,904

= **0.4 MFLOP**

AlexNet

	Input size		**Layer**				**Output size**				
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)	params (k)	flop (M)
conv1	3	227	64	11	4	2	64	56	784	23	73
pool1	64	56		3	2	0	64	27	182	0	0
conv2	64	27	192	5	1	2	192	27	547	307	224
pool2	192	27		3	2	0	192	13	127	0	0
conv3	192	13	384	3	1	1	384	13	254	664	112
conv4	384	13	256	3	1	1	256	13	169	885	145
conv5	256	13	256	3	1	1	256	13	169	590	100
pool5	256	13		3	2	0	256	6	36	0	0
flatten	256	6					9216		36	0	0

AlexNet

	Input size		Layer				Output size				
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)	params (k)	flop (M)
conv1	3	227	64	11	4	2	64	56	784	23	73
pool1	64	56		3	2	0	64	27	182	0	0
conv2	64	27	192	5	1	2	192	27	547	307	224
pool2	192	27		3	2	0	192	13	127	0	0
conv3	192	13	384	3	1	1	384	13	254	664	112
conv4	384	13	256	3	1	1	256	13	169	885	145
conv5	256	13	256	3	1	1	256	13	169	590	100
pool5	256	13		3	2	0	256	6	36	0	0
flatten	256	6					9216		36	0	0

Flatten output size = C_{in} x H x W

= 256 * 6 * 6

= **9216**

AlexNet

	Input size		**Layer**				**Output size**				
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)	params (k)	flop (M)
conv1	3	227	64	11	4	2	64	56	784	23	73
pool1	64	56		3	2	0	64	27	182	0	0
conv2	64	27	192	5	1	2	192	27	547	307	224
pool2	192	27		3	2	0	192	13	127	0	0
conv3	192	13	384	3	1	1	384	13	254	664	112
conv4	384	13	256	3	1	1	256	13	169	885	145
conv5	256	13	256	3	1	1	256	13	169	590	100
pool5	256	13		3	2	0	256	6	36	0	0
flatten	256	6					9216		36	0	0
fc6	9216		4096				4096		16	37,749	38

FC params $= C_{in} \times C_{out} + C_{out}$
$= 9216 * 4096 + 4096$
$= 37{,}725{,}832$

FC flops $= C_{in} \times C_{out}$
$= 9216 * 6409$
$= 37{,}748{,}736$

AlexNet

	Input size		Layer				Output size				
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)	params (k)	flop (M)
conv1	3	227	64	11	4	2	64	56	784	23	73
pool1	64	56		3	2	0	64	27	182	0	0
conv2	64	27	192	5	1	2	192	27	547	307	224
pool2	192	27		3	2	0	192	13	127	0	0
conv3	192	13	384	3	1	1	384	13	254	664	112
conv4	384	13	256	3	1	1	256	13	169	885	145
conv5	256	13	256	3	1	1	256	13	169	590	100
pool5	256	13		3	2	0	256	6	36	0	0
flatten	256	6					9216		36	0	0
fc6	9216		4096				4096		16	37,749	38
fc7	4096		4096				4096		16	16,777	17
fc8	4096		1000				1000		4	4,096	4

AlexNet

	Input size		Layer				Output size				
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)	params (k)	flop (M)
conv1	3	227	64	11	4	2	64	56	784	23	73
pool1	64	56		3	2	0	64	27	182	0	0
conv2	64	27	192	5	1	2	192	27	547	307	224
pool2	192	27		3	2	0	192	13	127	0	0
conv3	192	13	384	3	1	1	384	13	254	664	112
conv4	384	13	256	3	1	1	256	13	169	885	145
conv5	256	13	256	3	1	1	256	13	169	590	100
pool5	256	13		3	2	0	256	6	36	0	0
flatten	256	6					9216		36	0	0
fc6	9216		4096				4096		16	37,749	38
fc7	4096		4096				4096		16	16,777	17
fc8	4096		1000				1000		4	4,096	4

Determined by trial and error

AlexNet

	Input size		**Layer**				**Output size**				
Layer	C	H / W	filters	kernel	stride	pad	C	H / W	memory (KB)	params (k)	flop (M)
conv1	3	227	64	11	4	2	64	56	784	23	73
pool1	64	56		3	2	0	64	27	182	0	0
conv2	64	27	192	5	1	2	192	27	547	307	224
pool2	192	27		3	2	0	192	13	127	0	0
conv3	192	13	384	3	1	1	384	13	254	664	112
conv4	384	13	256	3	1	1	256	13	169	885	145
conv5	256	13	256	3	1	1	256	13	169	590	100
pool5	256	13		3	2	0	256	6	36	0	0
flatten	256	6					9216		36	0	0
fc6	9216		4096				4096		16	37,749	38
fc7	4096		4096				4096		16	16,777	17
fc8	4096		1000				1000		4	4,096	4

Max pooling inexpensive

AlexNet

Most of the **memory usage** is in the early convolution layers

Nearly all **parameters** are in the fully-connected layers

Most **floating-point ops** occur in the convolution layers