Fitzhugh-Nagumo model

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Neuron

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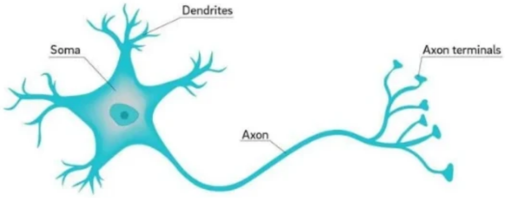
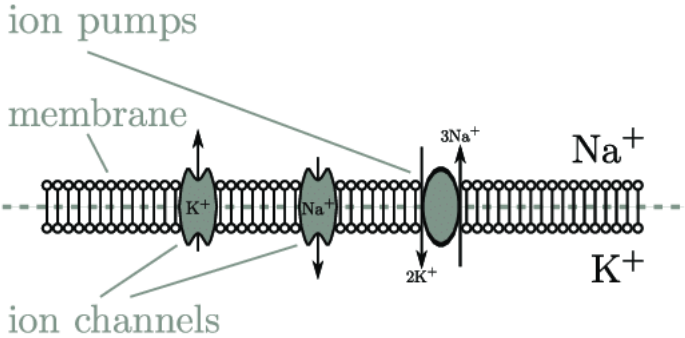
- Consists of a cell body (soma), dendrites, and a single axon.
- It can transmit electrical signals, called impulses, which travel in one direction.

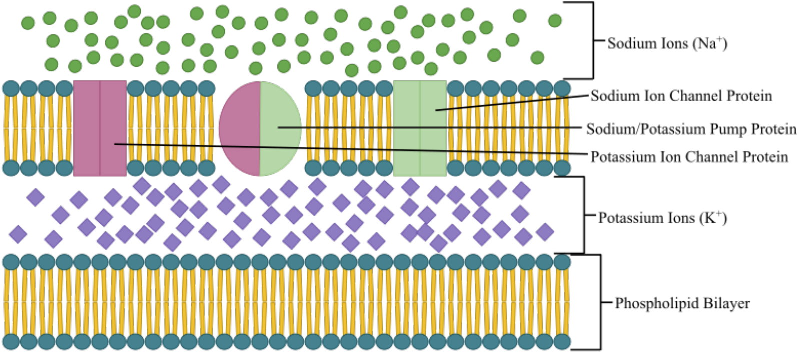


Membrane

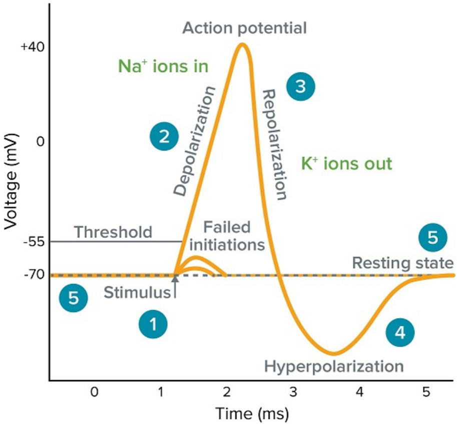
- Composed of a lipid bilayer with proteins embedded in it.



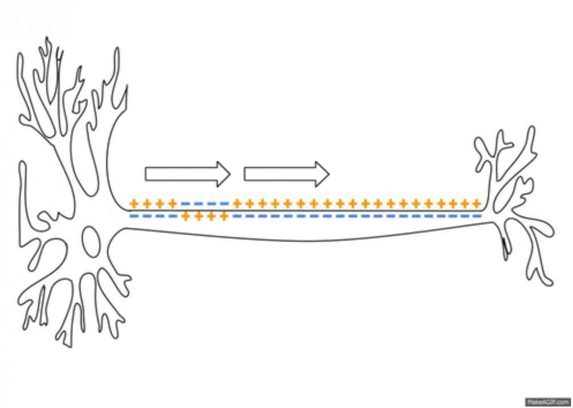
Resting potential



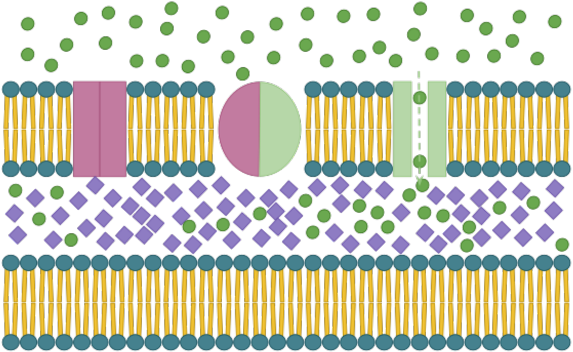
Threshold


Action potential
+40
Na+ ions in
3
2
Depolarization
Repolarization
0
K+ ions out
Voltage (mV)
Failed
initiations
Threshold
-55
5
Resting state
-70
Stimulus
5
1
4
Hyperpolarization
0
1
2
3
4
5
Time (ms)

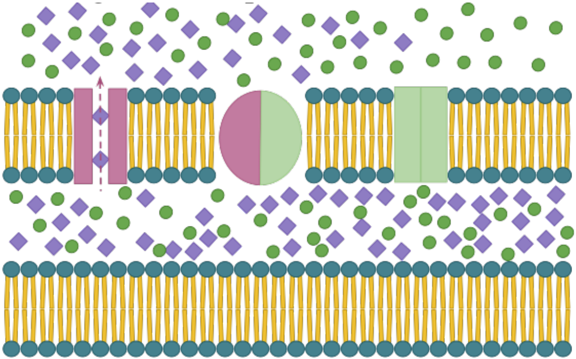
Action potential



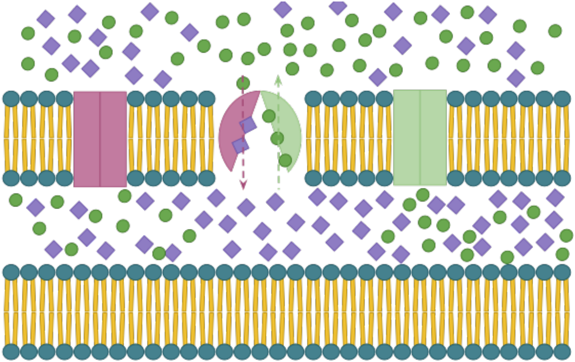
Depolarization



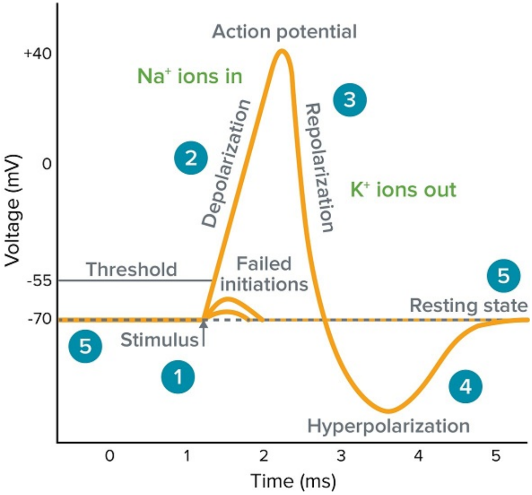
Repolarization



Hyperpolarization

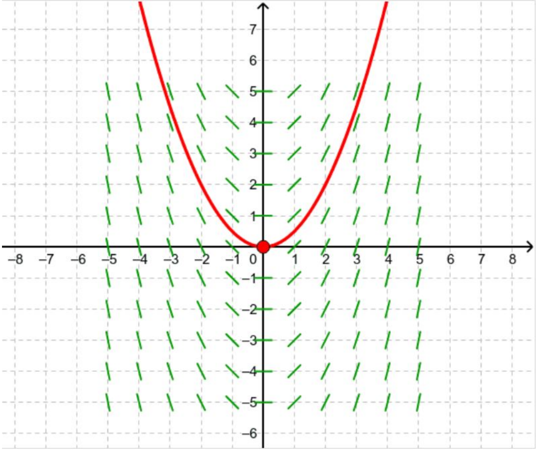


Neuron spike

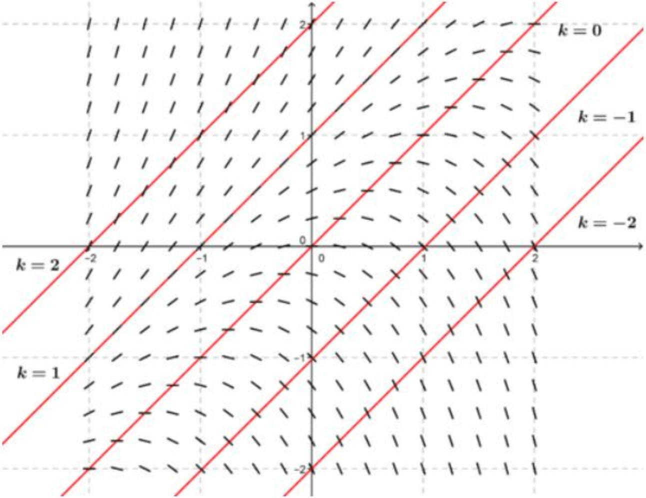


Slope fields

"Tiny little tangent lines"



Isoclines



Phase diagrams

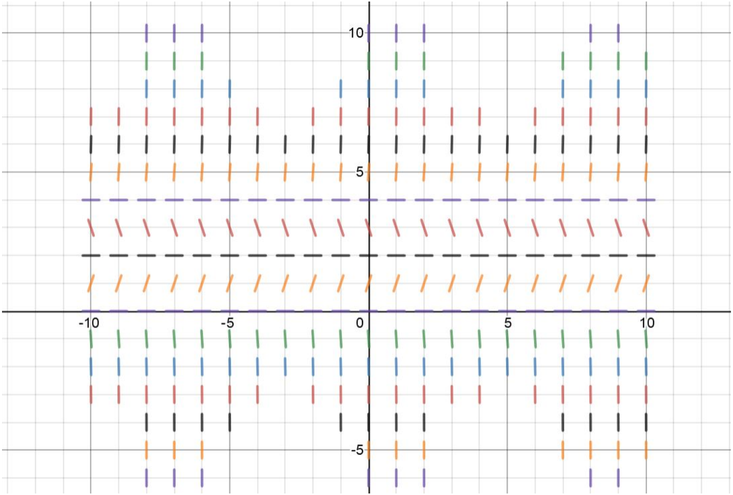
Phase diagrams are a way to visualise solutions to **autonomous** ODE

autonomous: slope doesn't change from left to right

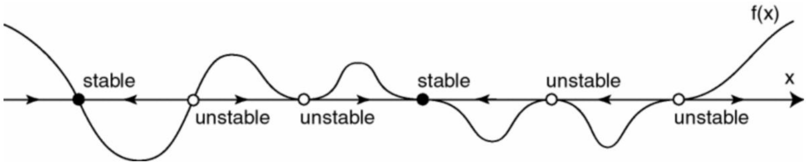
Let g(x,t)=dx/dt

$$g(t,x) = x(2-x)(4-x)$$

Phase diagrams



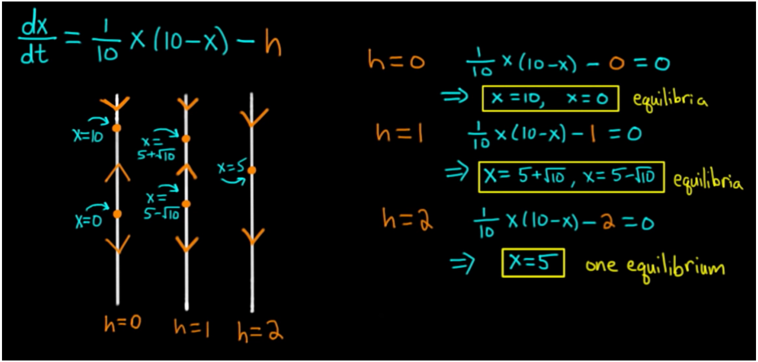
Phase portrait



Bifurcation

A bifurcation occurs when a small smooth change made to the parameter values (the bifurcation parameters) of a system causes a sudden 'qualitative' or topological change in its behavior.

Bifurcation



The old way: Hodgkin-Huxley model

- FutzHugh-Nagumo model is a simplification of Hodgkin-Huxley model of spike generations in aquid giant axions;

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  $$C_m \frac{dV_m}{dt} + g_K(V_m - V_K) + g_Na(V_m - V_Na) + g_l(V_m - V_l)$$

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- FutzHugh-Nagumo model is a simplification of Hodgkin-Huxley model of spike generations in aquid giant axions;
- The equations:

$$C_m \frac{dV_m}{dt} + g_K(V_m - V_K) + g_N a(V_m - V_N a) + g_l(V_m - V_l)$$

$$\frac{dn}{dt} = \alpha_n(V_m)(1-n) - \beta_n(V_m)n$$

$$\frac{dm}{dt} = \alpha_m(V_m)(1-m) - \beta_m(V_m)m$$

$$\frac{dh}{dt} = \alpha_h(V_m)(1-h) - \beta_h(V_m)h$$

Fitzhugh-Nagumo model

- Based on Van der Pol oscillator:

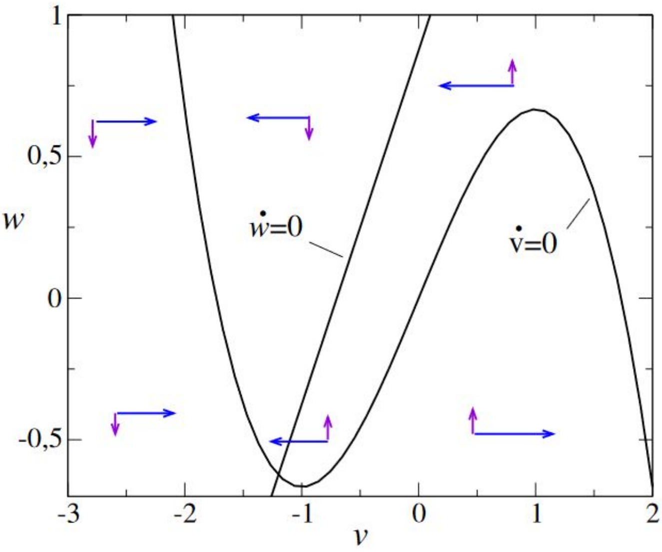
$$\frac{d^2x}{dt^2} + c(x^2 - 1)\frac{dx}{dt} + x = 0$$

- The Two Equations:

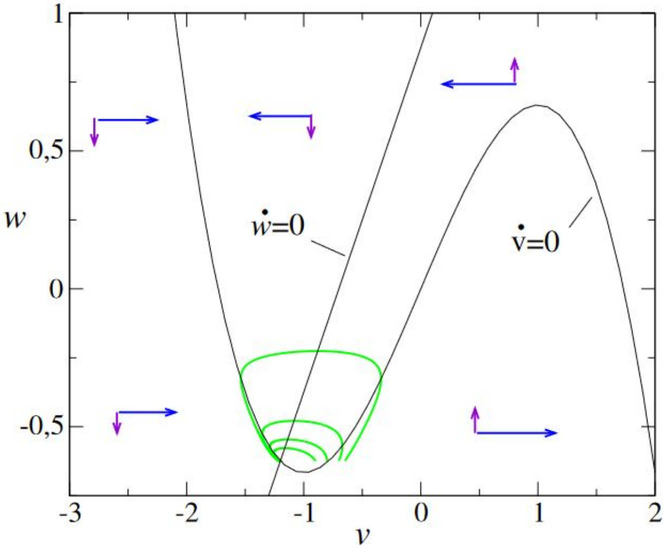
$$\frac{dV}{dt} = V - \frac{V^3}{3} - W + I$$

$$\frac{dW}{dt} = \frac{1}{c}(V + a - bW)$$

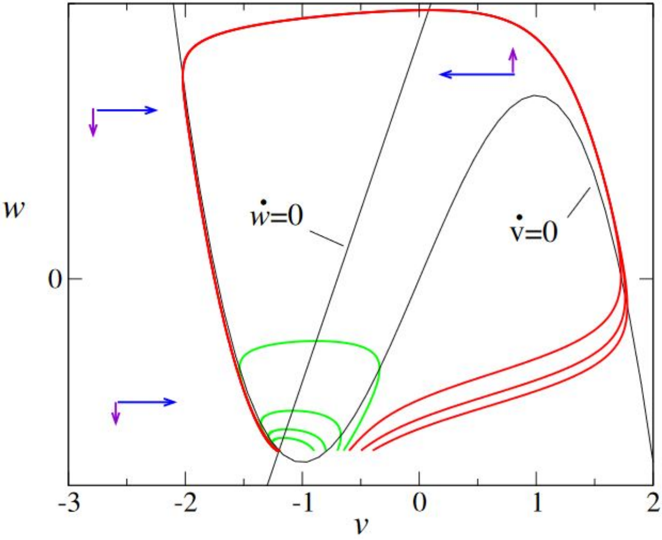
FitzHugh-Nagumo model: nullclines



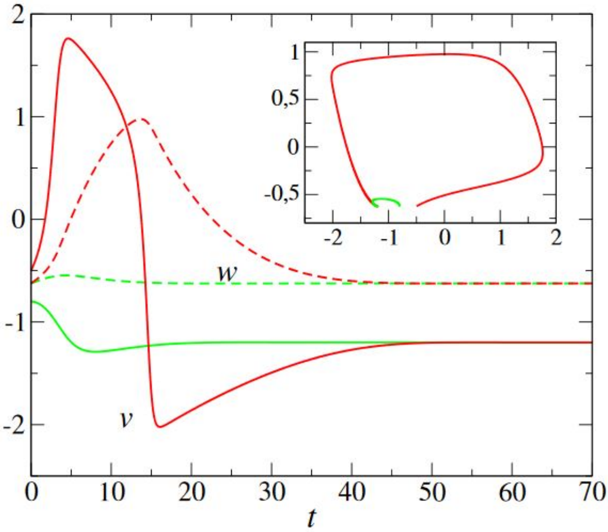
FitzHugh-Nagumo: weak pulse



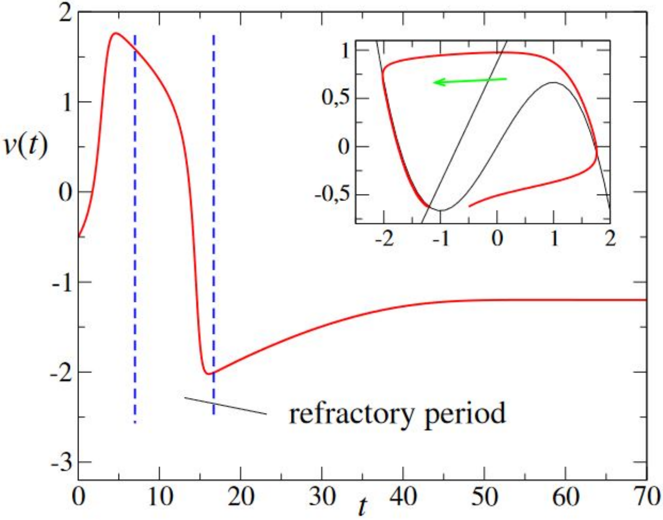
FitzHugh-Nagumo: strong pulse



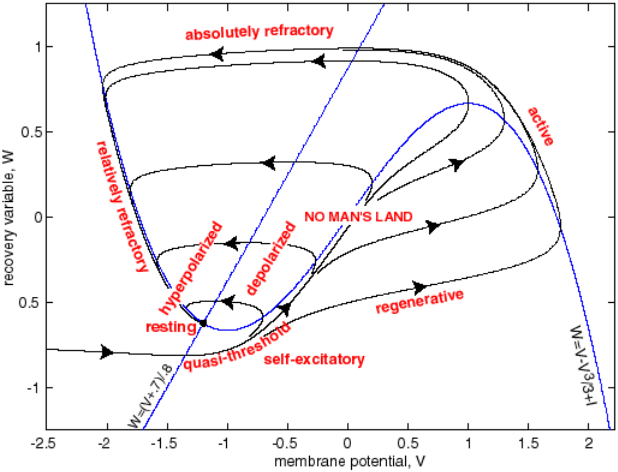
FitzHugh-Nagumo: spike response



FitzHugh-Nagumo: spike response



FutzHugh-Nagumo model: Physiological state diagram



FitzHugh-Nagumo: all-or-none spikes

See Yourself

Sources:

- Biological knowledge pt.1
- Biological knowledge pt.2
- Neuron
- Membrane
- Axon Propagation
- Neuron Spike
- Math 1
- Math 2
- Scholarpedia: FitzHugh-Nagumo model
- FitzHugh-Nagumo model
- Slope field graphic
- Isocline graphic
- Slope field Graphic
- Python code