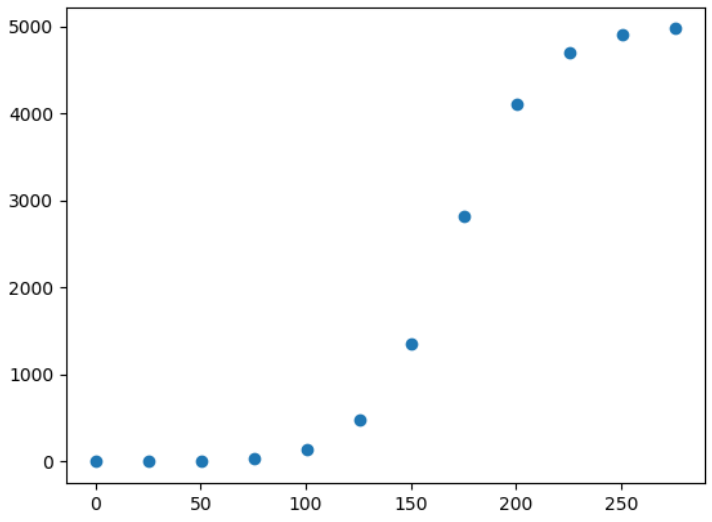
PFE 8

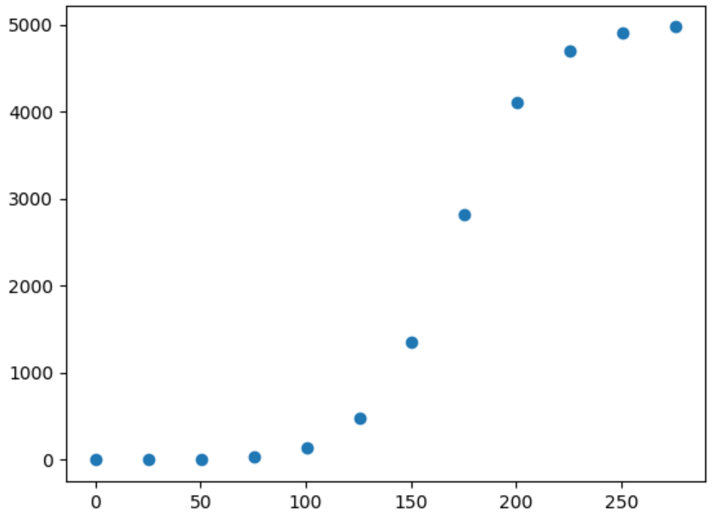
Palubicki

Data Models



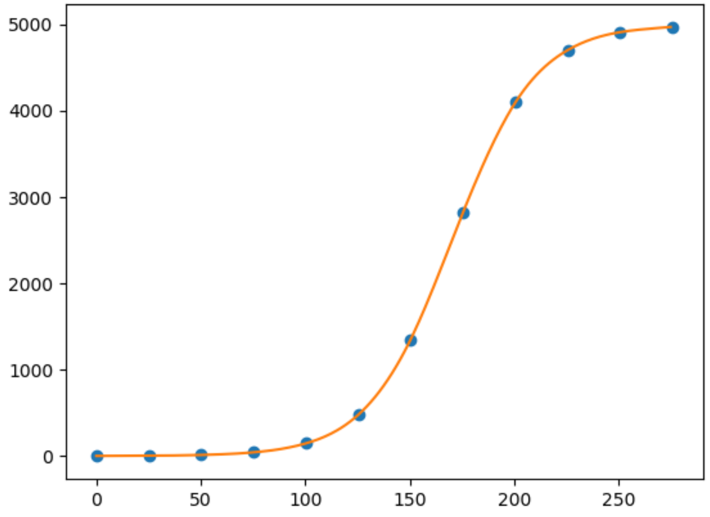
Data Sample

Data Models

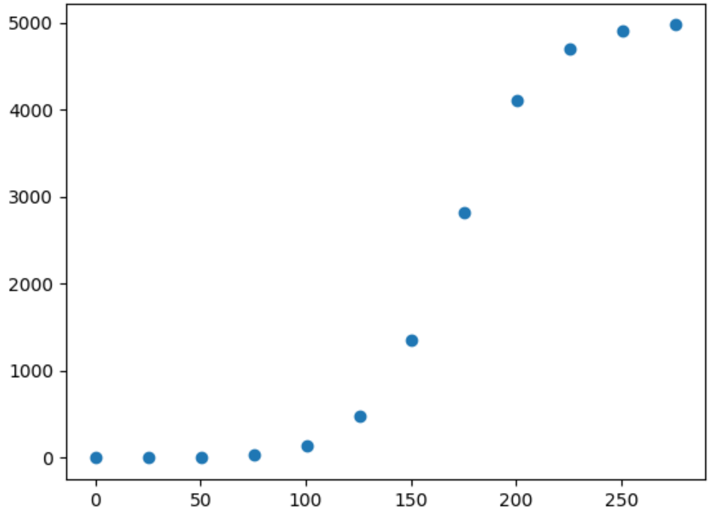


Data Sample

Interpolation



Data Models



Data Sample

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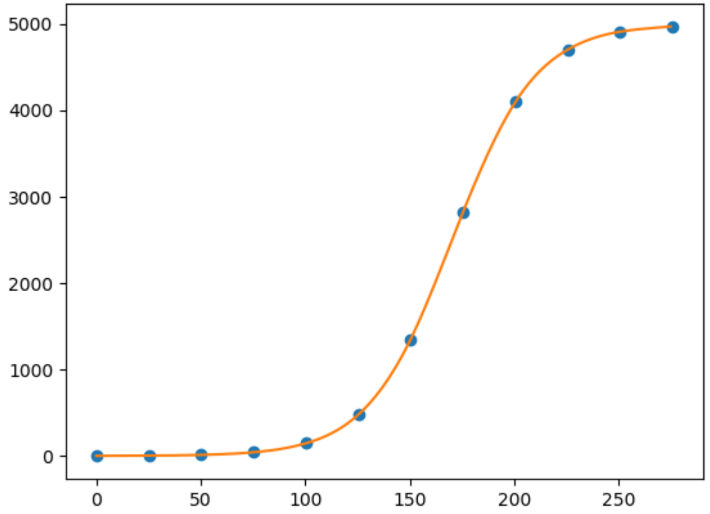
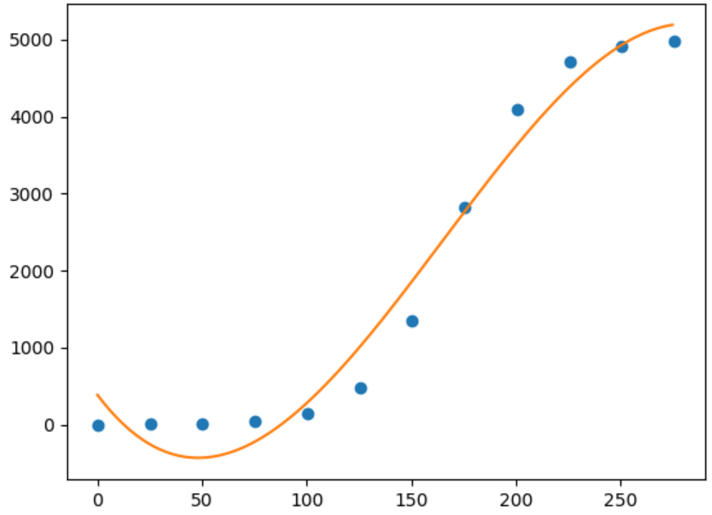
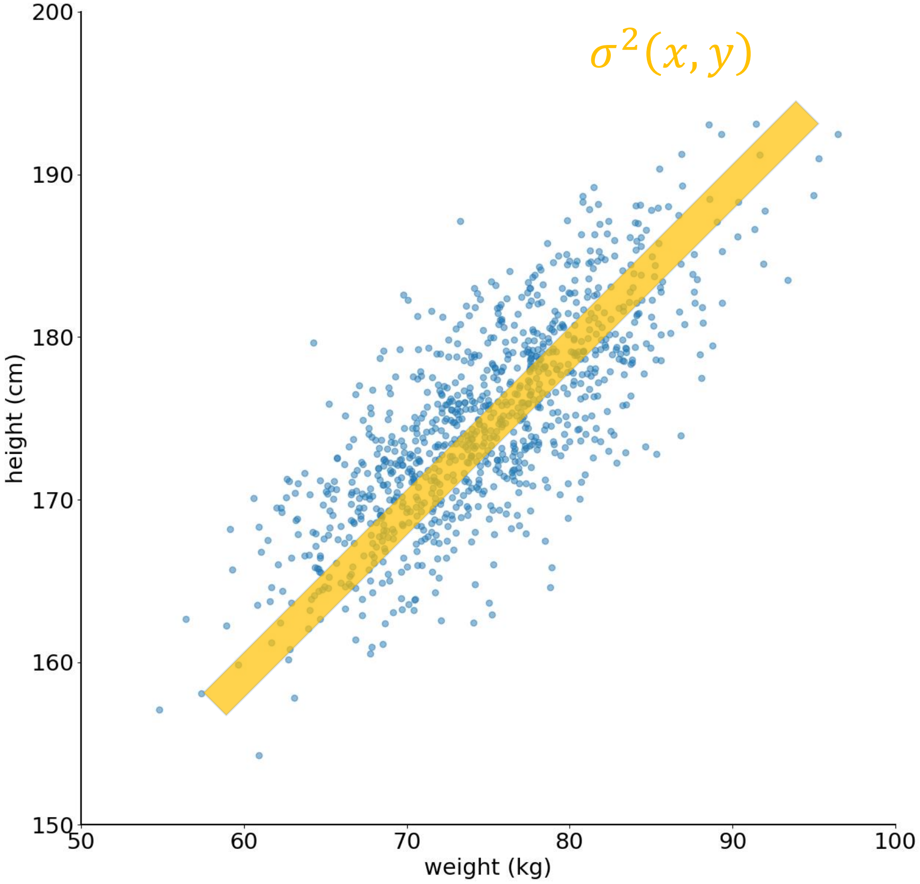


Approximation



Covariance

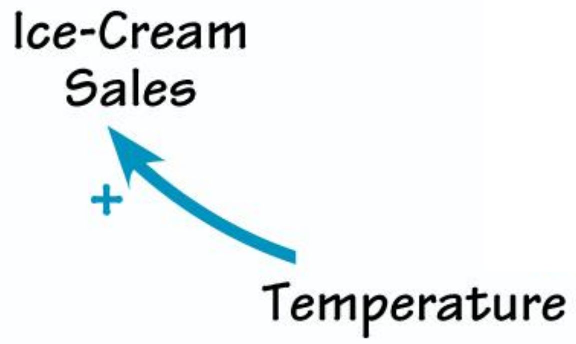
- Quantify correlations between variables
- E.g. Weight is correlated with height



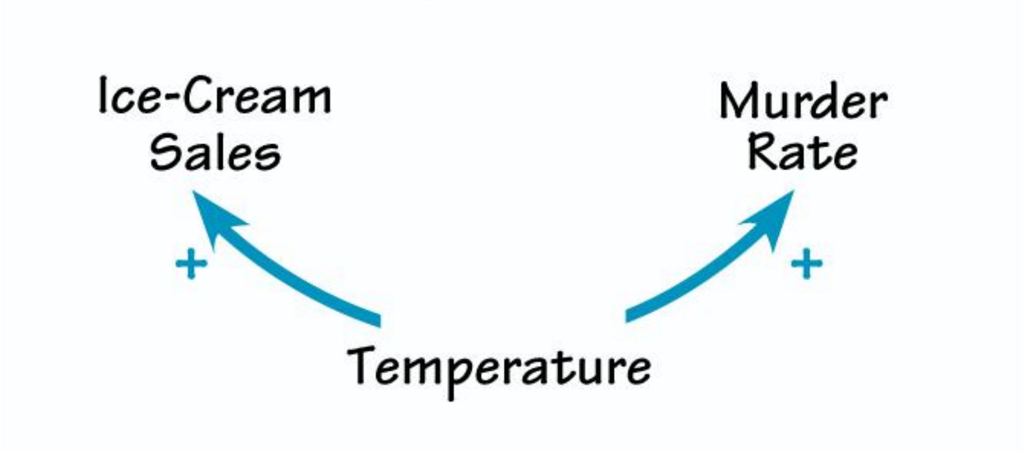
Causality vs. Correlation

- Causality is an **abstraction** that indicates how the world progresses.
- What causes to go from one state to another.
- It is an expressive method to describe patterns and essentially what we would like to extract from a good data model.

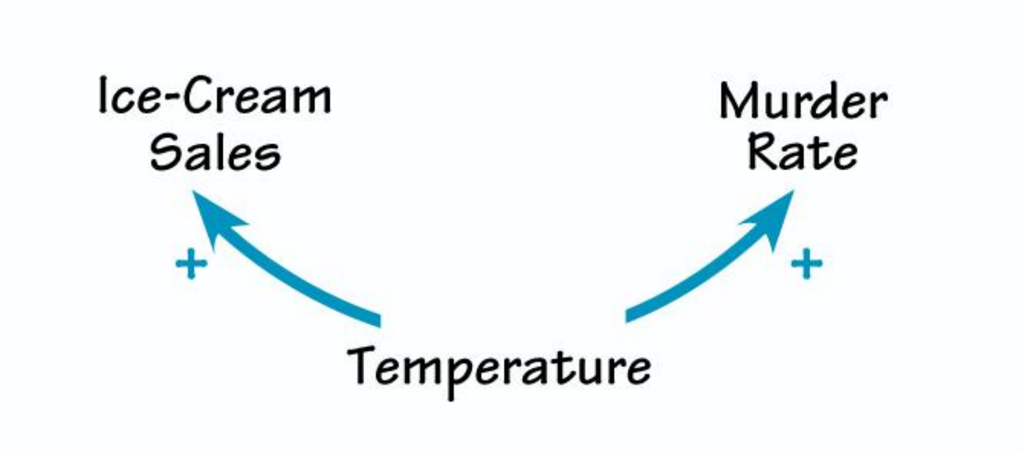
Causality vs. Correlation



Causality vs. Correlation



Causality vs. Correlation



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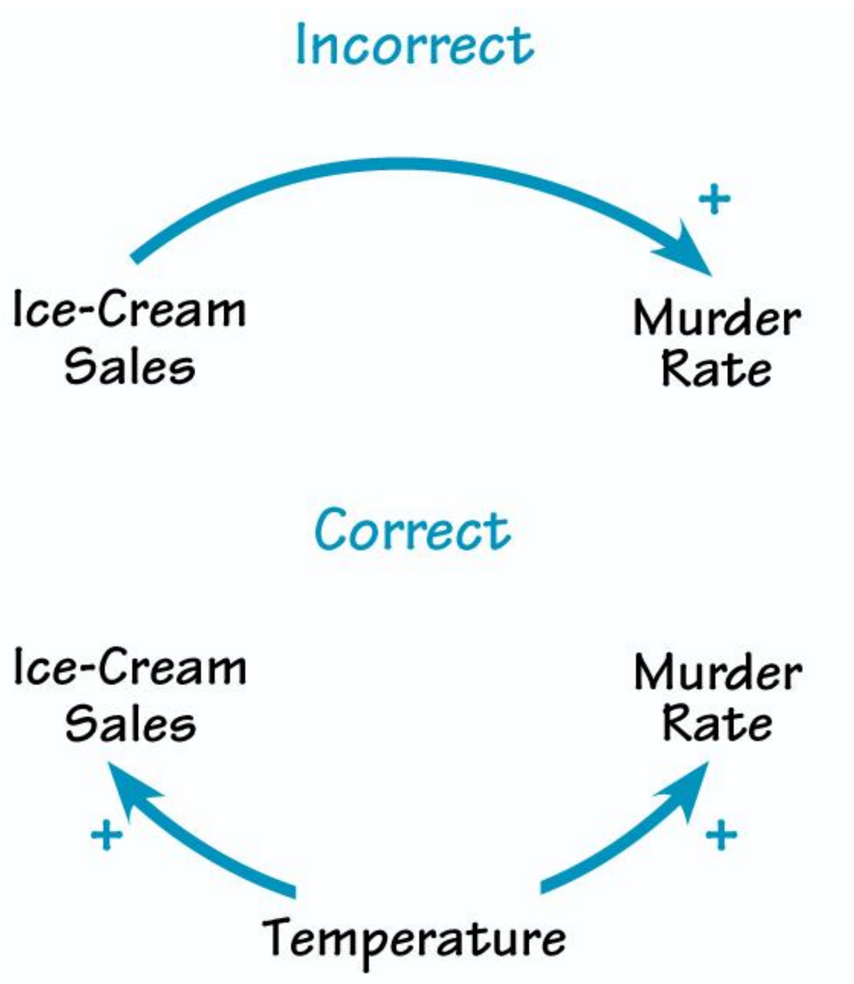
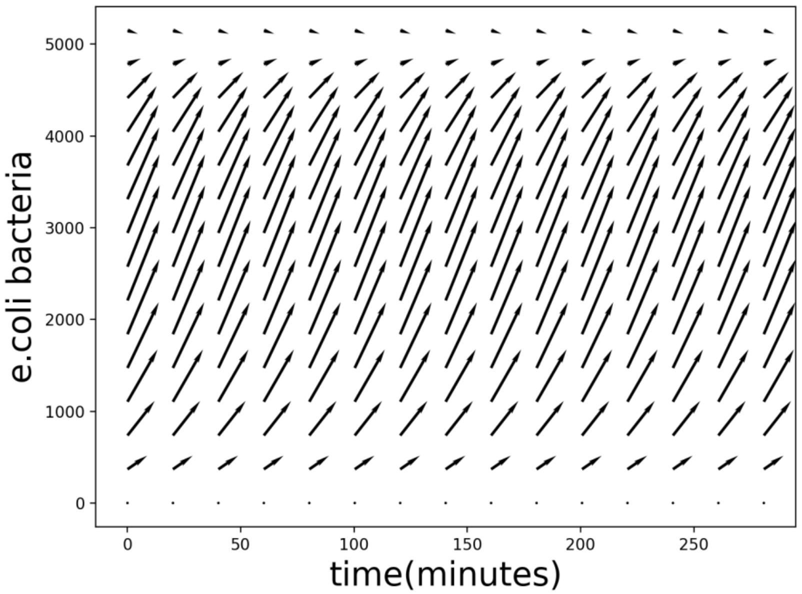
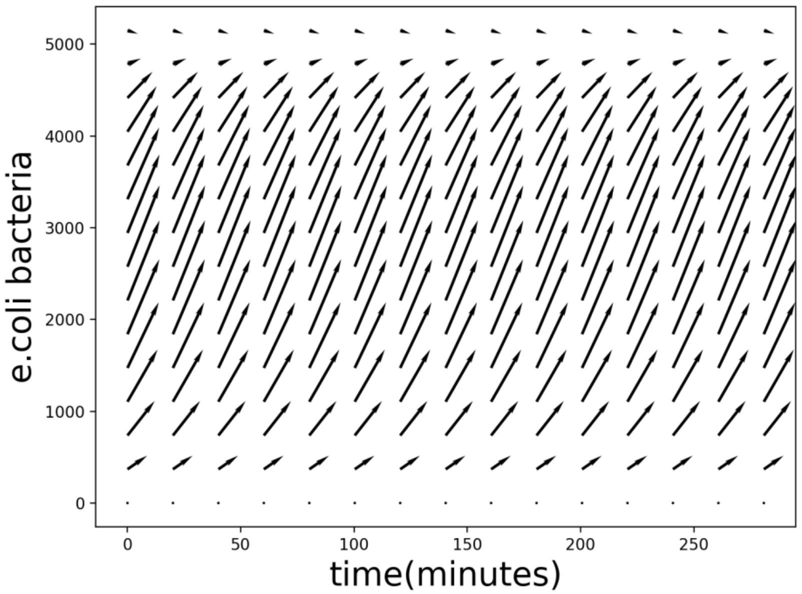


Differential Equations

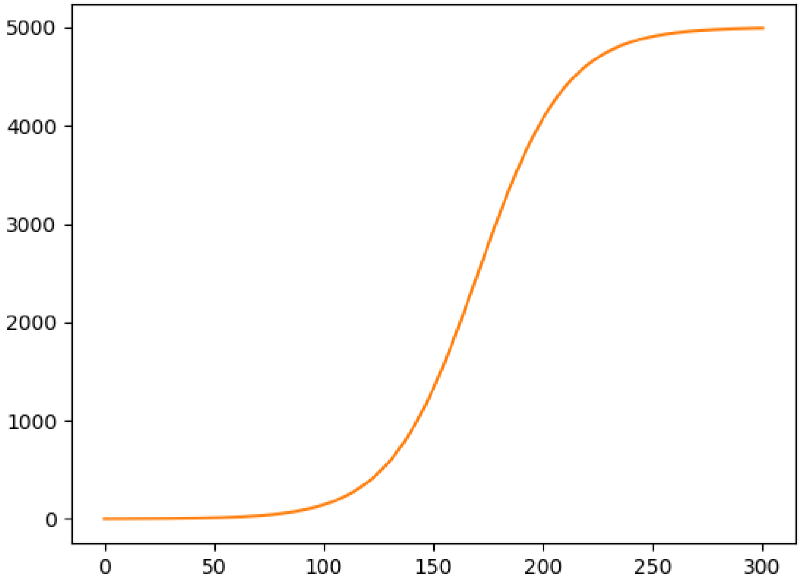


Differential Equations

Differential Equations

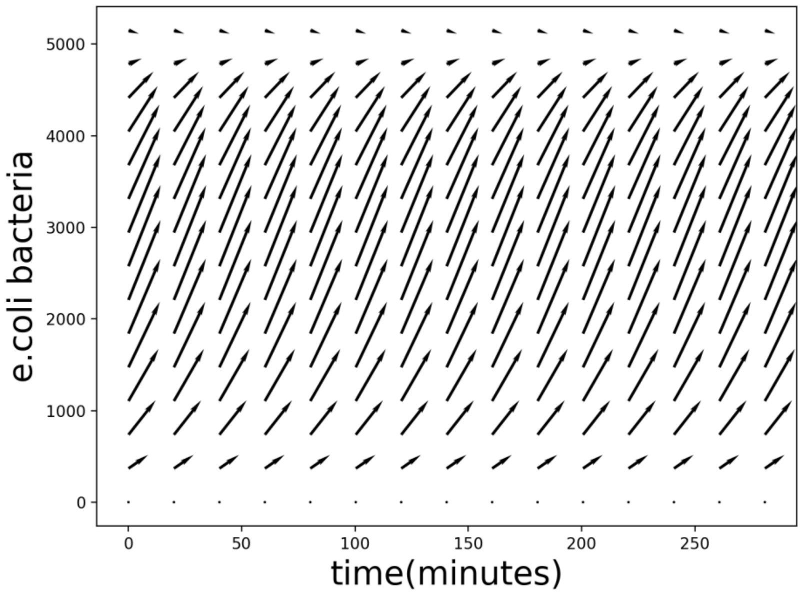


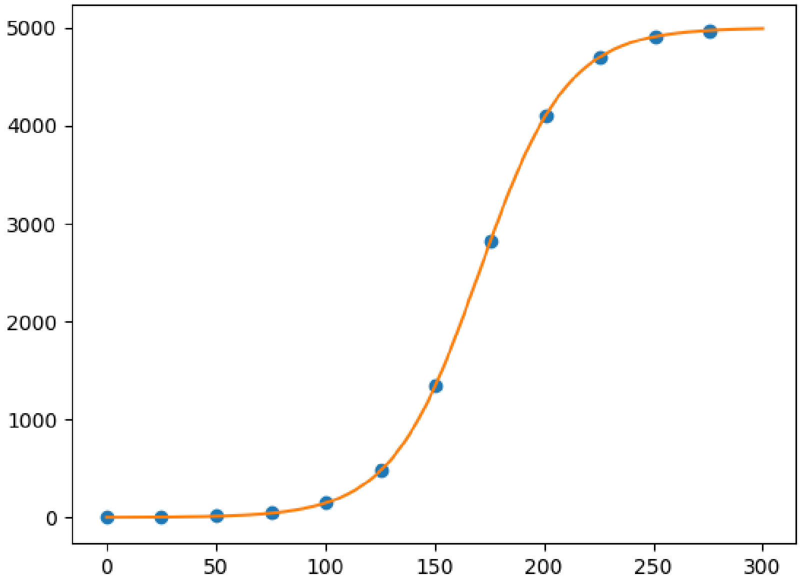
Differential Equations



Model

Differential Equations





Explicit causality

Population growth depends on whole population size but limited by food availability

Model

Can be evaluated with experimental data

SIR Model

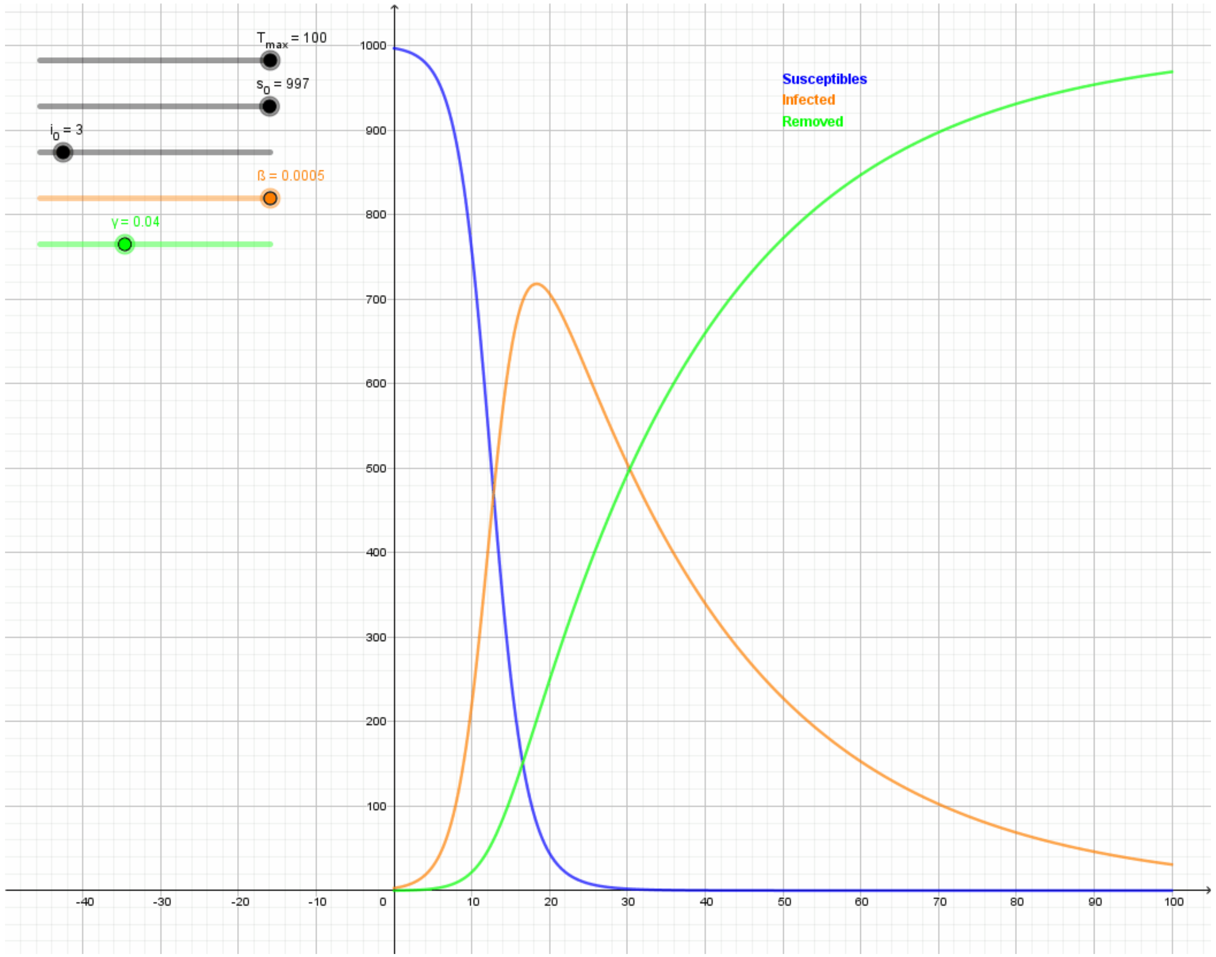
- **N:** total population
- **S(t):** number of people susceptible on day t
- **I(t):** number of people infected on day t
- **R(t):** number of people recovered on day t
- **β:** expected amount of people an infected person infects per day
- **D:** number of days an infected person has and can spread the disease
- **γ:** the proportion of infected recovering per day (γ = 1/D)

$$\frac{dS}{dt} = -\beta \cdot I \cdot \frac{S}{N}$$
$$\frac{dI}{dt} = \beta \cdot I \cdot \frac{S}{N} - \gamma \cdot I$$
$$\frac{dR}{dt} = \gamma \cdot I$$

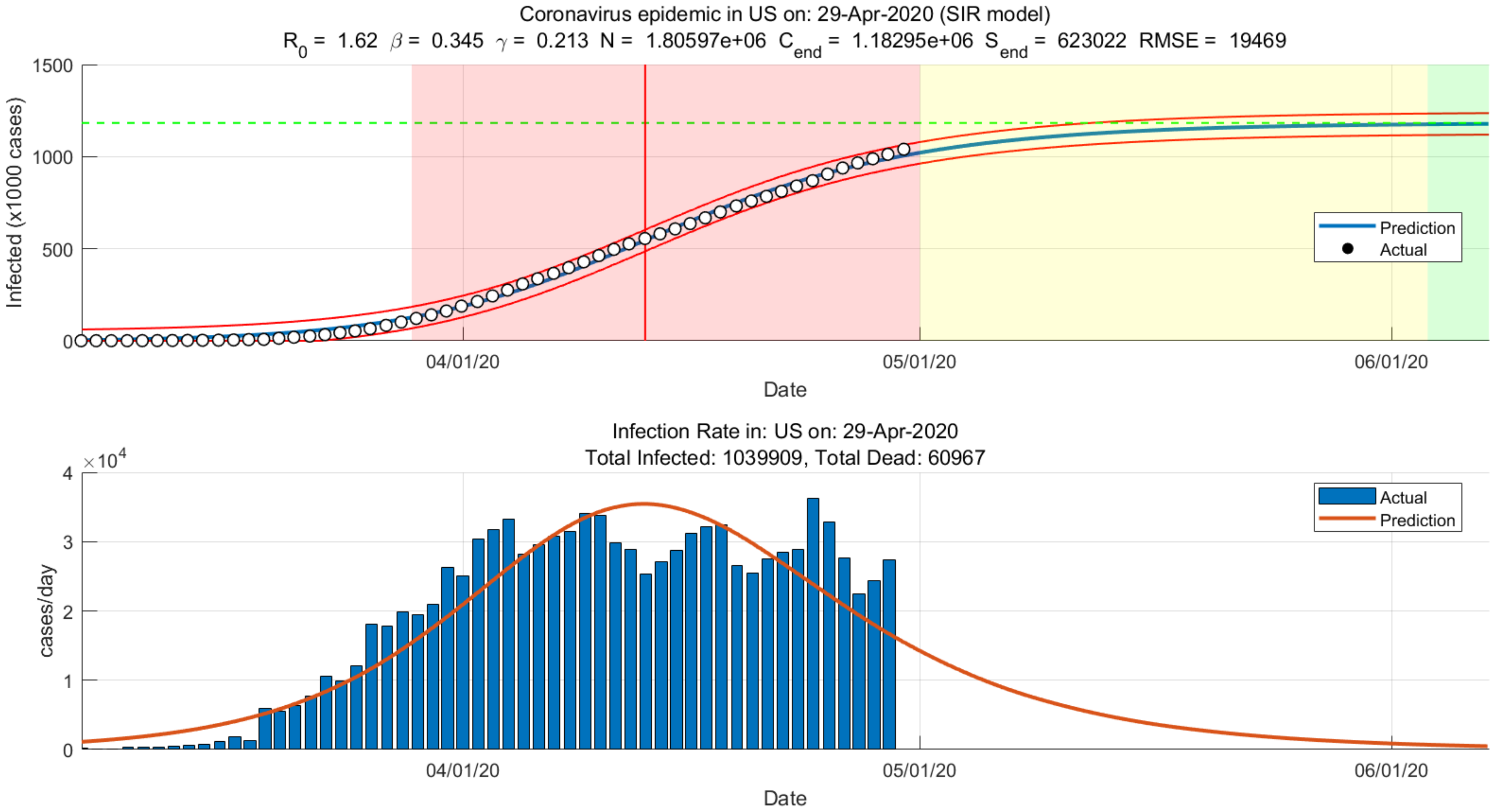
$$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0,$$

$$S(t) + I(t) + R(t) = \text{constant} = N,$$

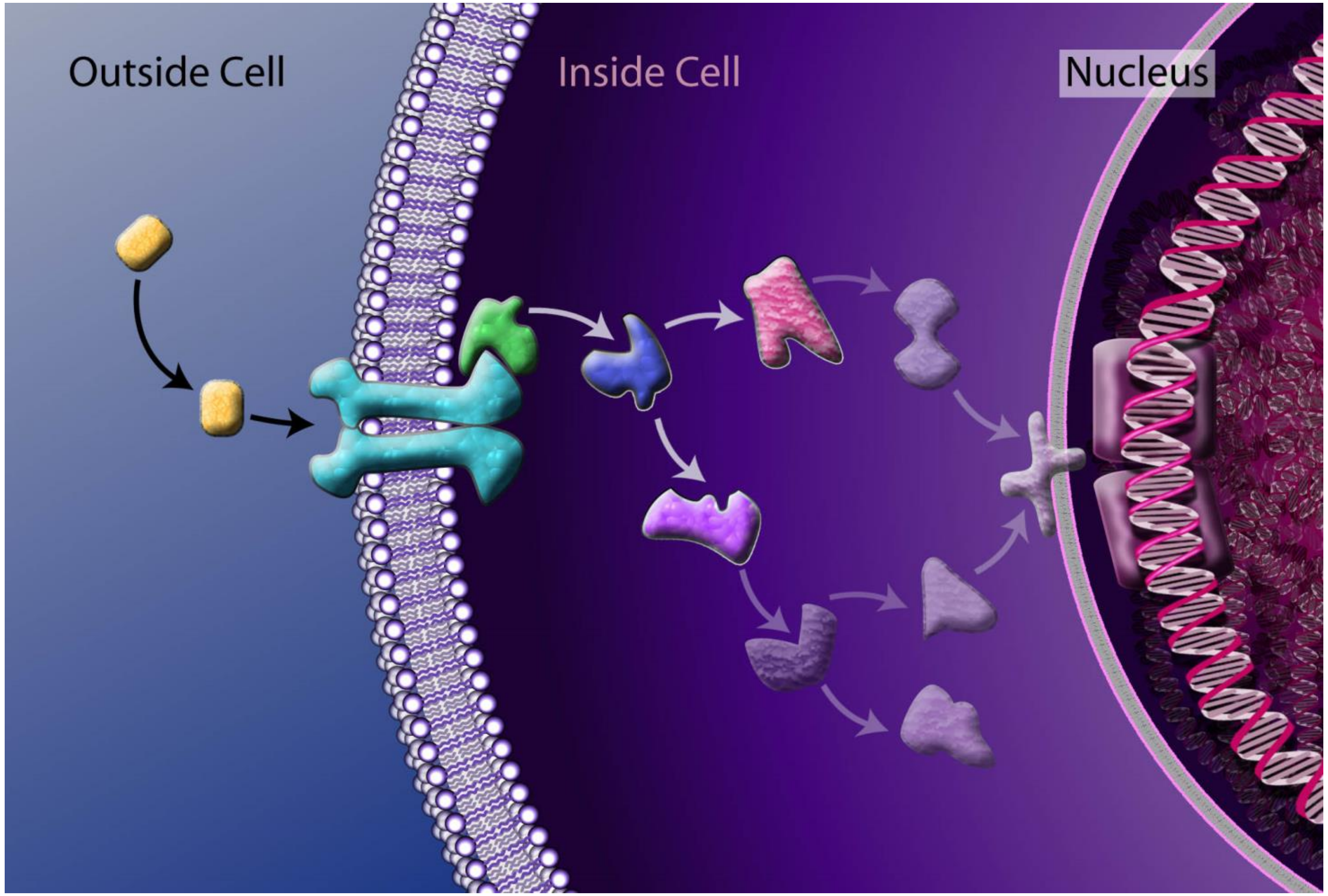
SIR Model Parameter Space Exploration



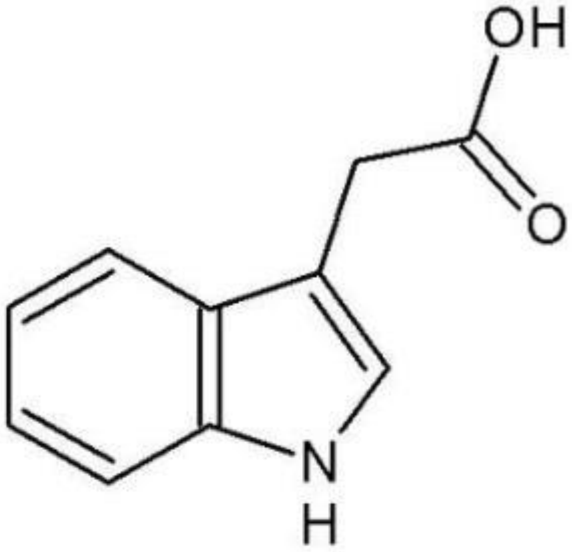
SIR Model Prediction



Information processing in organisms



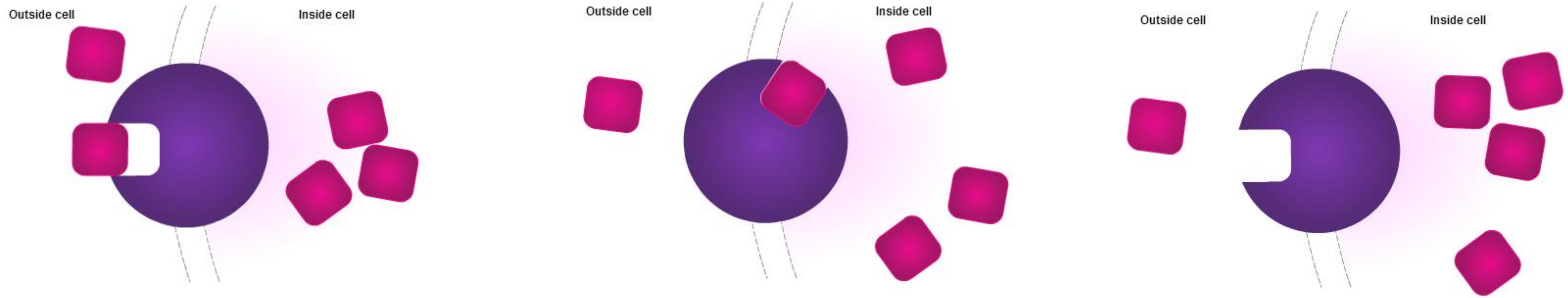
Signals



Hormones

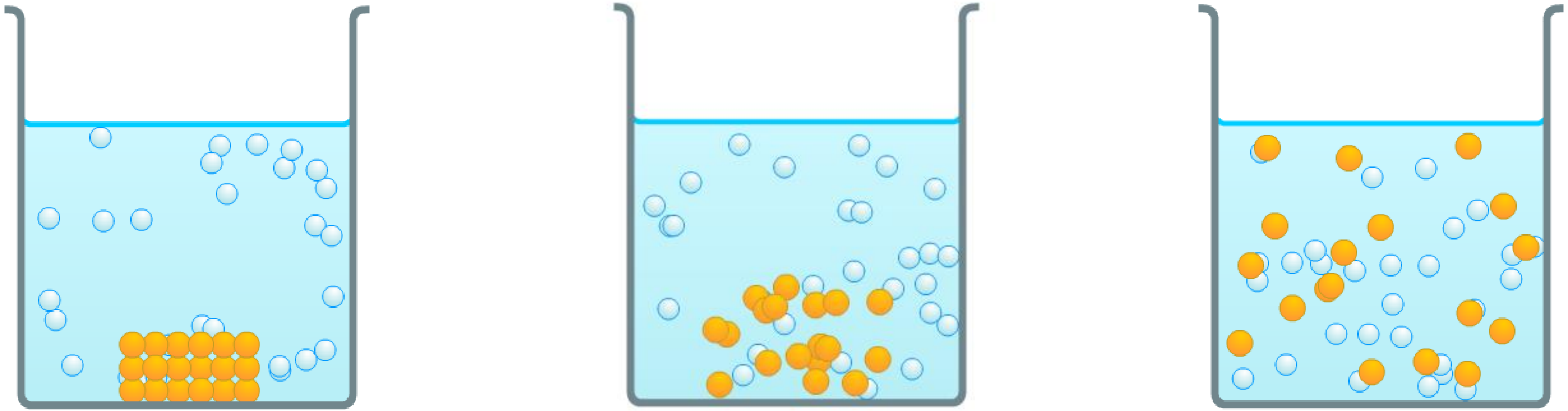
Proteins

Active/Passive transport



http://www.abpischools.org.uk/

Diffusion



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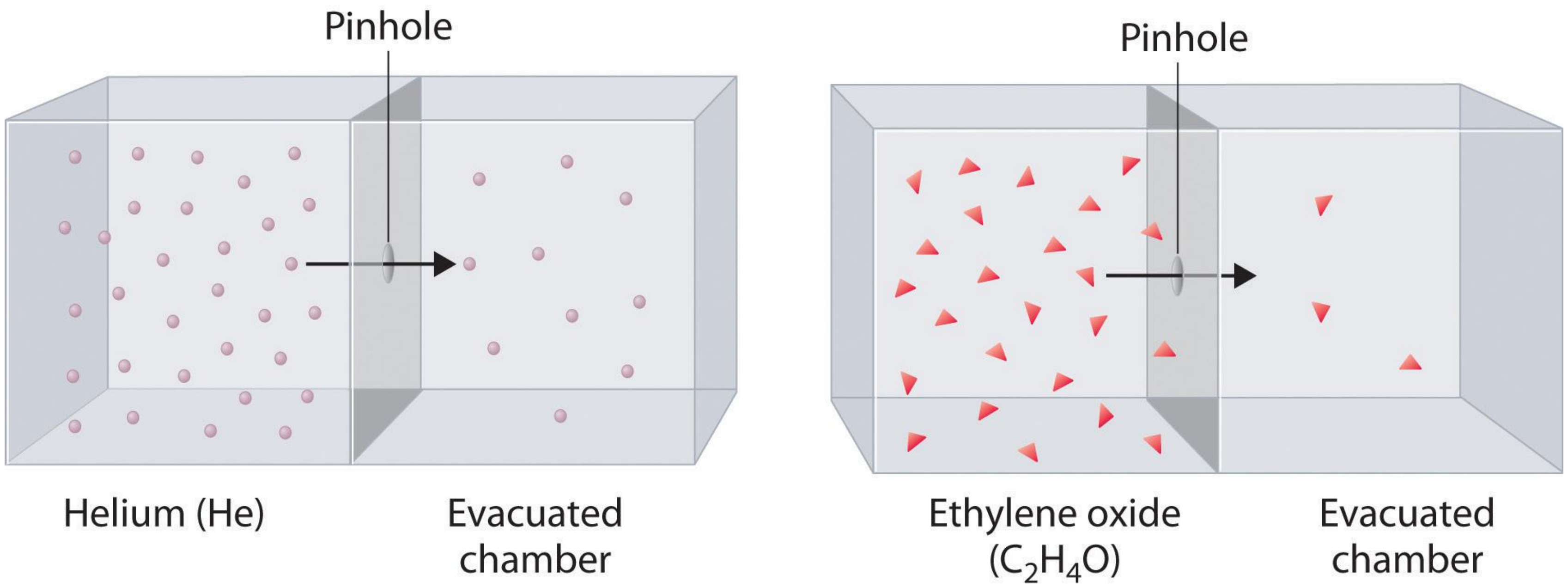
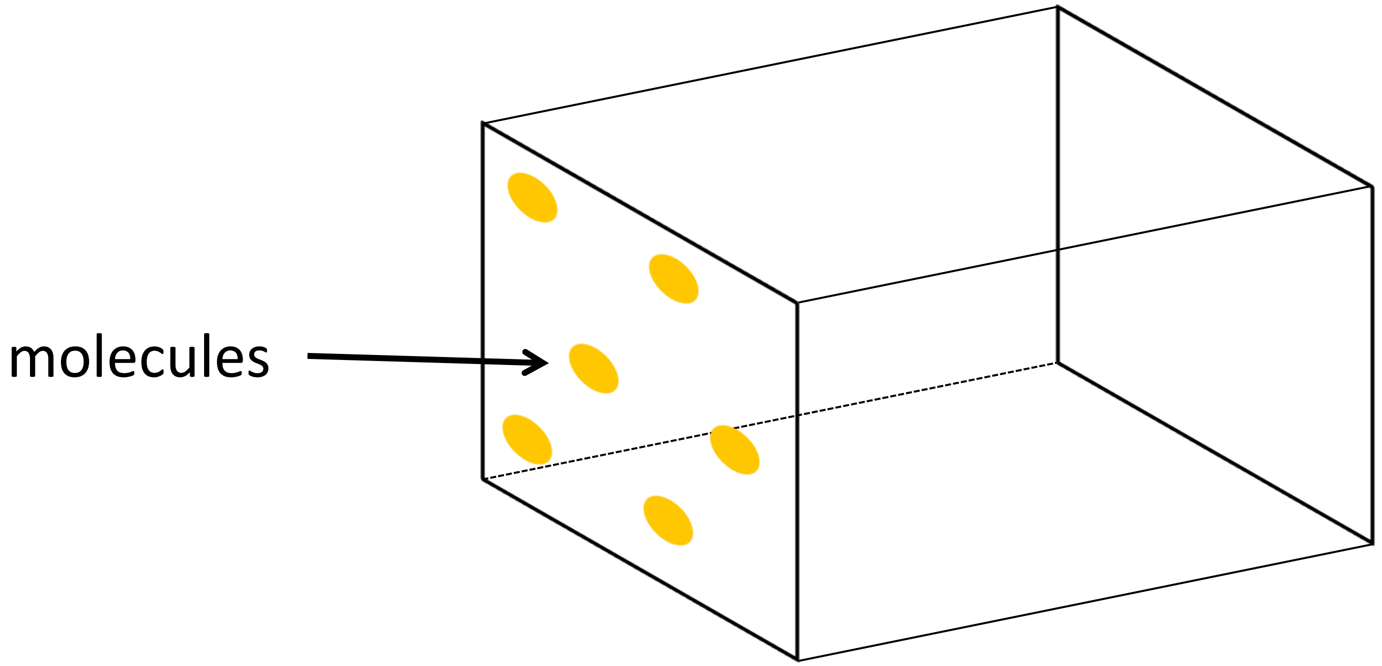
Grahams Law

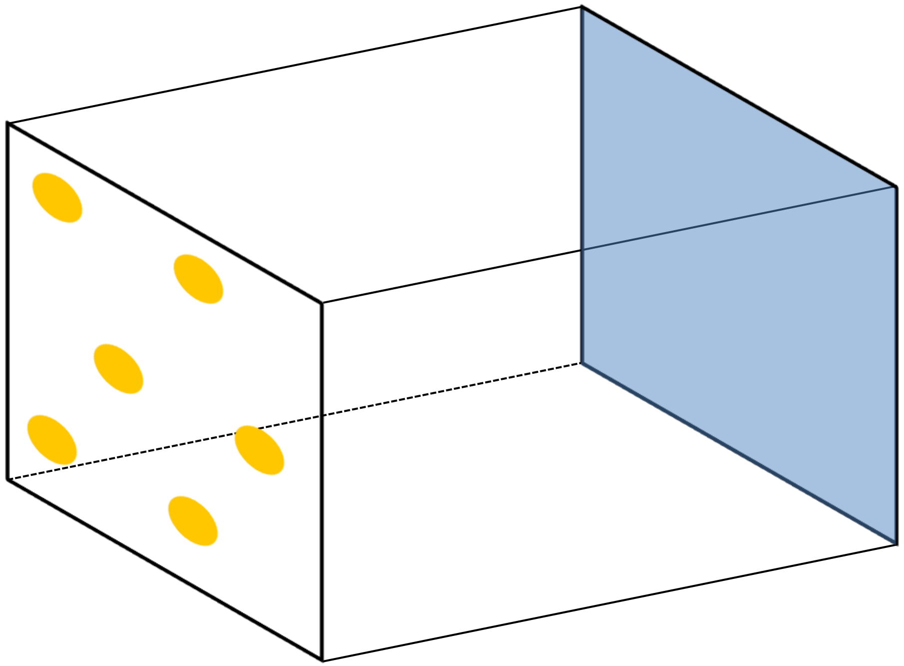


$$\frac{Rate_A}{Rate_B} = \sqrt{\frac{Molar\ Mass_B}{Molar\ Mass_A}}$$

Diffusion



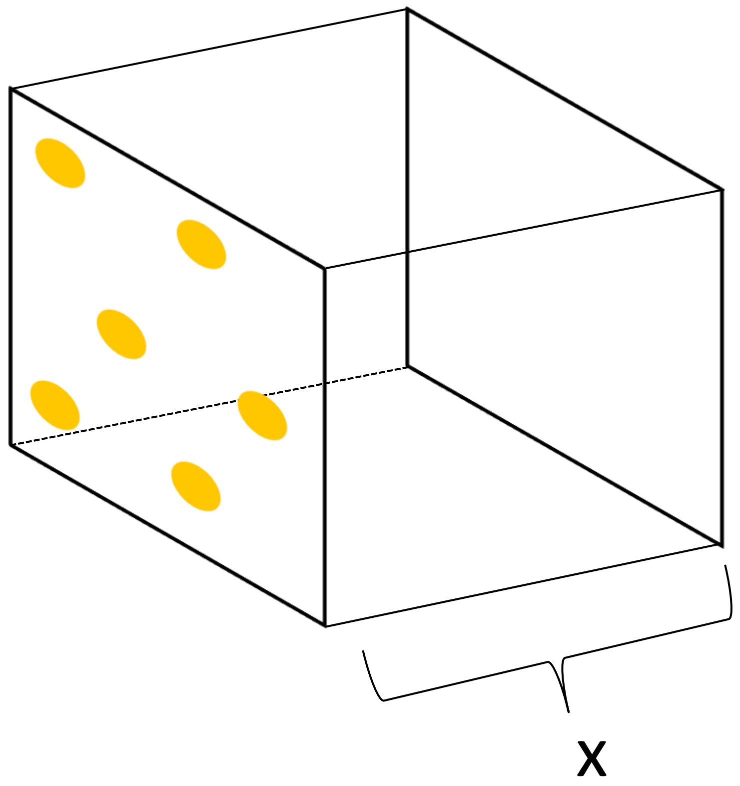
Diffusion



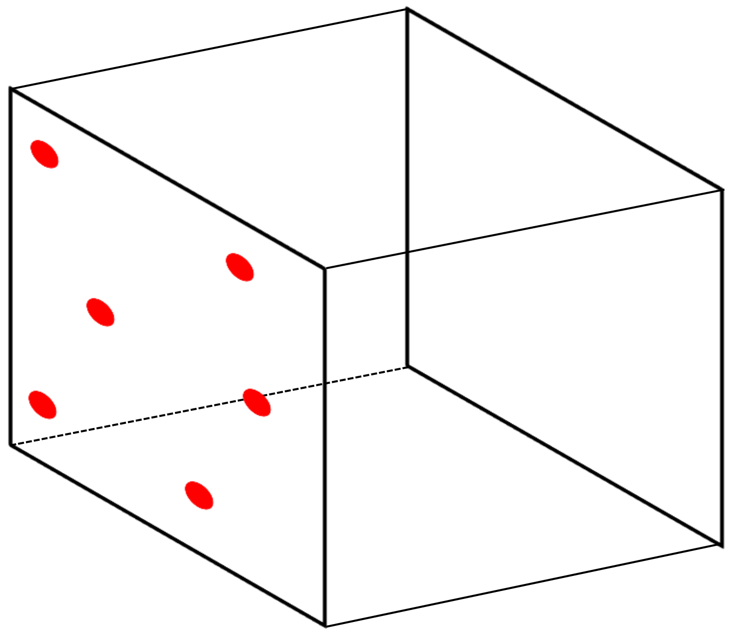
What parameters influence how many molecules move from the front side of the cube to the backside?

Diffusion

Distance x

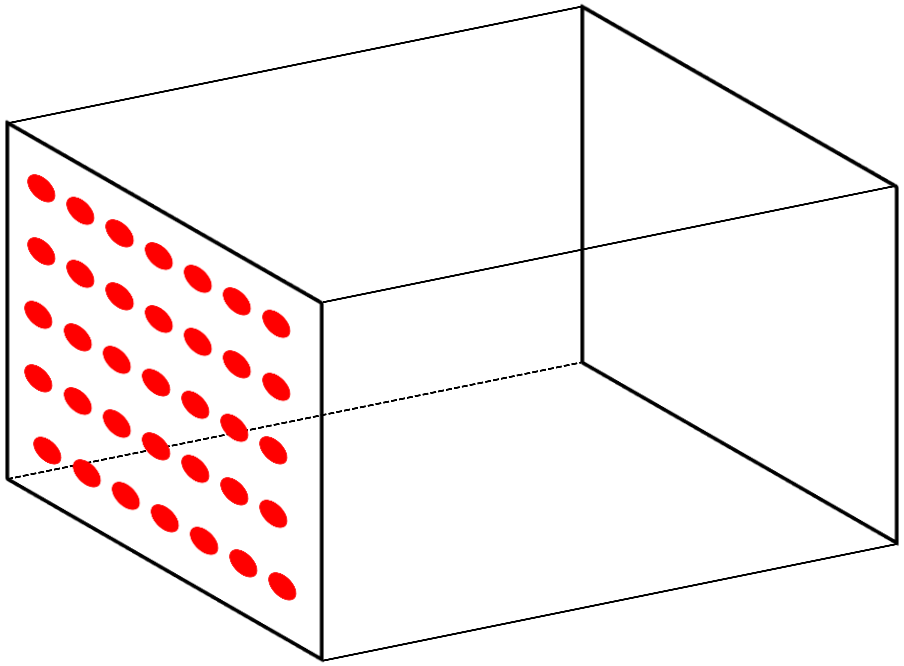


Diffusion



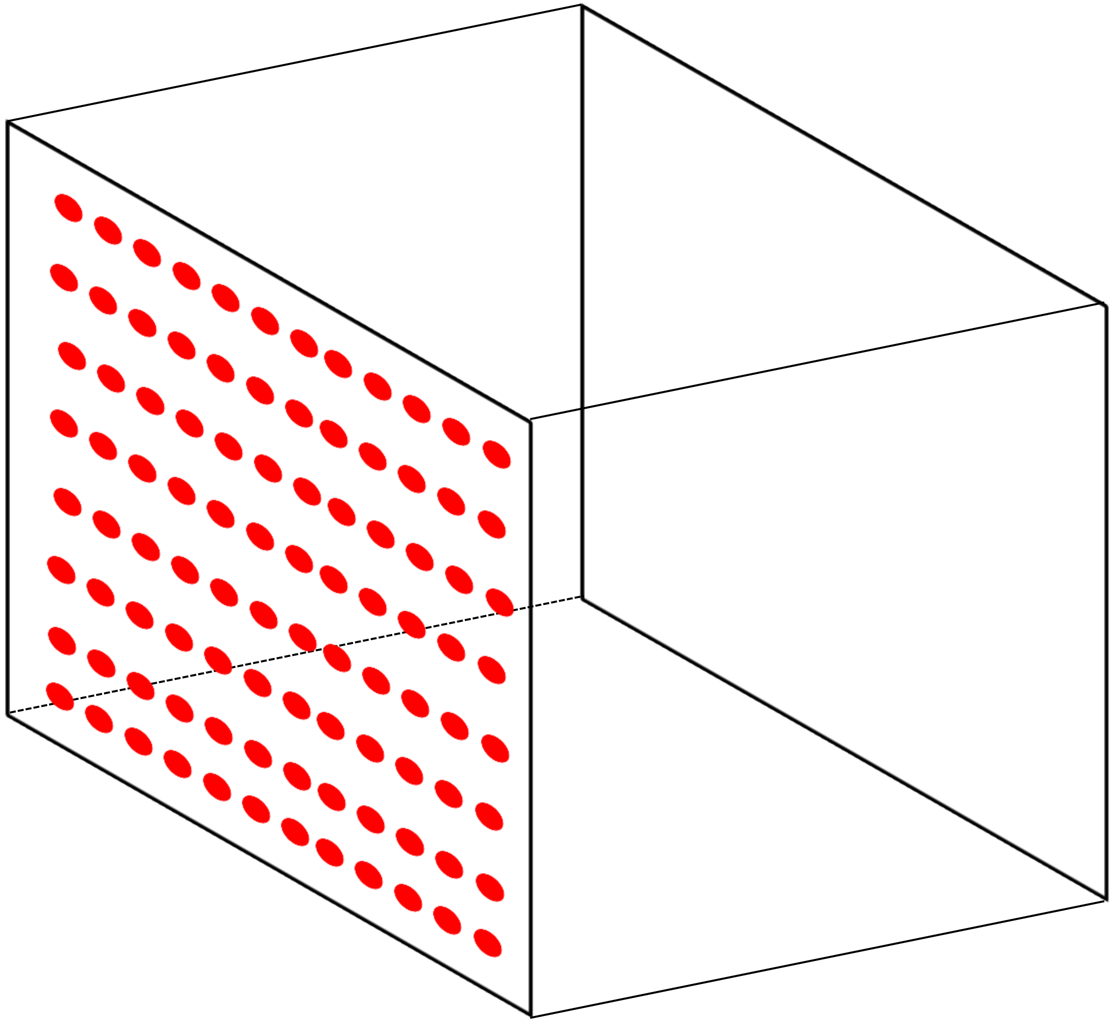
”Molar mass” D
(diffusion coefficient)

Diffusion



Density (concentration) c

Diffusion



Area A

Ficks First Law

The rate of change of molecules over time →

$$\dot{V} = \frac{(c_1 - c_2) \cdot A \cdot D}{x}$$

Ficks First Law

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$$\frac{\dot{V}}{A} = \frac{(c_1 - c_2)}{x} D$$

Ficks First Law

$$\dot{V} = \frac{(c_1 - c_2) \cdot A \cdot D}{x}$$

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Flux Gradient

Ficks First Law

$$\dot{V} = \frac{(c_1 - c_2) \cdot A \cdot D}{x}$$

$$\frac{\dot{V}}{A} = -D\nabla c$$

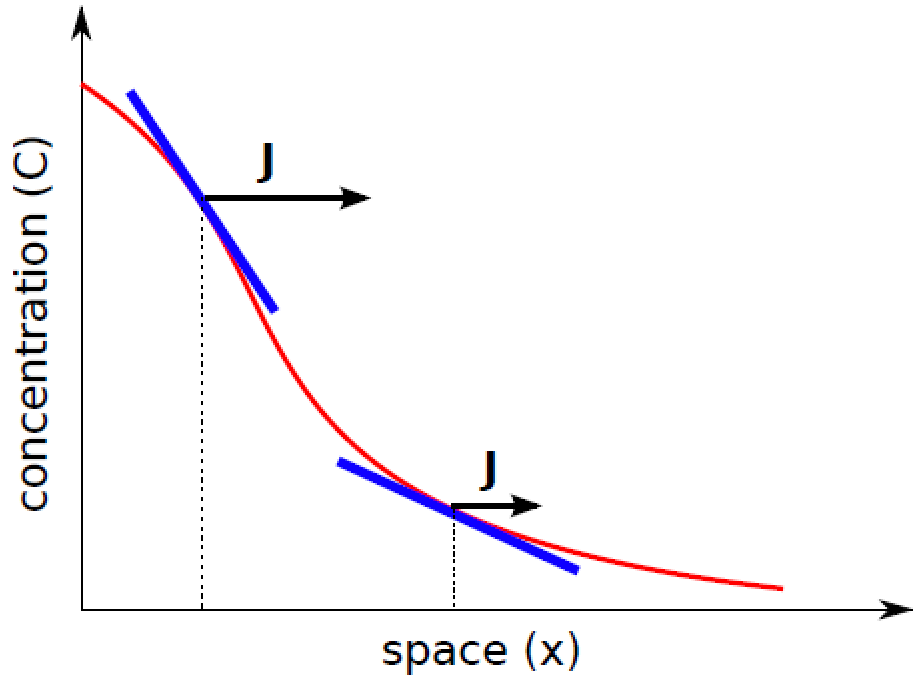
Flux Gradient

Ficks First Law

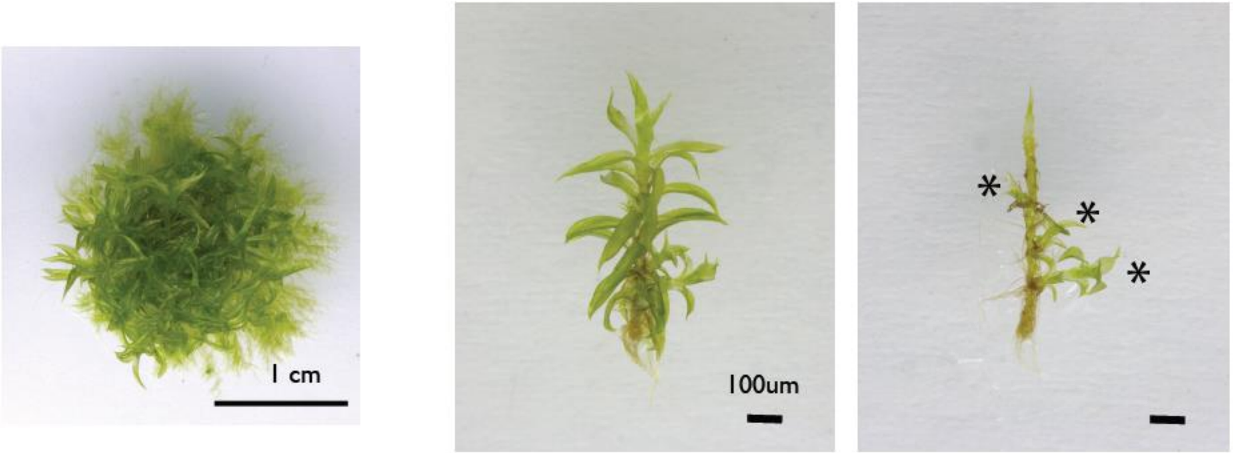
$$\dot{V} = \frac{(c_1 - c_2) \cdot A \cdot D}{x}$$

$$J = -D\nabla c$$

Flux Gradient

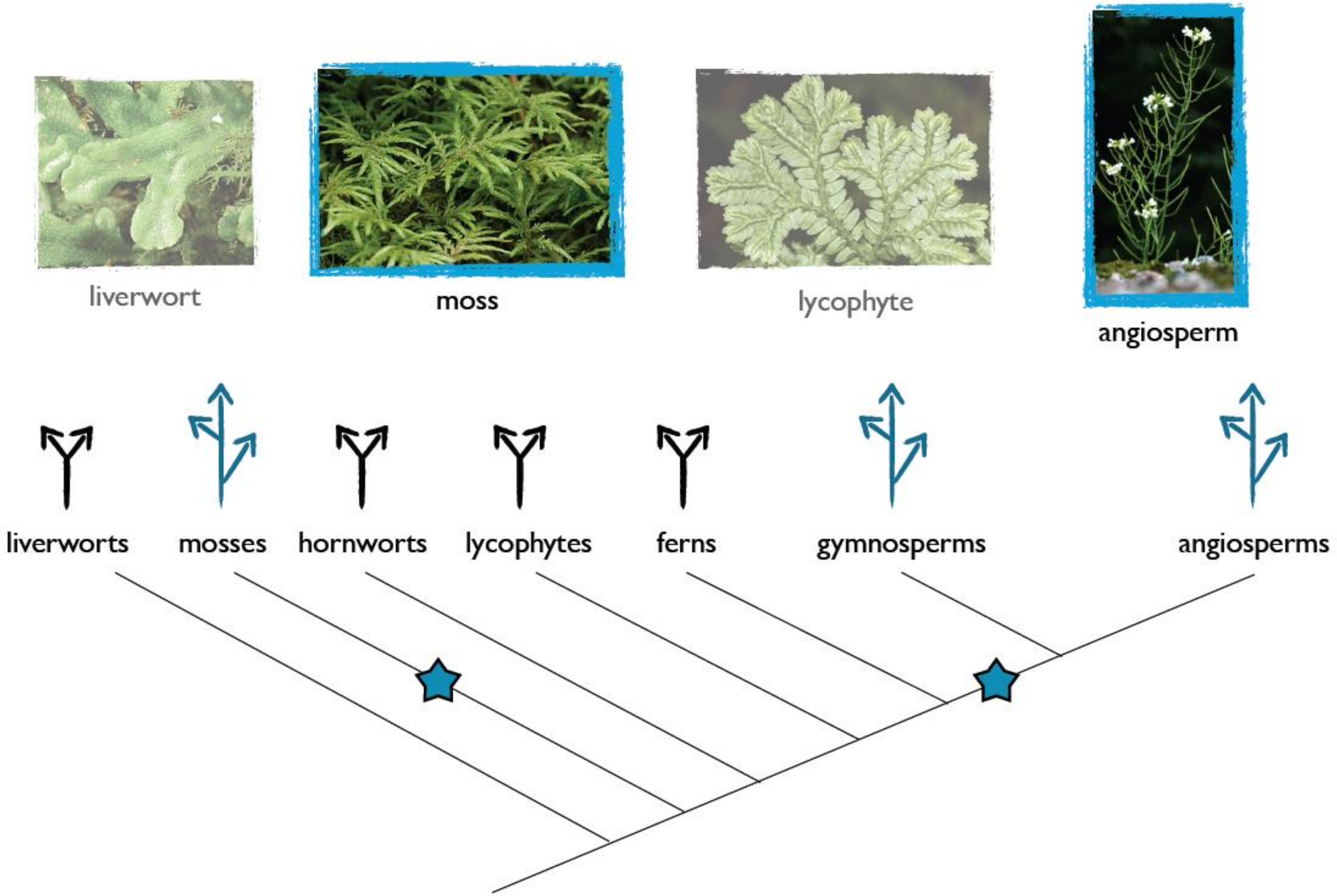


Modeling Example: how is lateral branching controlled in Physcomitrella patens?

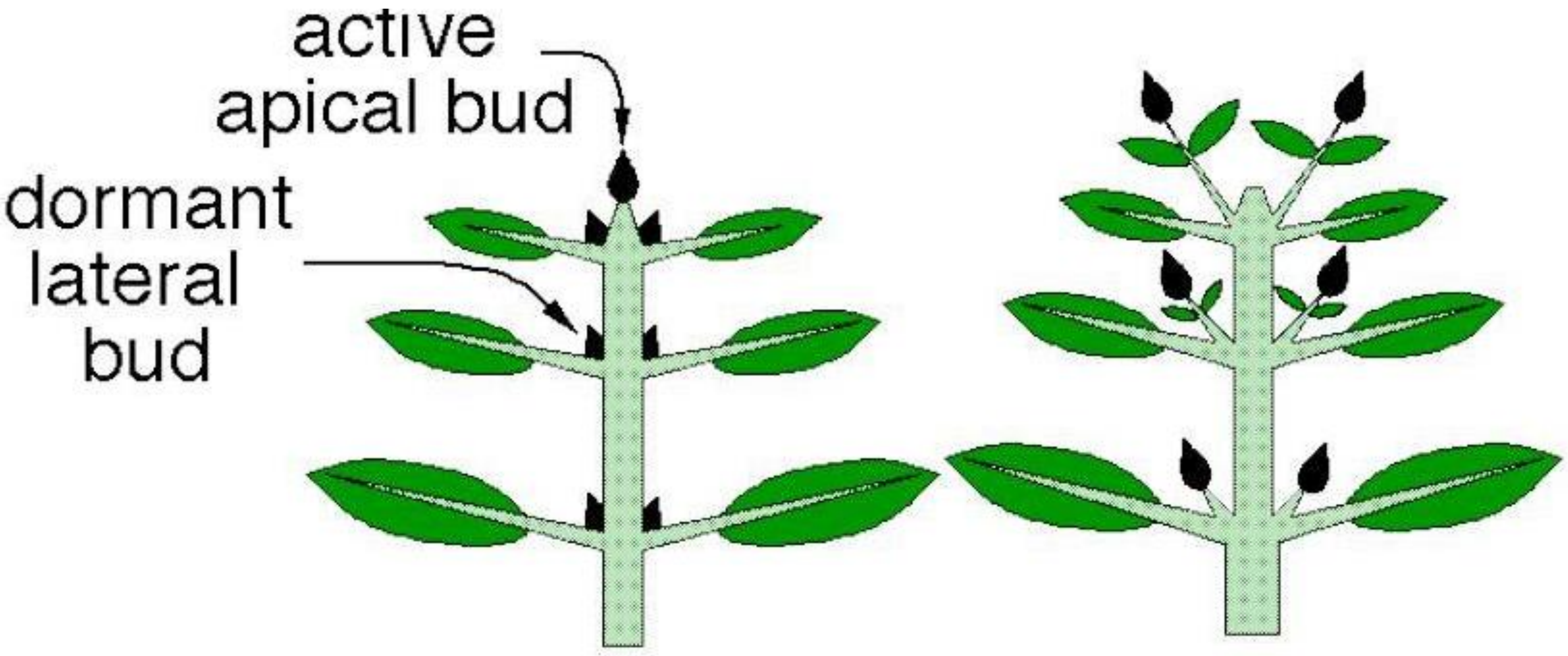


Coudert et al. Three ancient hormonal cues co-ordinate shoot branching in a moss. eLife 2015

Independent evolution of lateral branching

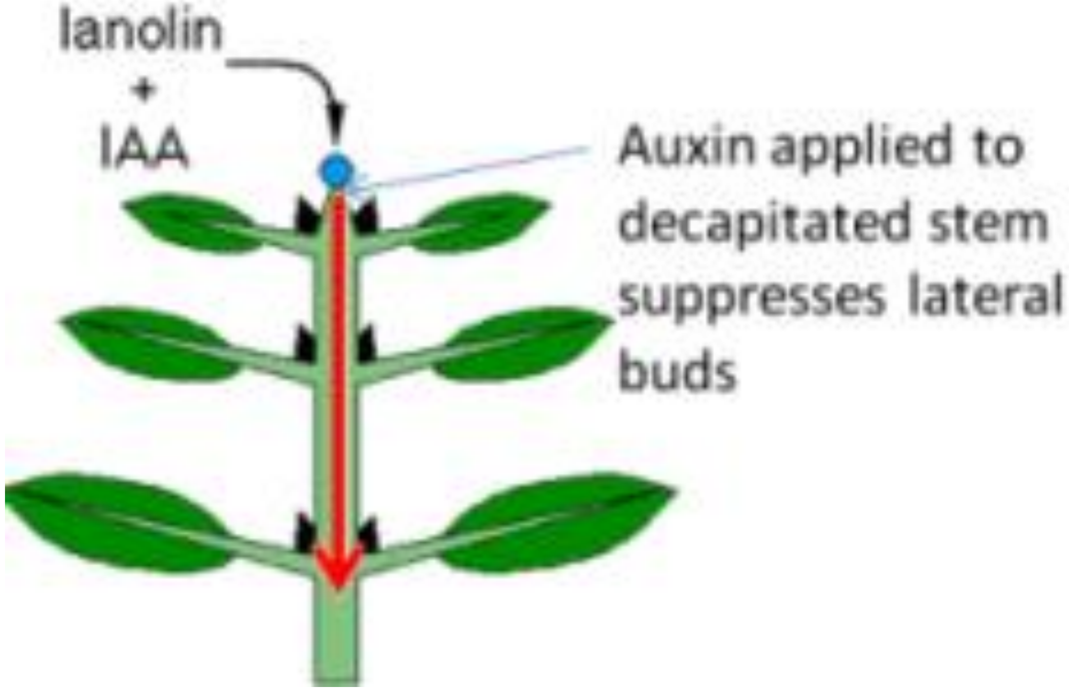


Apical Dominance



with apical bud **decapitated**

Auxin suppresses lateral buds



Apical dominance in moss

Decapitation experiment performed in the moss *Splachnum ampullaceum*

Treatment	Percentage of bud reactivation
Intact	0
Decapitated + water	84
Decapitated + IAA (1 mgm./ml.)	0

von Maltzhan et al., Nature 1959

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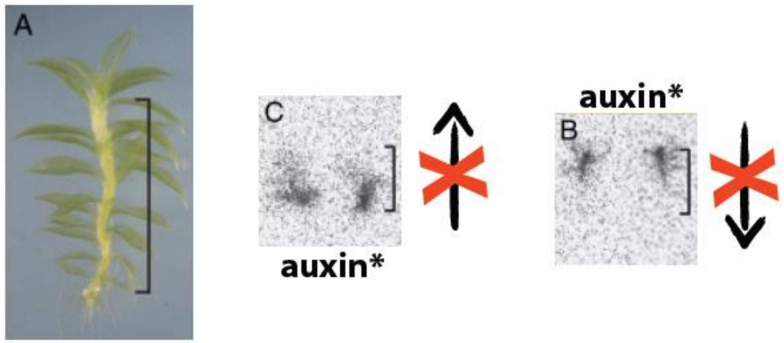
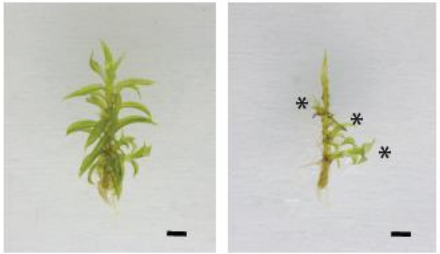
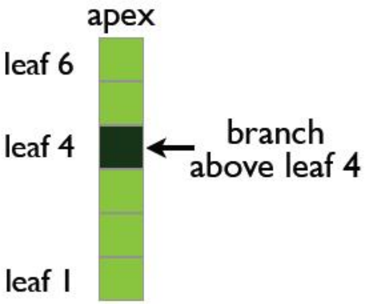
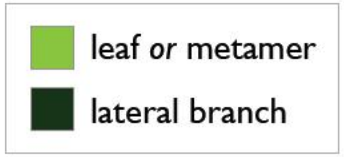


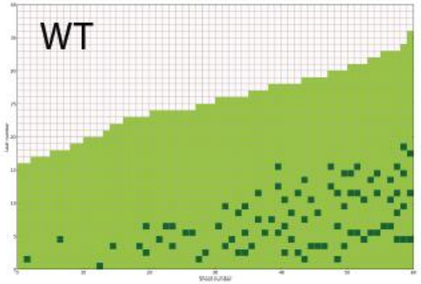
Fujita et al., Evol. Dev. 2008

Branching inhibited by the apex

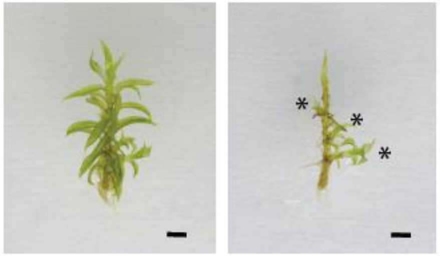


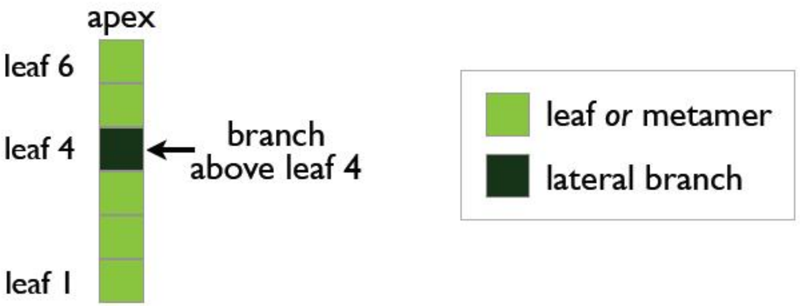


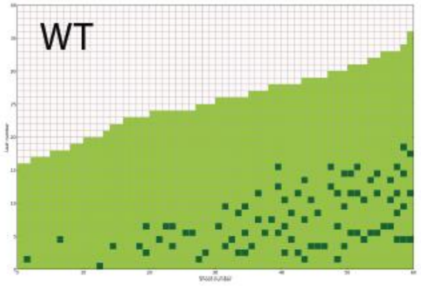


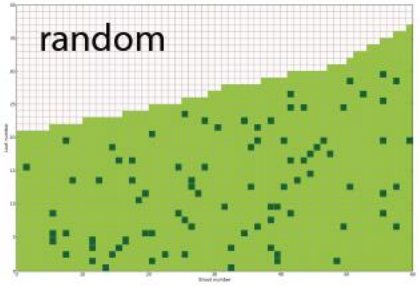


Branching inhibited by the apex









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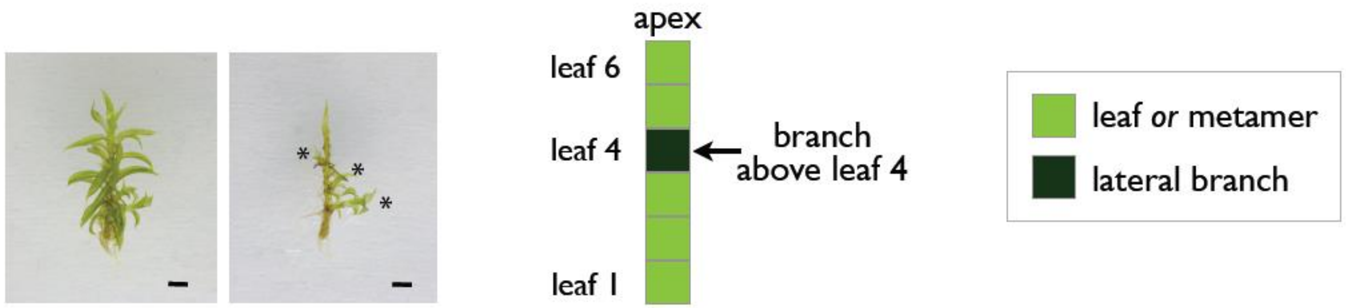
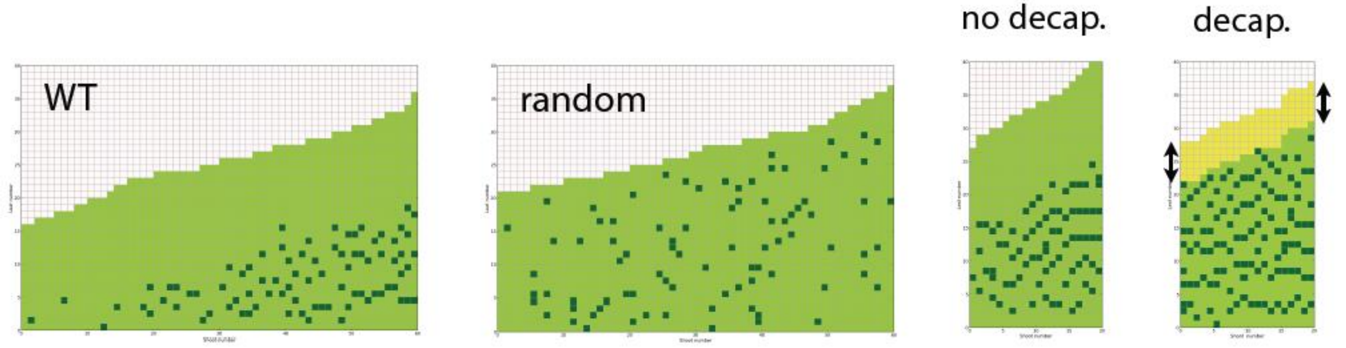
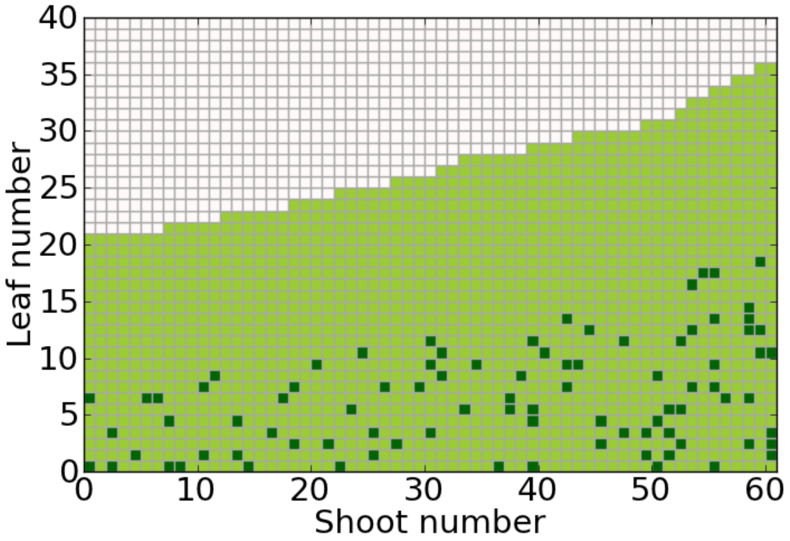


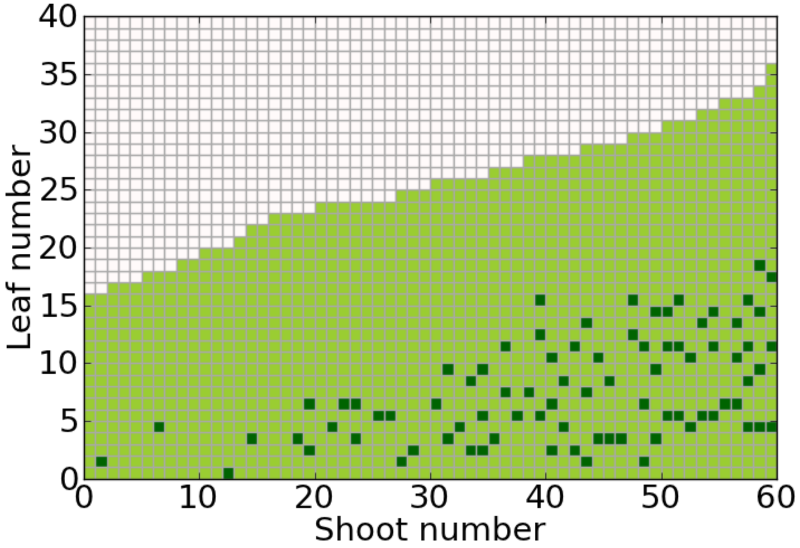


branching pattern after 1 week

Average distance between branches



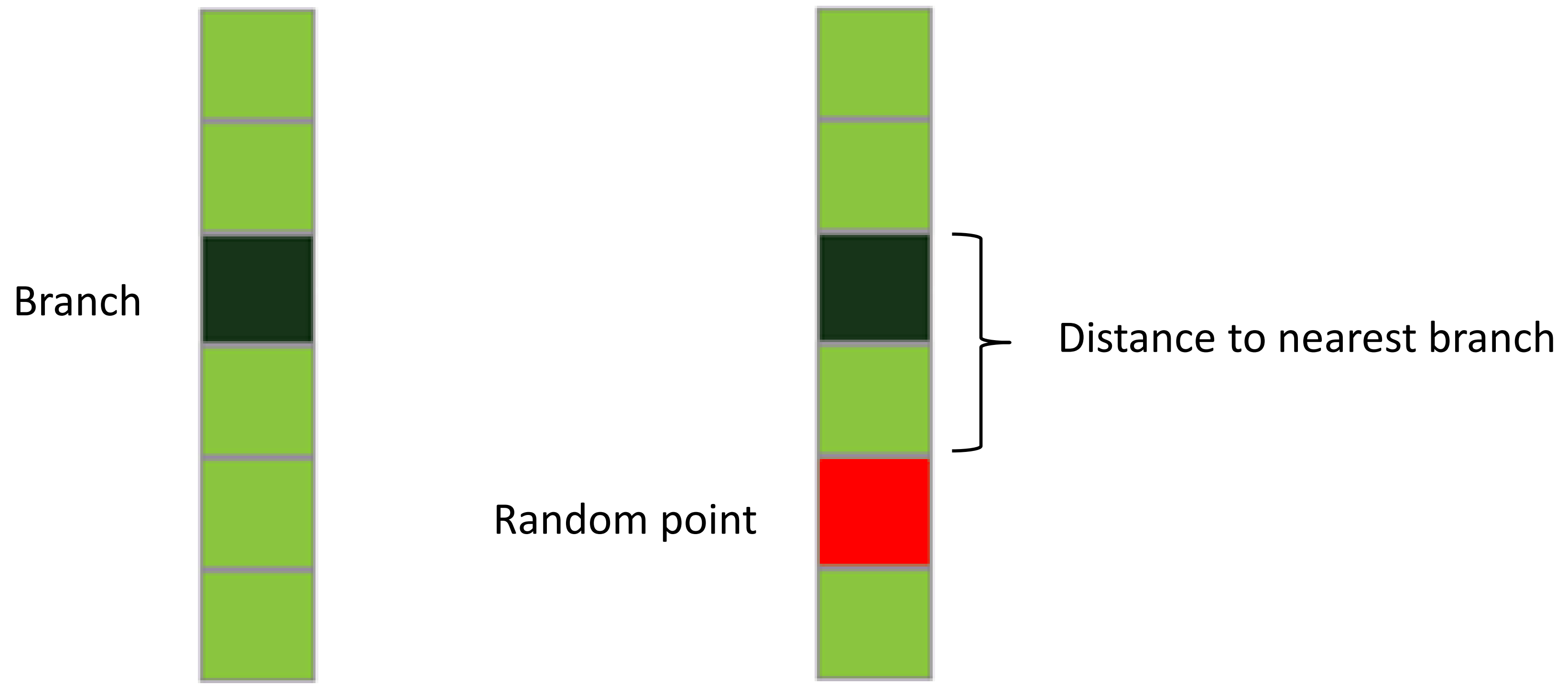
Random model:
Ave. distance between neighbors: 3.13



Wild-type:
Ave. distance between neighbors: 4.48

Branch distribution is over-dispersed?

- Calculate minimum distance of random points in the branching zone to real branches



Branch distribution is over-dispersed

- Calculate Hopkins index:

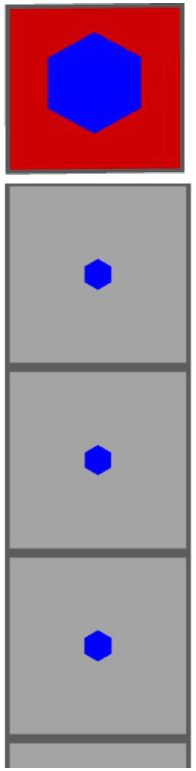
$$H = \frac{\langle min_i(\|x - b_i\|)\rangle_x}{\langle min_i(\|b_j - b_i\|)\rangle_j}$$

Wild-type data Hopkins Index: **0.81** ($\sigma = 0.09$)

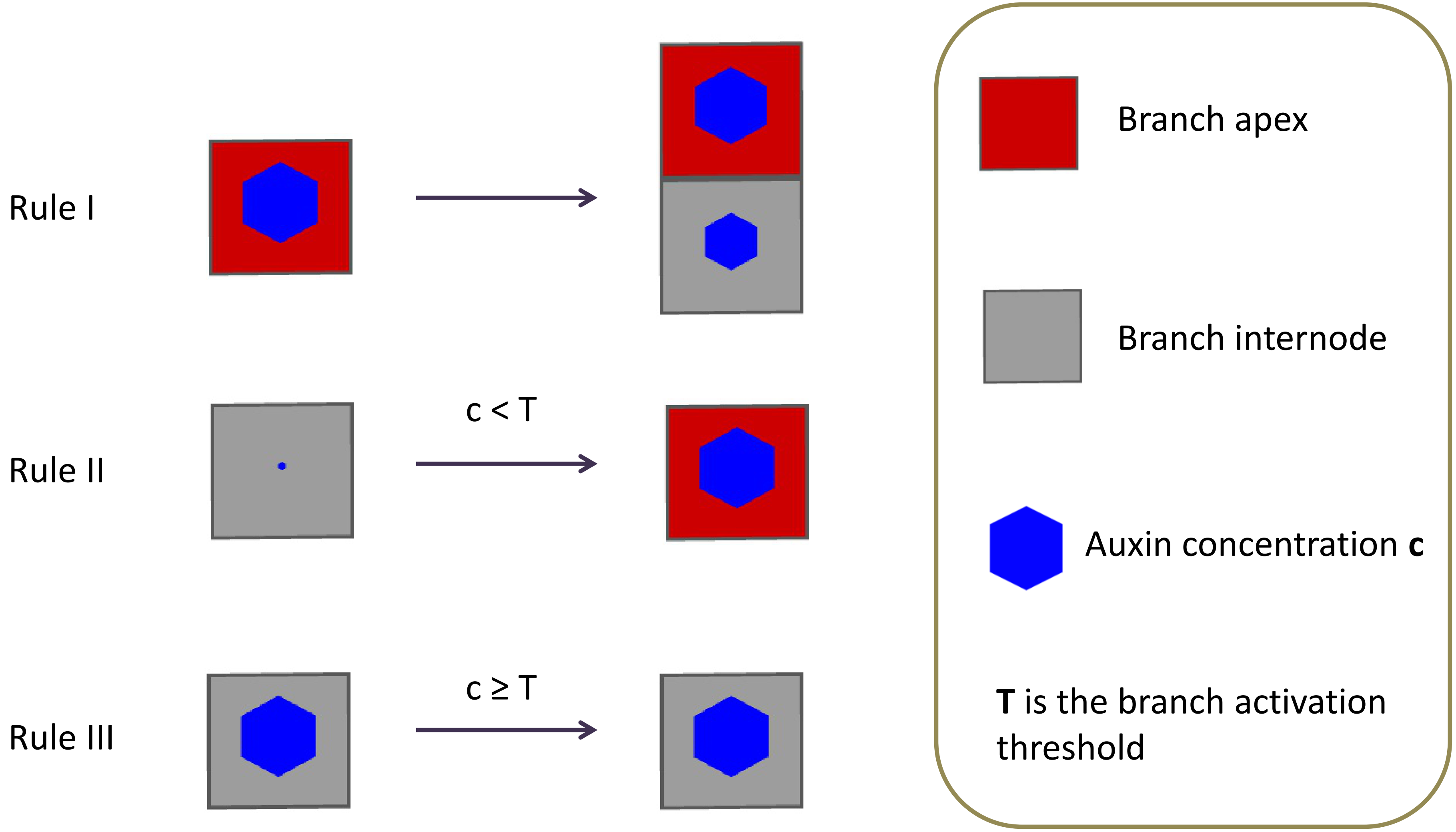
Moss Branching Model

- Biologically
  - Apices are sources of auxin production that produce further metamers
  - Branch initiation is controlled by auxin concentration
  - How is auxin moving through the moss shoot?

- Mathematically
  - Represent mosses as a set of connected compartments
  - Each compartment expresses the local concentration of auxin

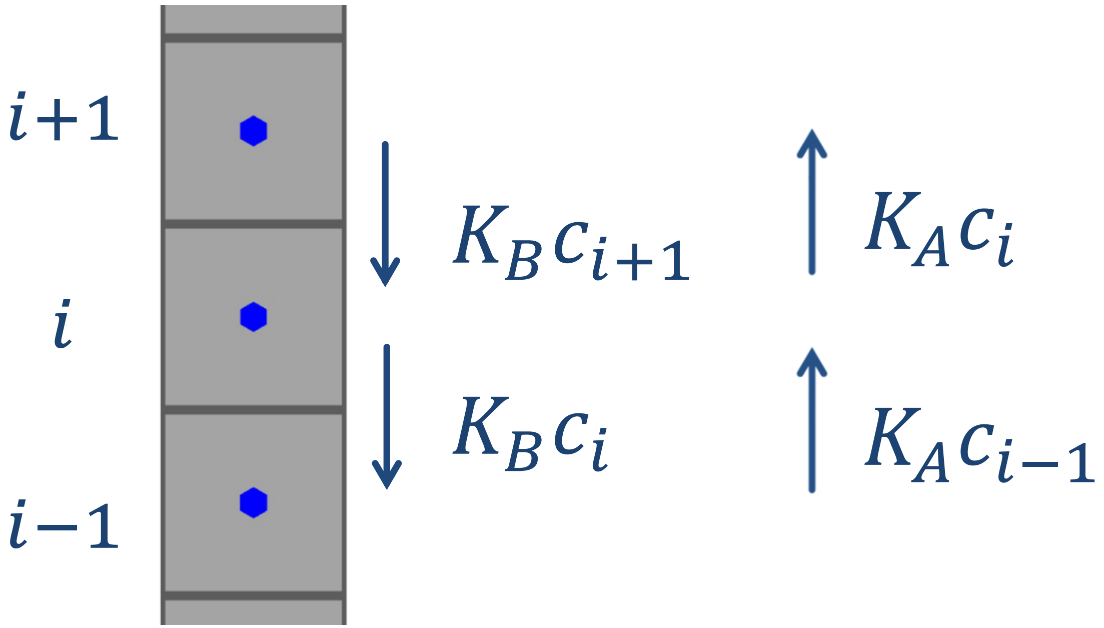


Development in the Moss Branching Model



Directional movement of auxin in the moss branching model

$$\frac{dc_i}{dt} = K_B(c_{i+1} - c_i) + K_A(c_{i-1} - c_i) - \nu c_i$$

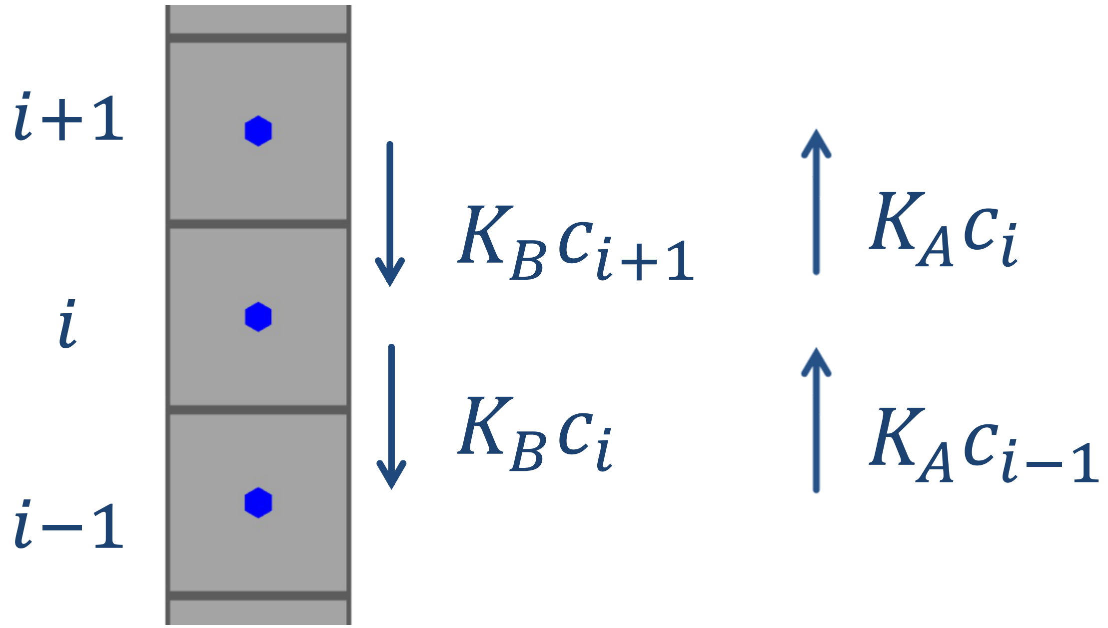


Directional movement of auxin in the moss branching model

$$\frac{dc_i}{dt} = K_B(c_{i+1} - c_i) + K_A(c_{i-1} - c_i) - \nu c_i$$

$K_B \approx K_A$

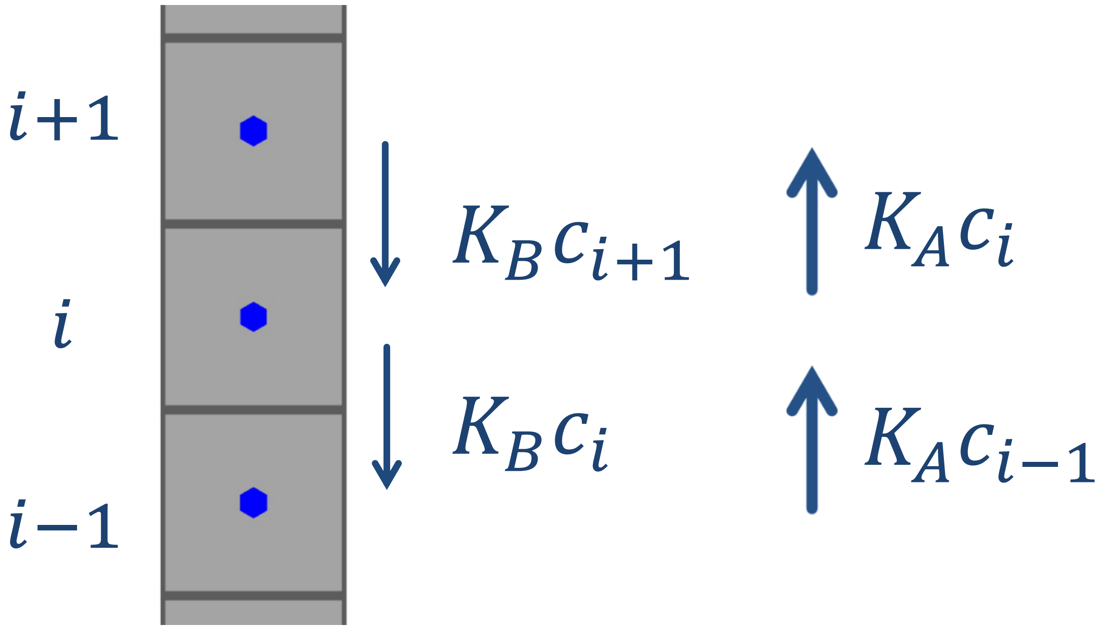
Diffusion



Directional movement of auxin in the moss branching model

$$\frac{dc_i}{dt} = K_B(c_{i+1} - c_i) + K_A(c_{i-1} - c_i) - \nu c_i$$

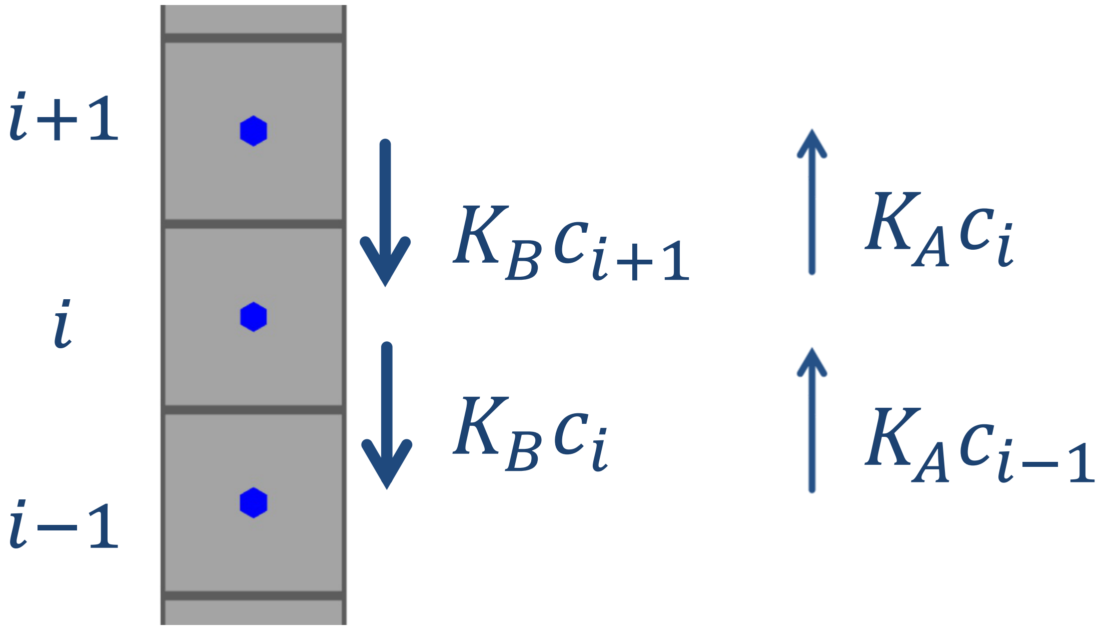
$K_B \ll K_A$



Directional movement of auxin in the moss branching model

$$\frac{dc_i}{dt} = K_B(c_{i+1} - c_i) + K_A(c_{i-1} - c_i) - \nu c_i$$

$$K_B \gg K_A$$

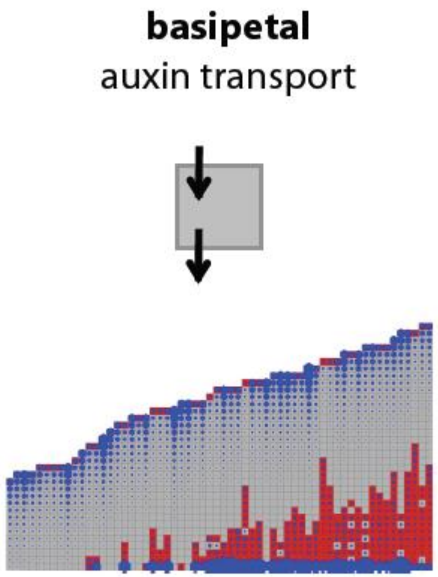


Input Parameters

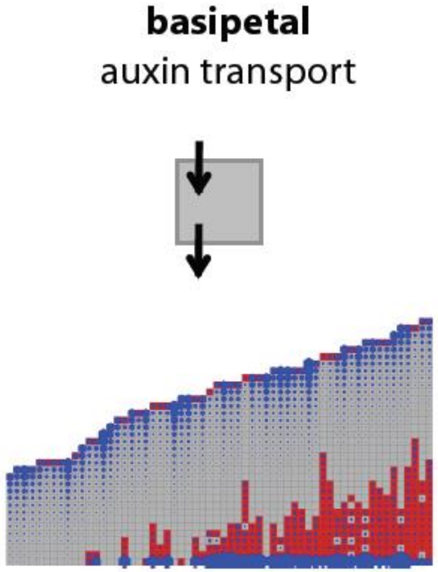
- H_{apex} is the target auxin concentration of the terminal apex (stochastic parameter)
- H is the target auxin concentration of lateral apices (stochastic parameter)
- T is the branching threshold (stochastic parameter)
- ν is the auxin concentration decay rate
- K_A and K_B determine the rate of directional auxin transport

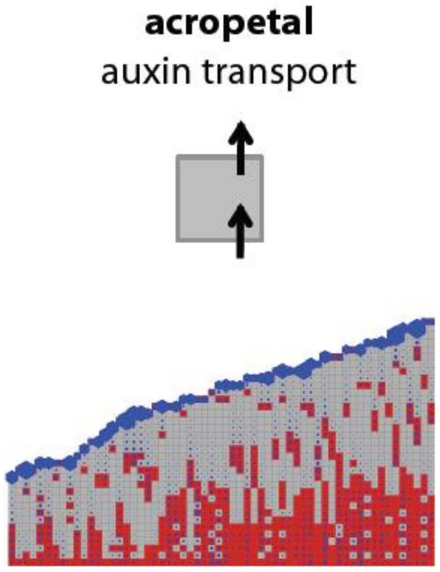
Development and Auxin Transport

Directional movement of auxin

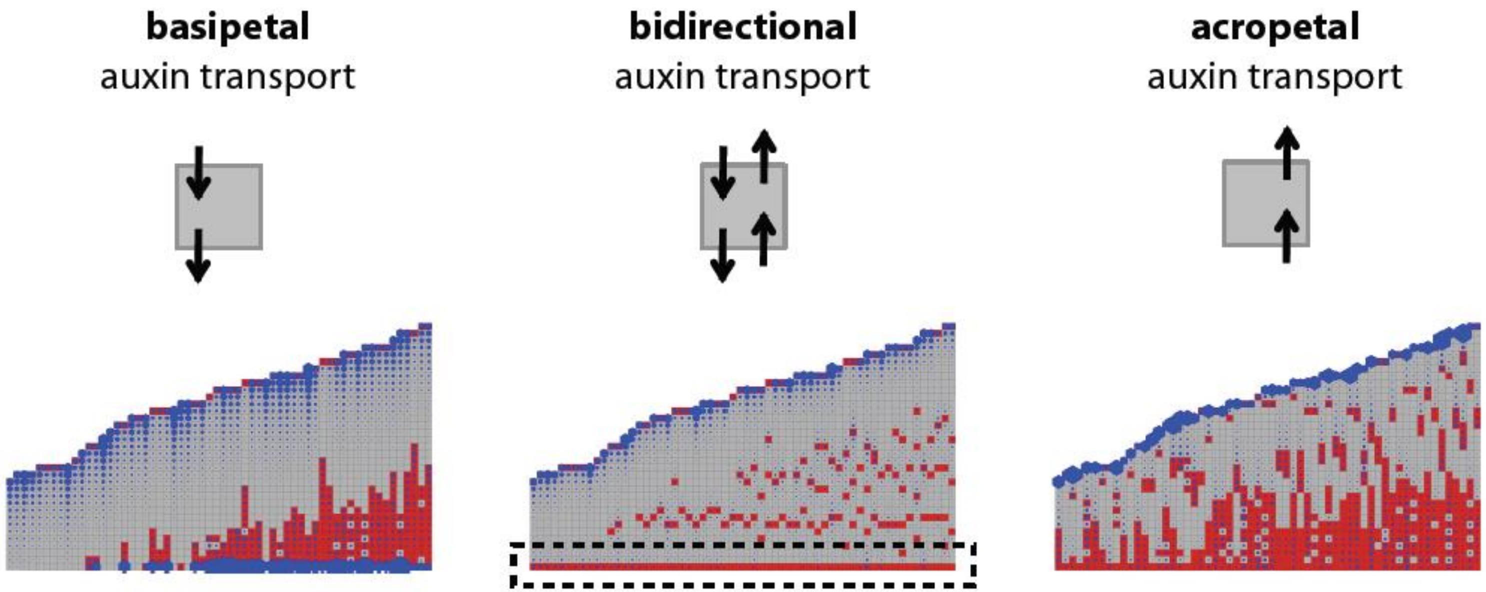


Directional movement of auxin

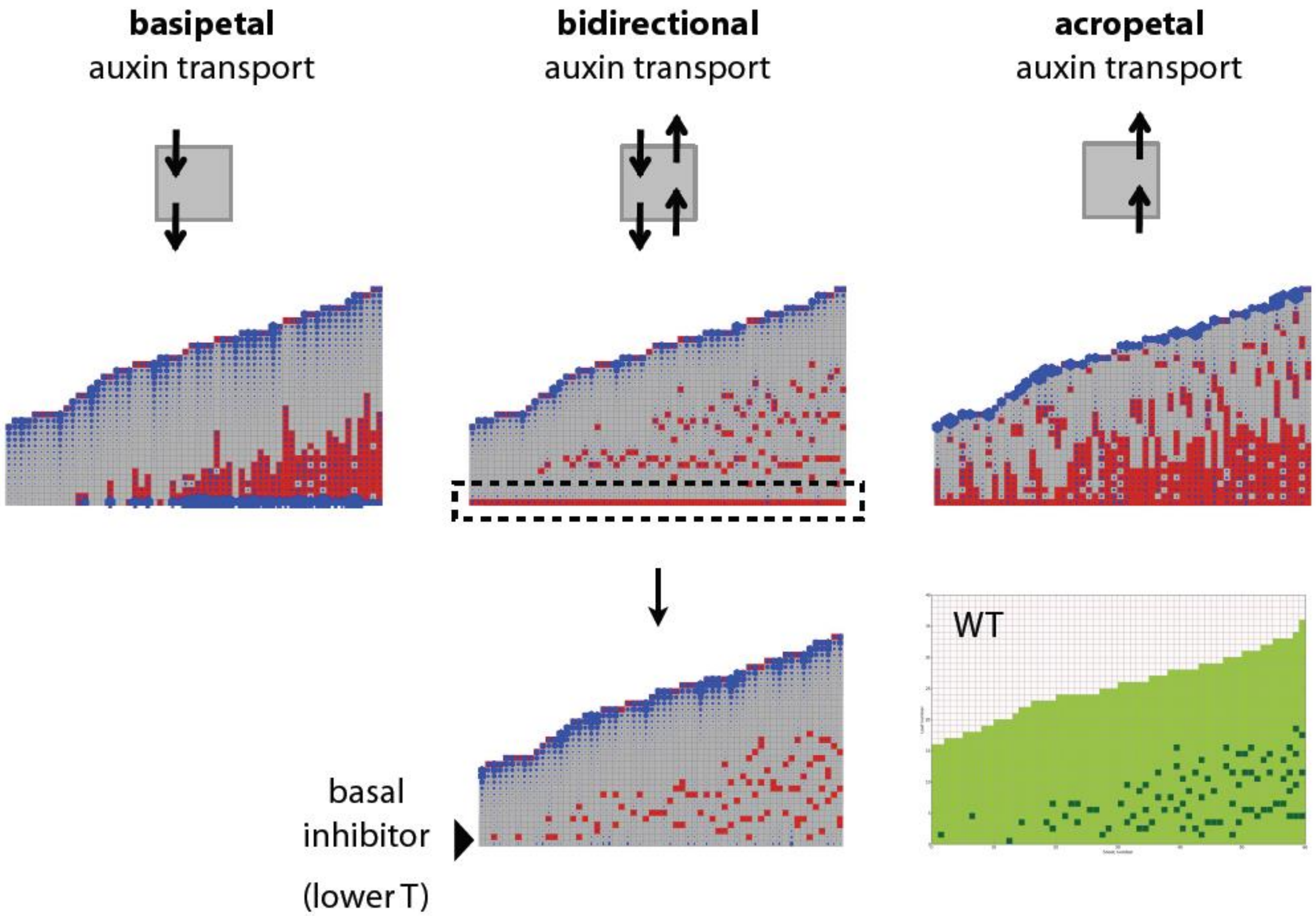




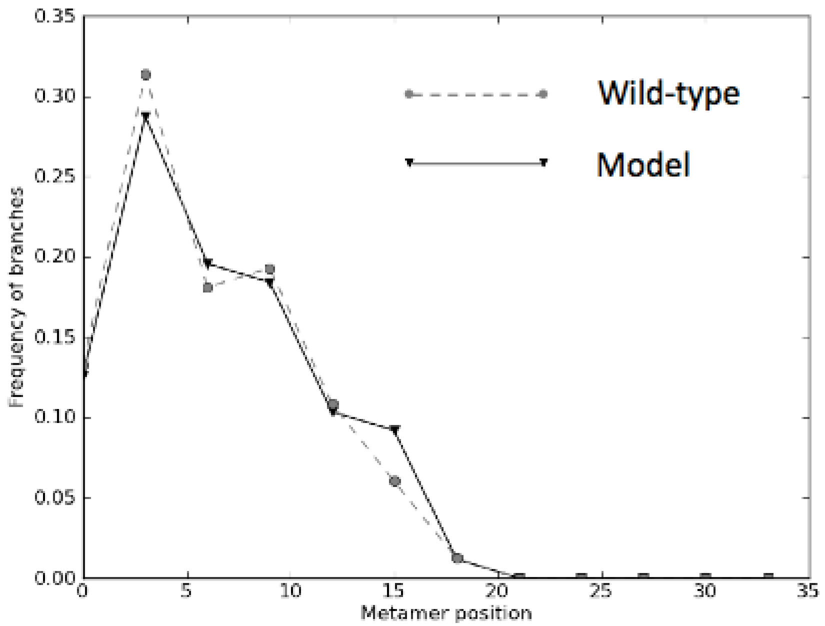
Directional movement of auxin

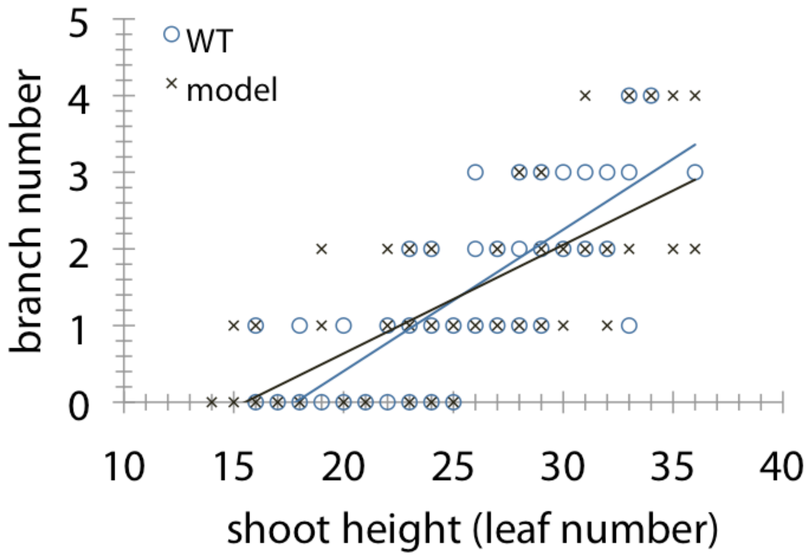


Bidirectional transport model captures real data



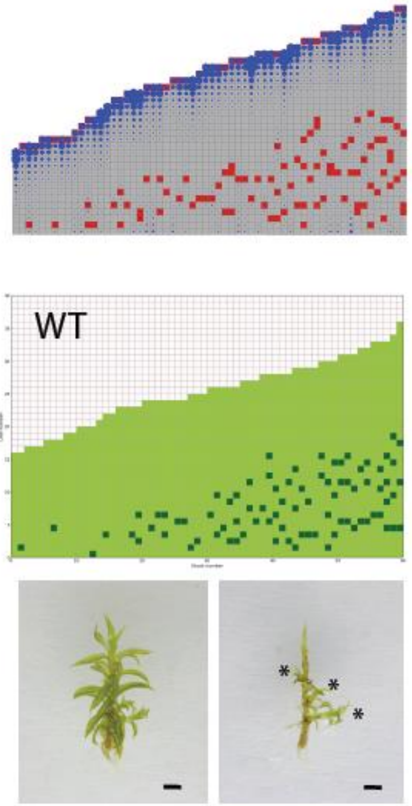
Quantitative analysis: bidirectional model



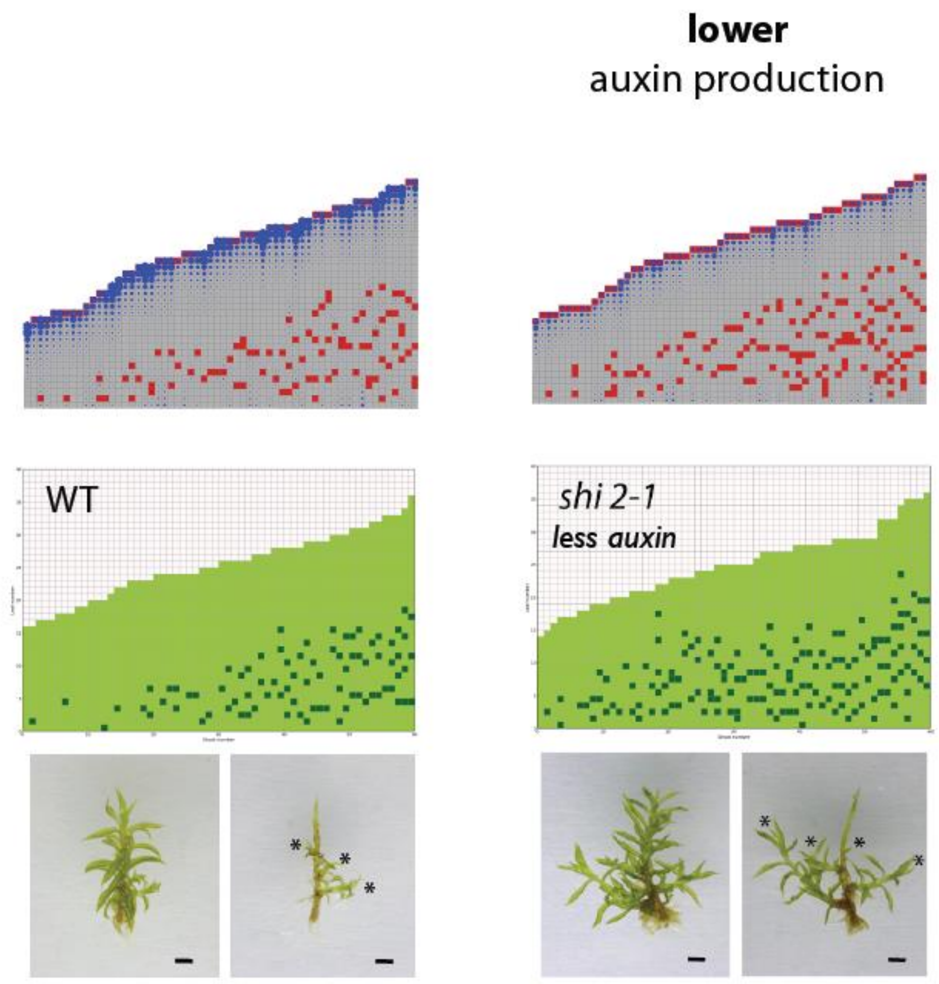


	Model	Wild-type
Branch number	87	83
Distance between branches	4.46	4.48
Apical inhibition zone	18.0	17.9

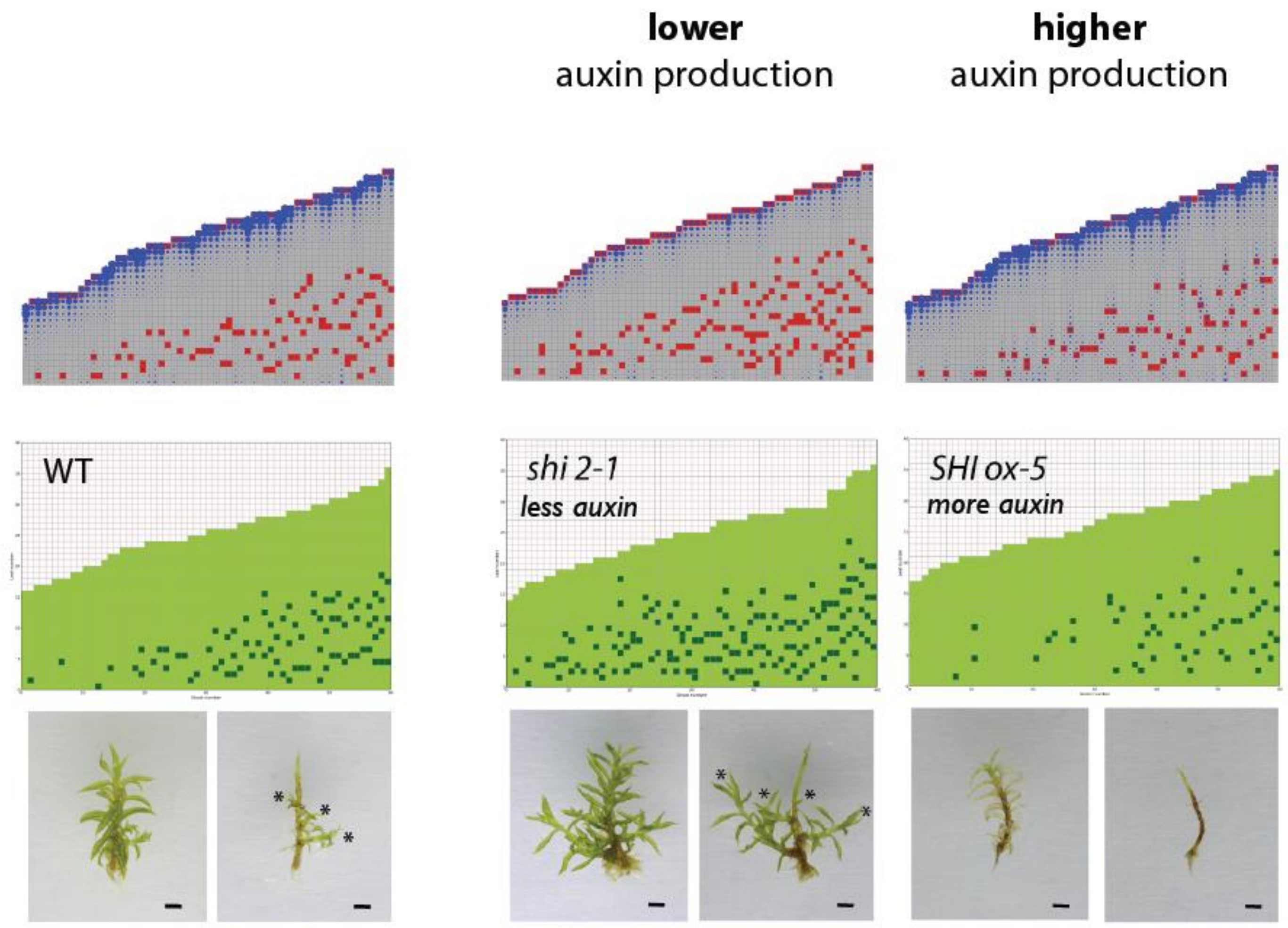
Branching patterns in altered auxin level mutants



Branching patterns in altered auxin level mutants



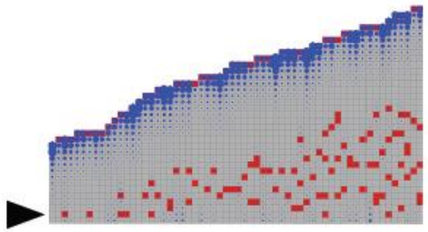
Branching patterns in altered auxin level mutants



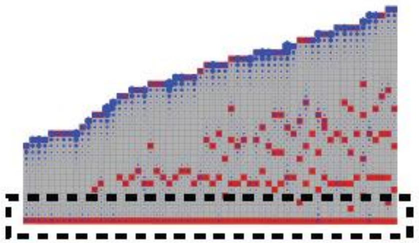
Strigolactone deficient mutant

WT <u>with</u> basal inhibitor

lower basal threshold



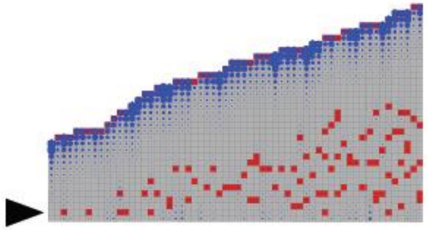
WT <u>without</u> basal inhibitor

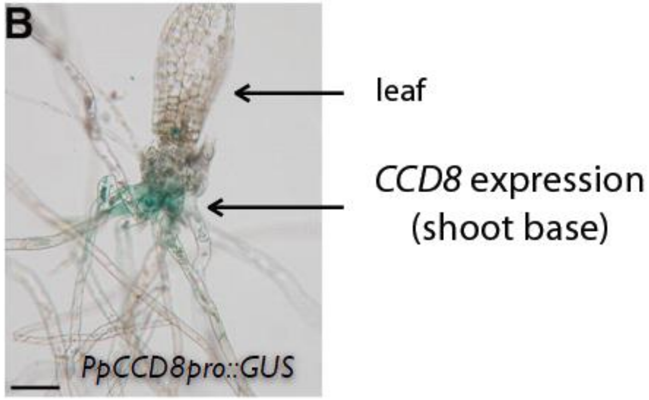


Strigolactone deficient mutant

WT with basal inhibitor

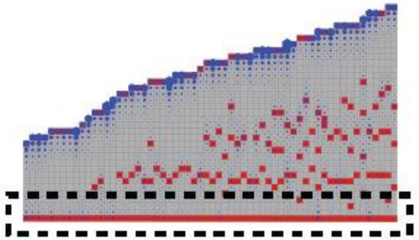
lower basal threshold

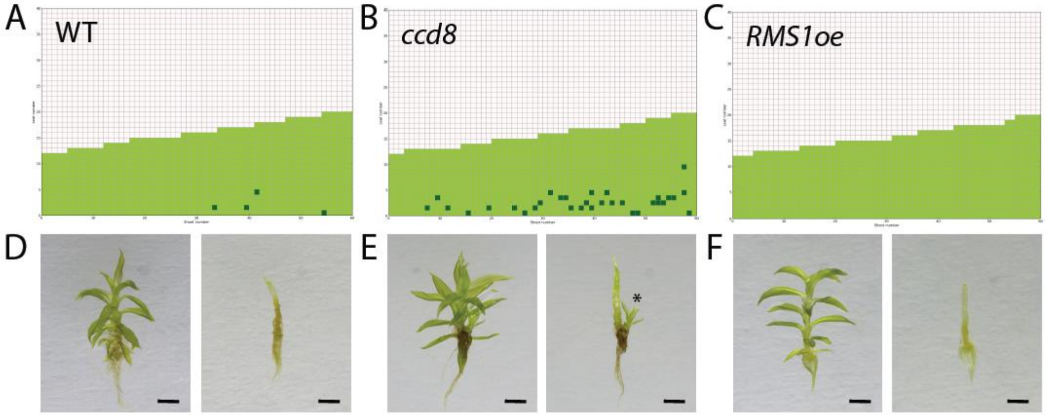




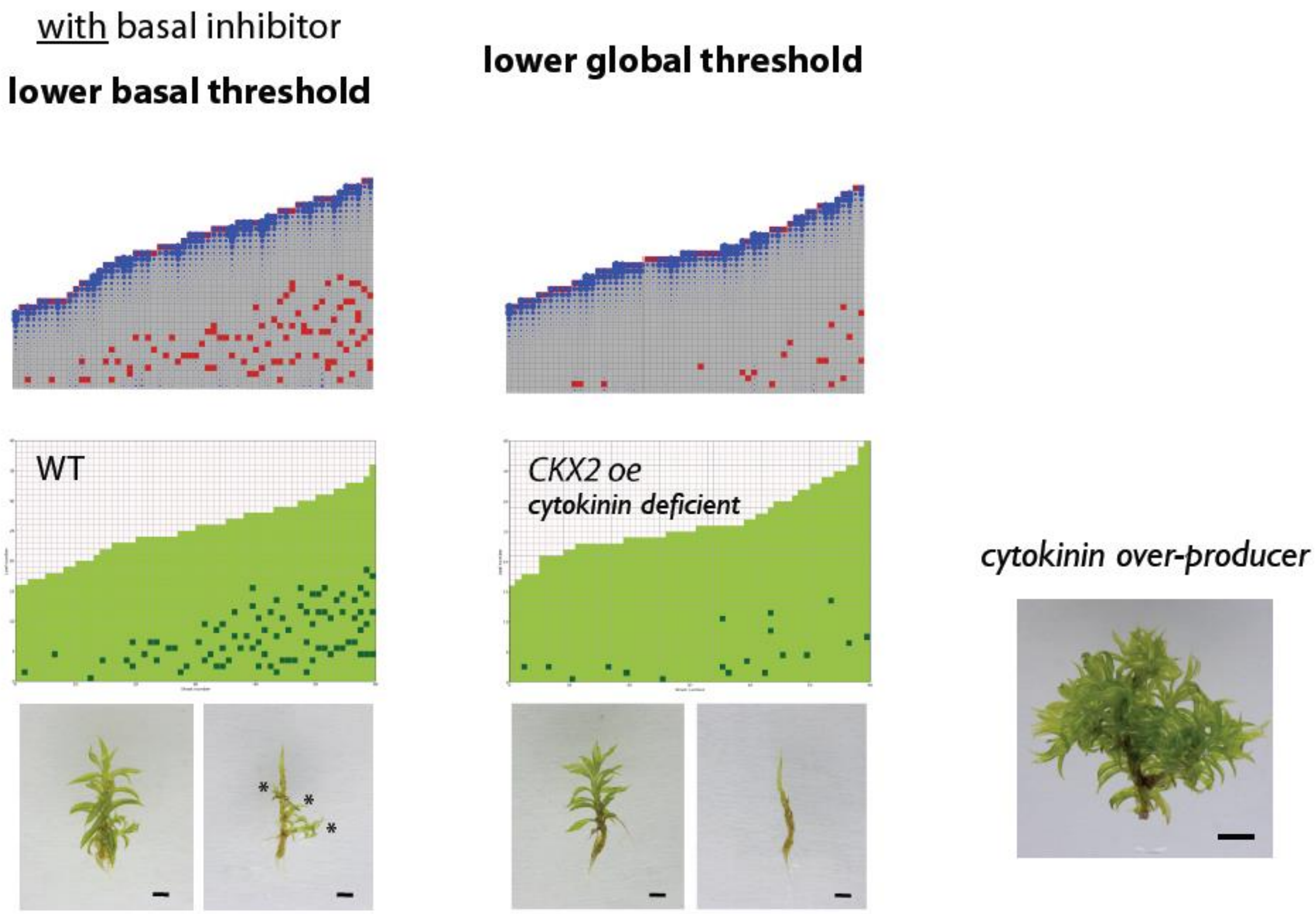
Proust *et al.*, *Development* 2011

WT without basal inhibitor

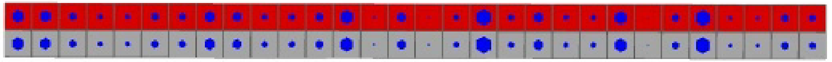




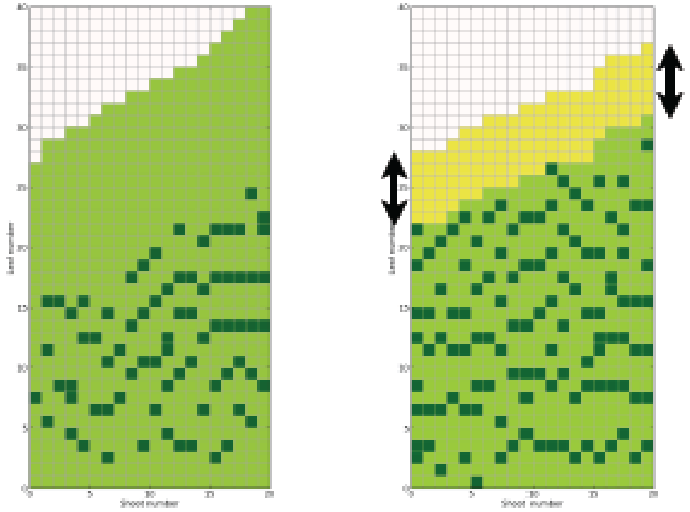
Cytokinin deficient mutant



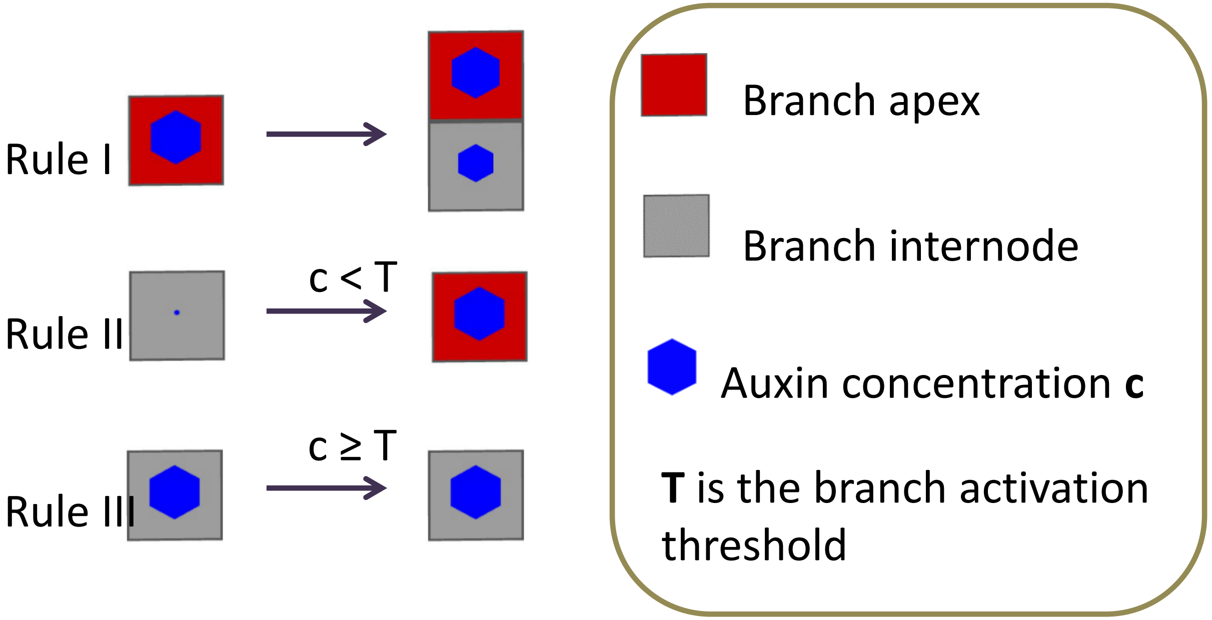
Decapitation simulation



no decap. decap.



branching pattern after 1 week

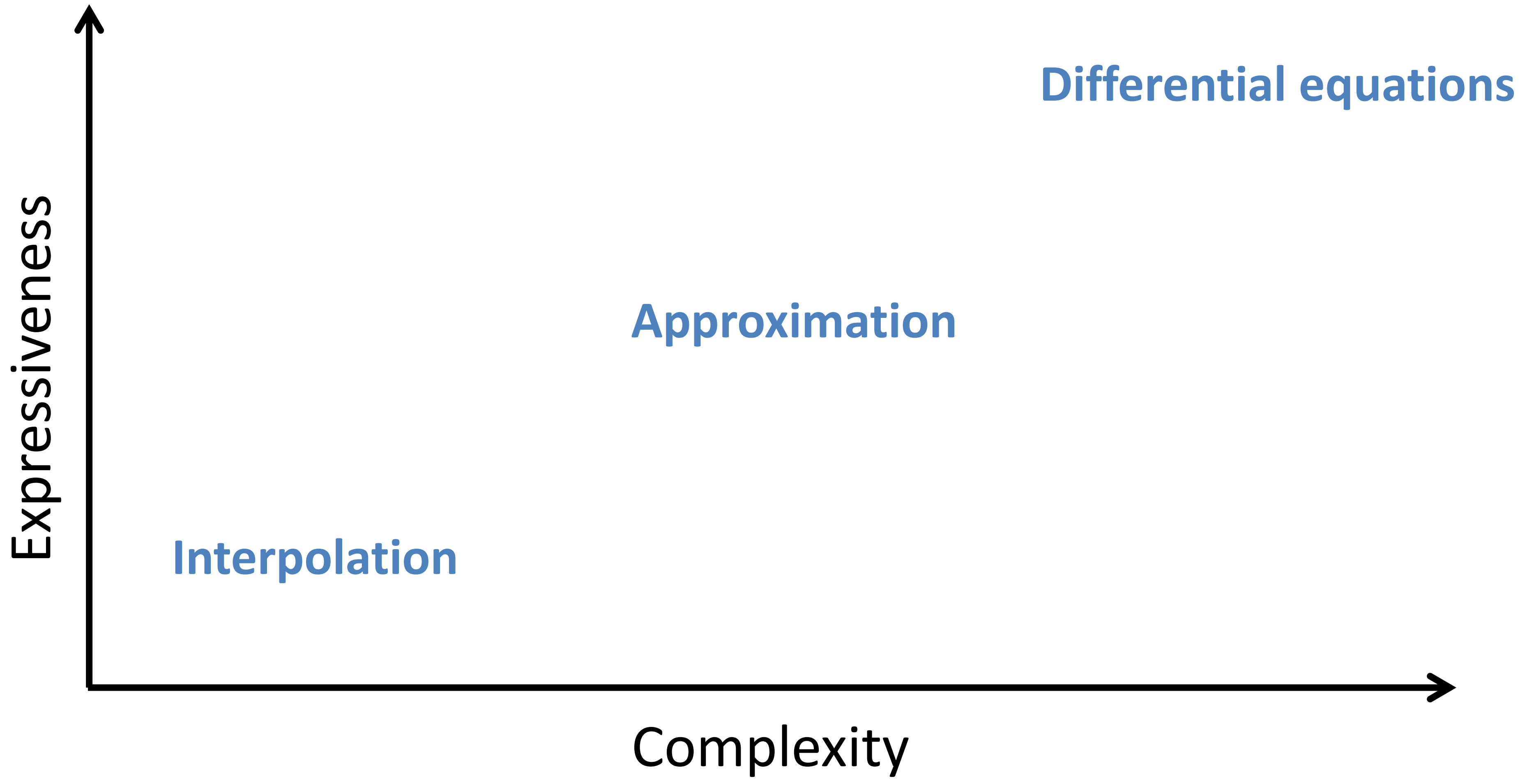


$$\frac{dc_i}{dt} = K_B(c_{i+1} - c_i) + K_A(c_{i-1} - c_i) - \nu c_i$$

Causality

- Parameter space of simple model captures branching patterns of mutant and transgenic lines:
  - Denser branching at low auxin levels
  - Sparser branching at high auxin levels
  - **Local** and **global thresholds** of branching capture effects of **cytokinin** and **strigolactones**
- Auxin, cytokinin and strigolactones control major aspects of branching patterns in *Physcomitrella patens*

Data Models


Differential equations
Approximation
Interpolation
Expressiveness
Complexity

Group Presentation 25th May during lecture

- Present an example of a diffusion model from any topic (max 10 minutes, slides, topics must be unique, 3 people). The example must have a python implementation associated with it that you include and describe in the presentation. Explain the python libraries used, the diffusion model and the results.
- https://docs.google.com/spreadsheets/d/1HnV9assGZBdS1AaV_JTCz2LXRKe0TH0cFdyWYaMSDRc/edit?usp=sharing