

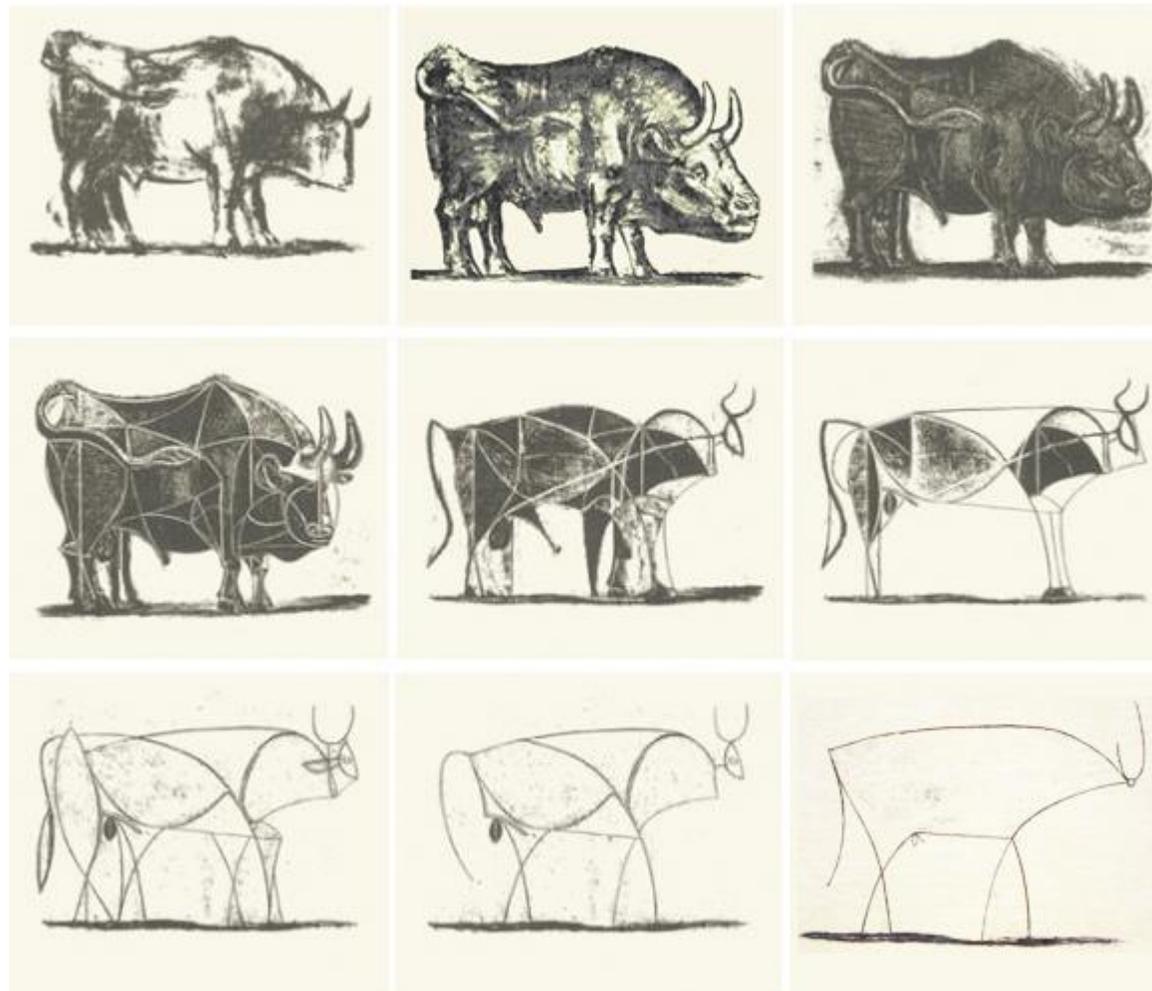
Symulacja Procesów Biologicznych

dr Wojciech Pałubicki

Symulacja procesów biologicznych

- **Symulacja** to operacja jakiegoś **modelu** w czasie
- **Model**: uproszczony opis świata
 - Za pomocy narzędzi analitycznych (matematyka, języki programistyczne)

Symulacja procesów biologicznych



„Byk”, Picasso

Symulacja procesów biologicznych

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I have deeply regretted that I did not proceed far enough at least to understand something of the great leading principles of mathematics; for men thus endowed seem to have an extra sense.

— Charles Darwin

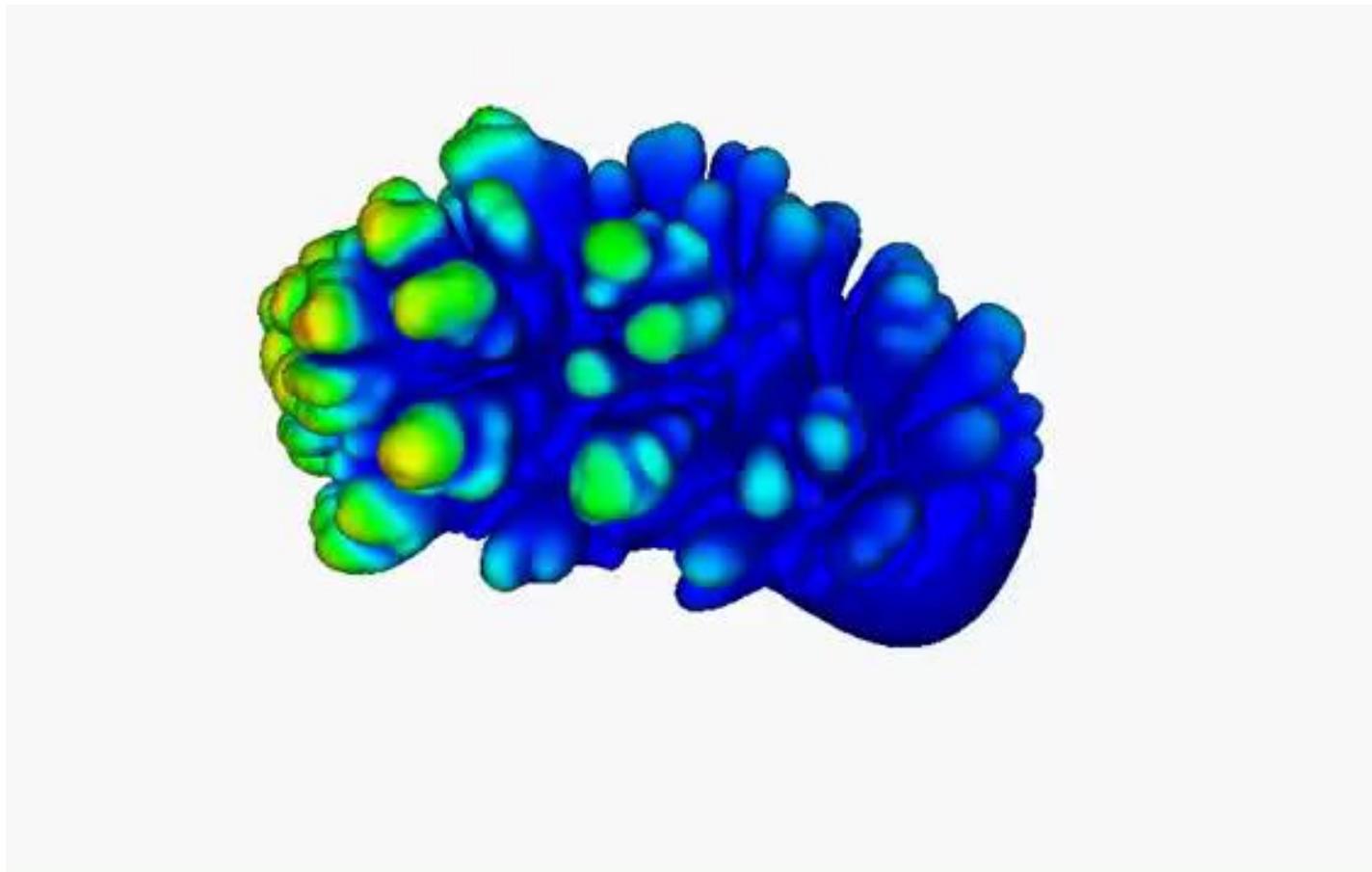
Wzrost drzewa mango



Wzrost grzyba

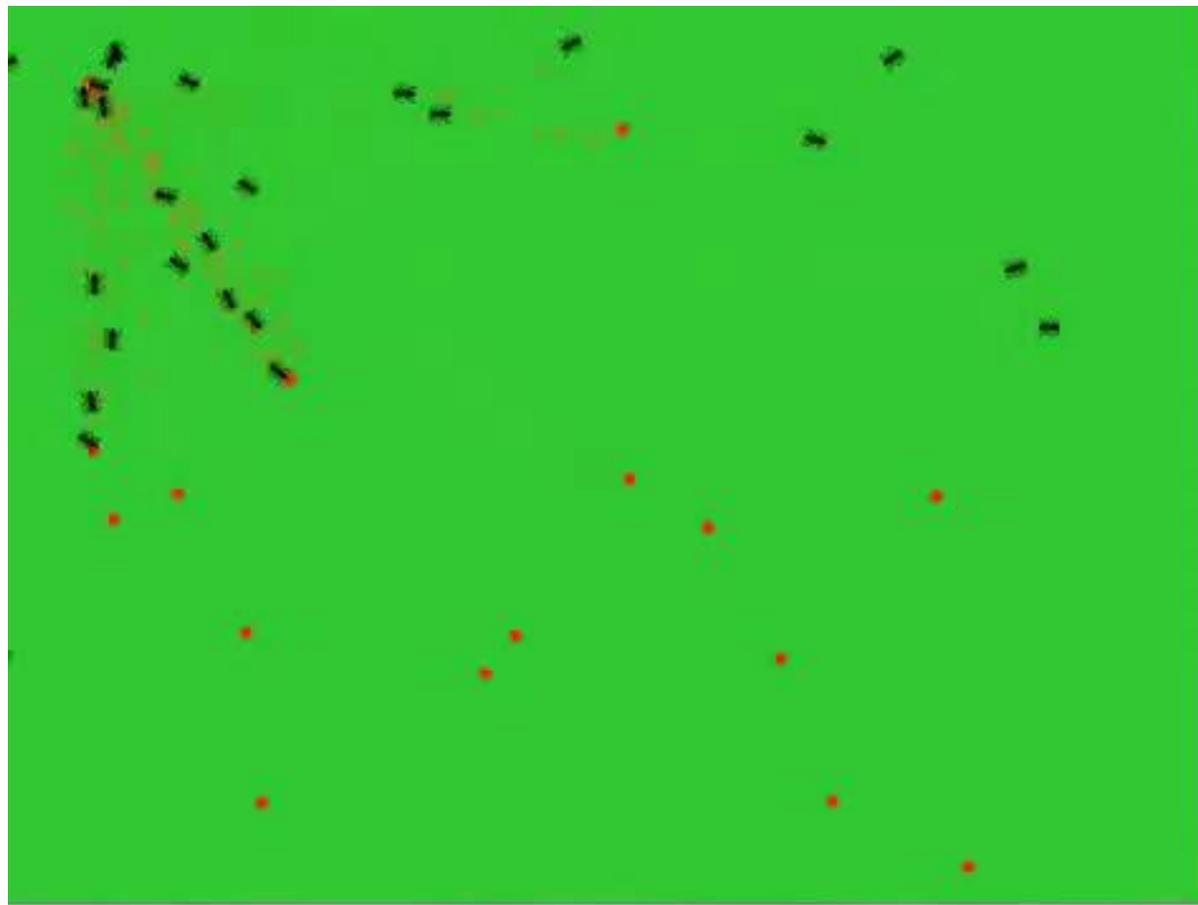
Diffusion Limited Aggregation
with a Cellular Automaton Machine
by Rozan Martin

Wzrost koralali

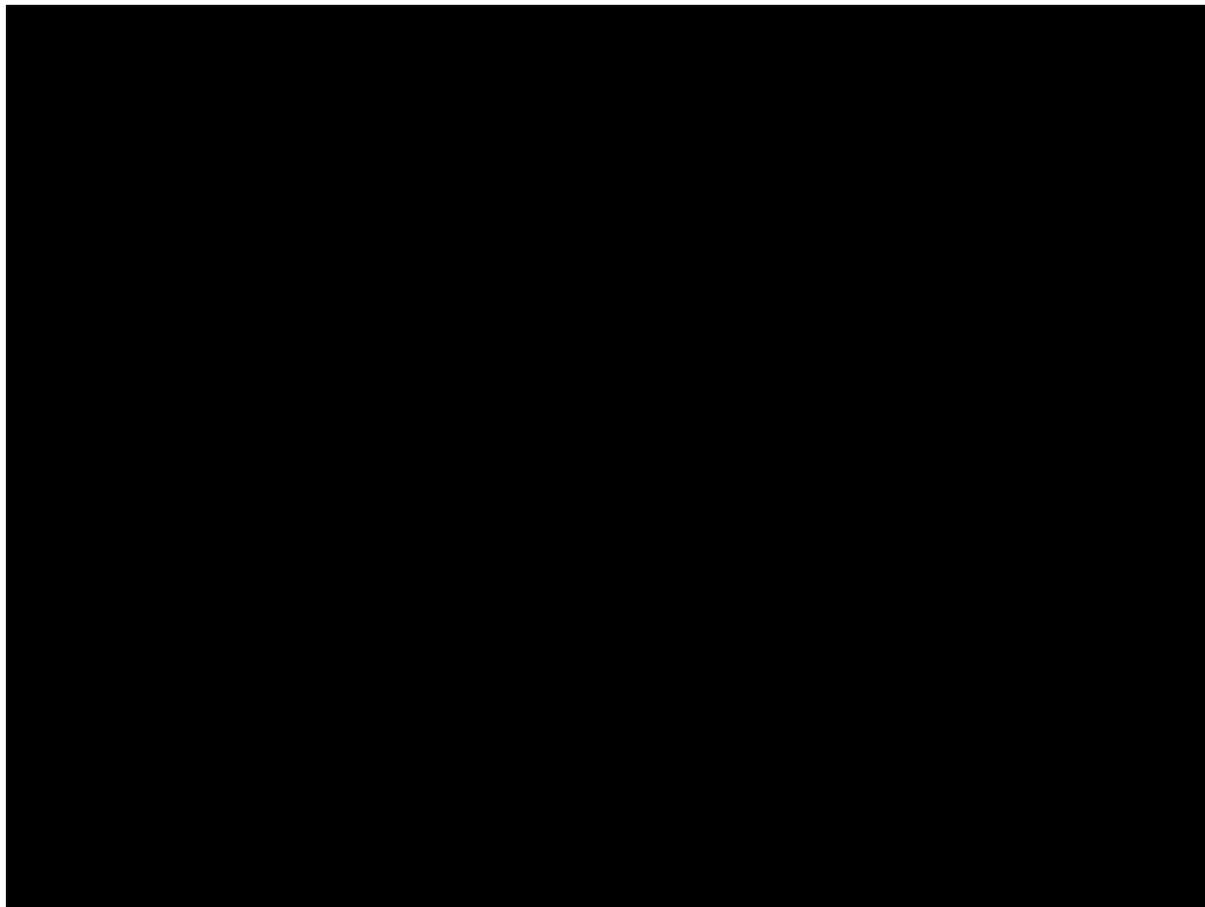


[Chindapol et al. 2013](#)

Ścieżki mrówek



Lot ptaków



Artykulacja pływania

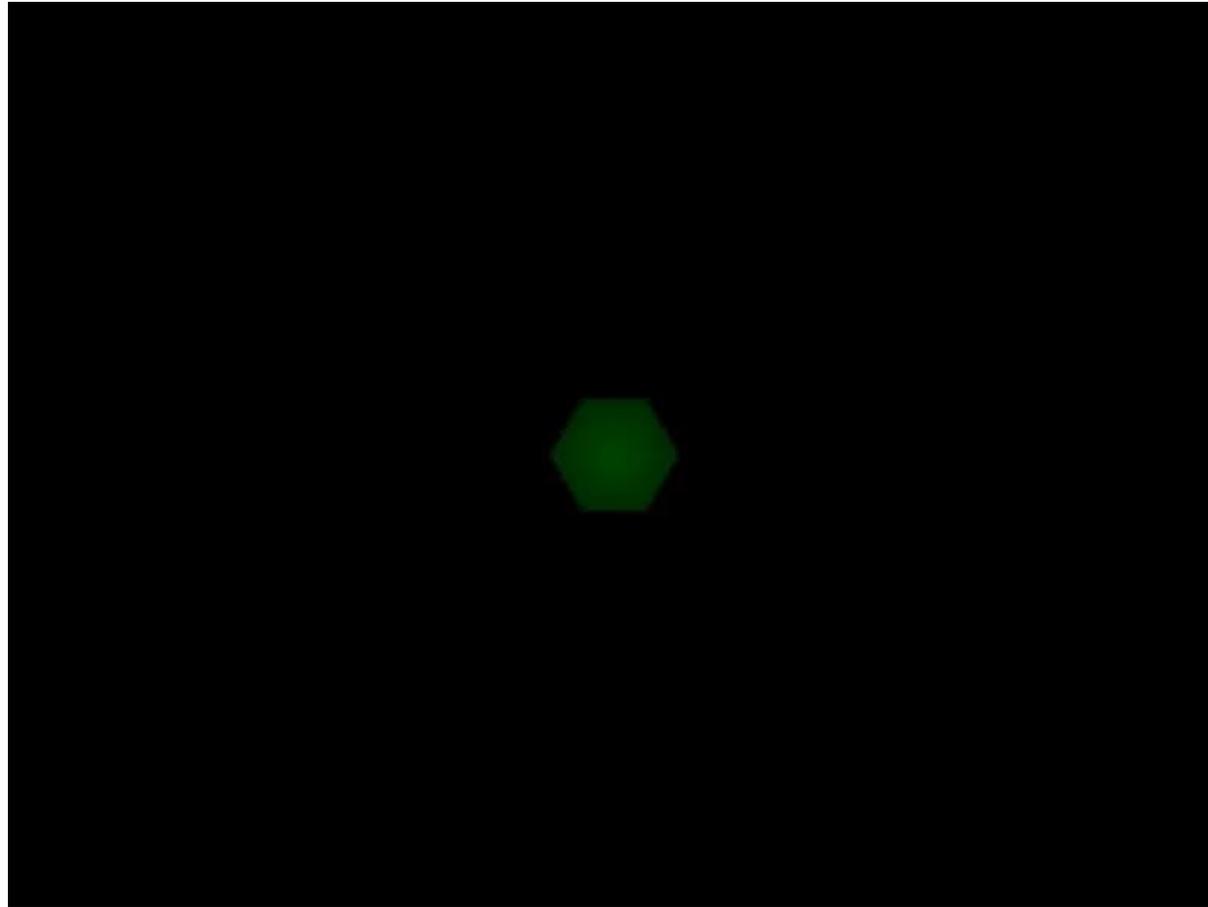
Articulated Swimming Creatures

Jie Tan Yuting Gu Greg Turk Karen Liu

Georgia Institute of Technology

<http://www.cc.gatech.edu/~jtan34/project/articulatedSwimmingCreatures.pdf>

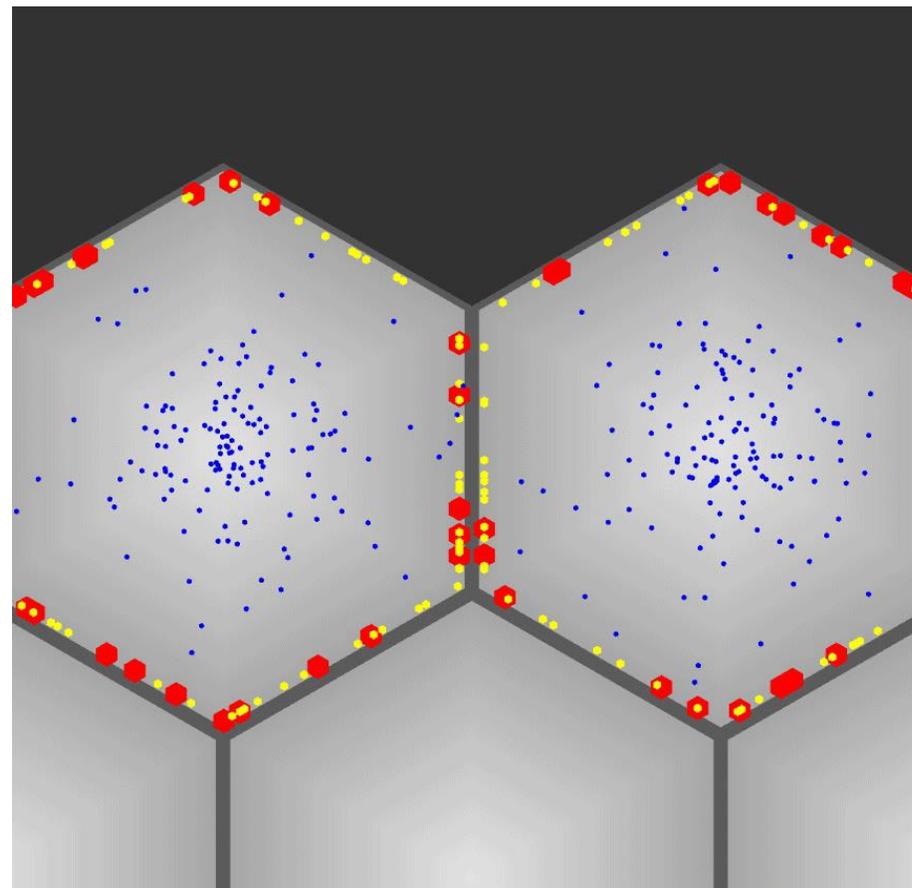
Ulistnienie roślin



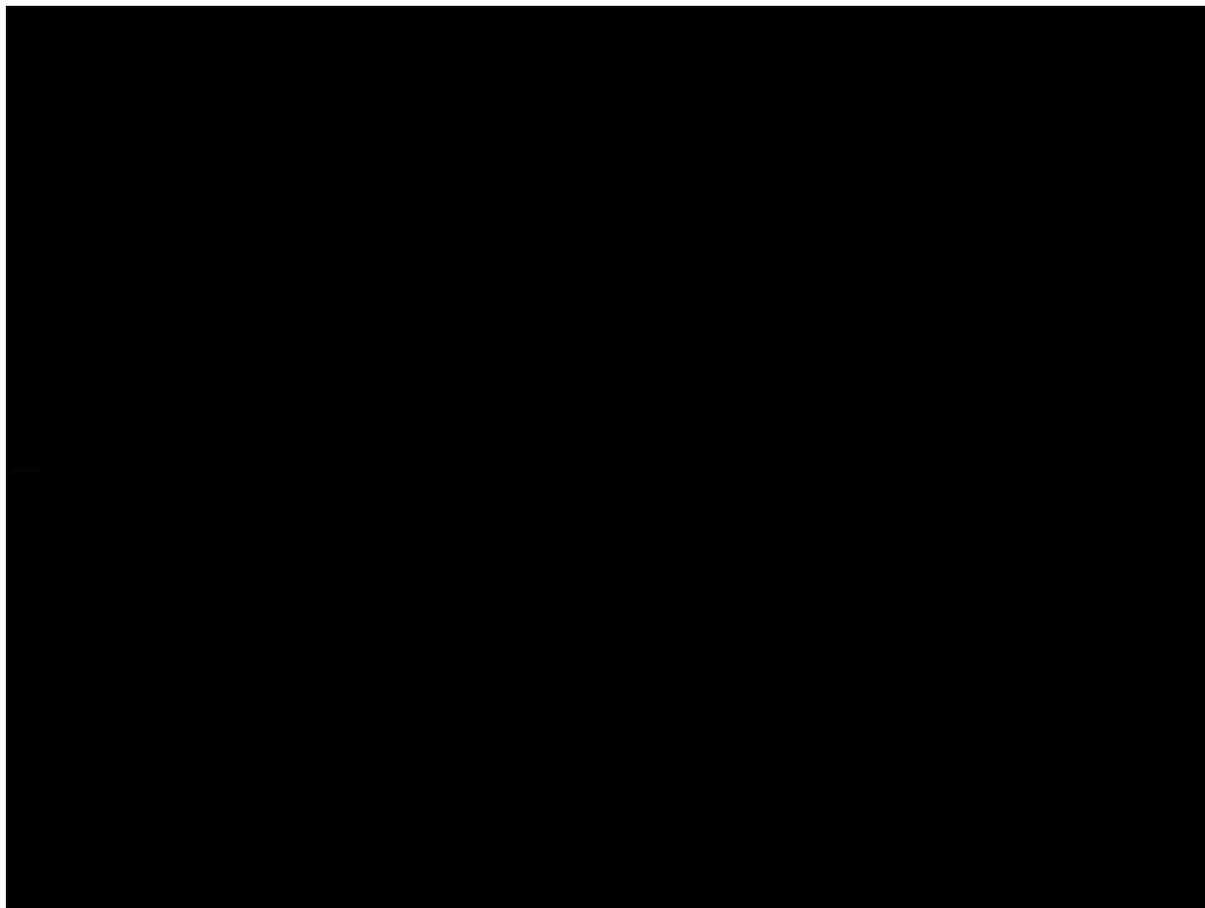
[Smith et al. 2006](#)

Transport białek w komórkach

- Auxin in cytoplasm
- Auxin in apoplast
- Efflux transporter
- Influx transporter



Zwijanie białek



<https://www.youtube.com/watch?v=iaHHgEoa2c8>

Symulacja fizyczna

Interactive Wood Combustion of
Botanical Tree Models

Online Submission ID: 0217



Skale Modelowania

Populacja



Skale Modelowania

Populacja

Organizm



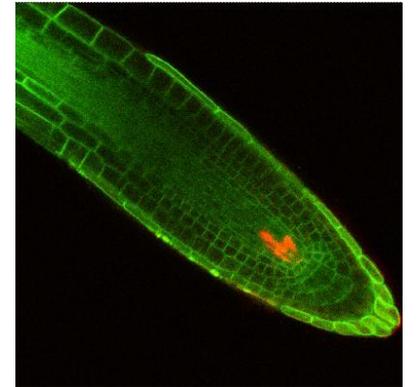
Skale Modelowania



Populacja

Organizm

Organ



Skale Modelowania

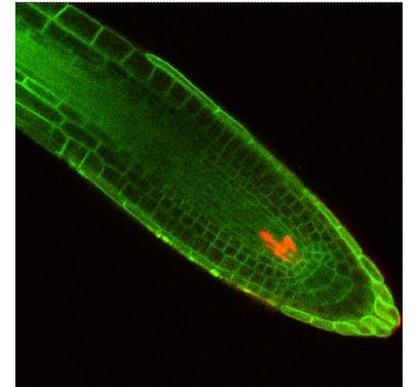
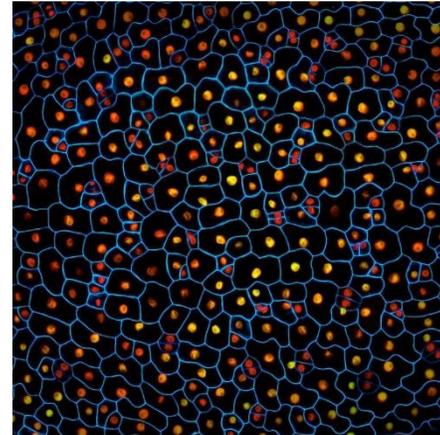


Populacja

Organizm

Organ

Tkanka



Skale Modelowania



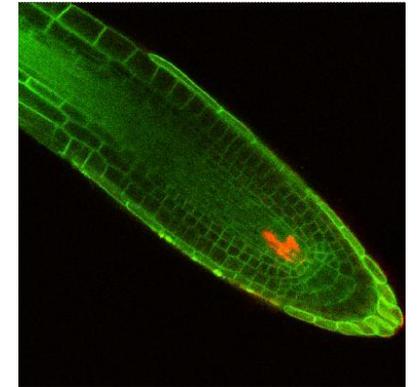
Populacja



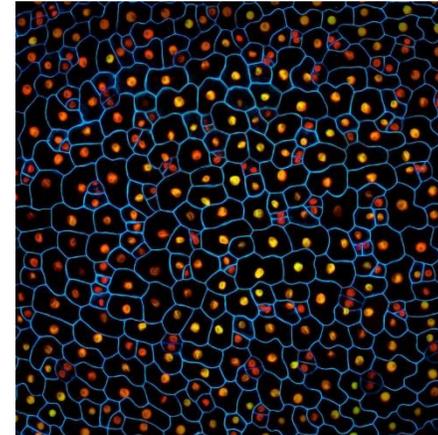
Organizm



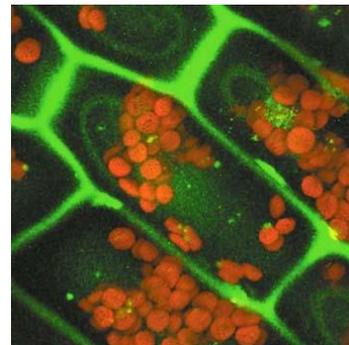
Organ



Tkanka



Komorka



Dlaczego Modelować?

- Wizualizacja pomysłów

Dlaczego Modelować?

- Wizualizacja pomysłów
- Komunikacja pomysłów

Dlaczego Modelować?

- Wizualizacja pomysłów
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- Artystyczne narzędzia
- ...

Cel tych zajęć

- Poznać najbardziej popularne matematyczne narzędzia modelowania

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- Umiejętność implementacji modeli za pomocą języków programistycznych / bibliotek → stwarzanie symulacji
- Przeprowadzić analizę danych symulacji jak i danych empirycznych

Zaliczanie SPB

- 3 projekty i prezentacje
- Egzamin
- Uczestnictwo na zajęciach

Różne Sposoby Modelowania

Komórkowe automaty

Różne Sposoby Modelowania

L-Systemy

Komórkowe automaty

Różne Sposoby Modelowania

L-Systemy

Komórkowe automaty

Modele agentowe

Różne Sposoby Modelowania

Modele cząstkowe

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Różne Sposoby Modelowania

Modele wierzchołkowe

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Różne Sposoby Modelowania

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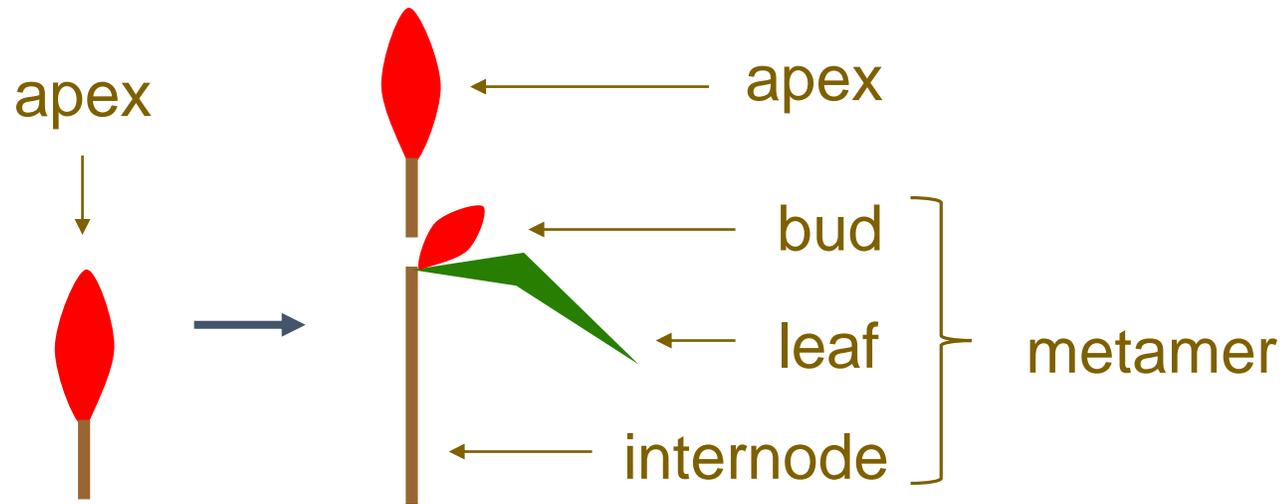
Modele skończonych elementów

Modele tensorowe

L-Systemy

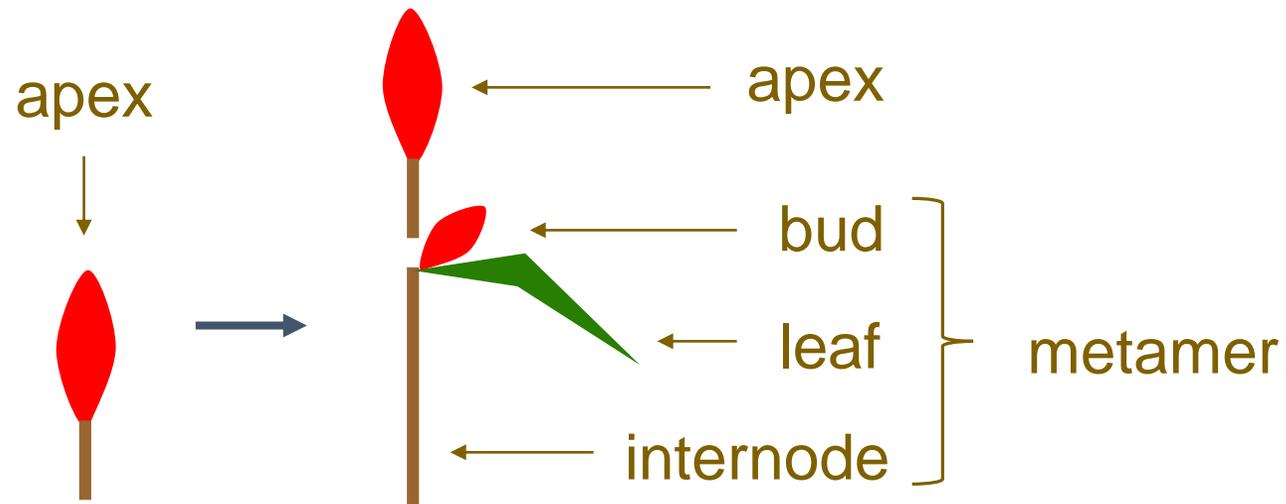
Jak wyrazić rozwój
matematycznie?

The fundamental developmental scheme



rule

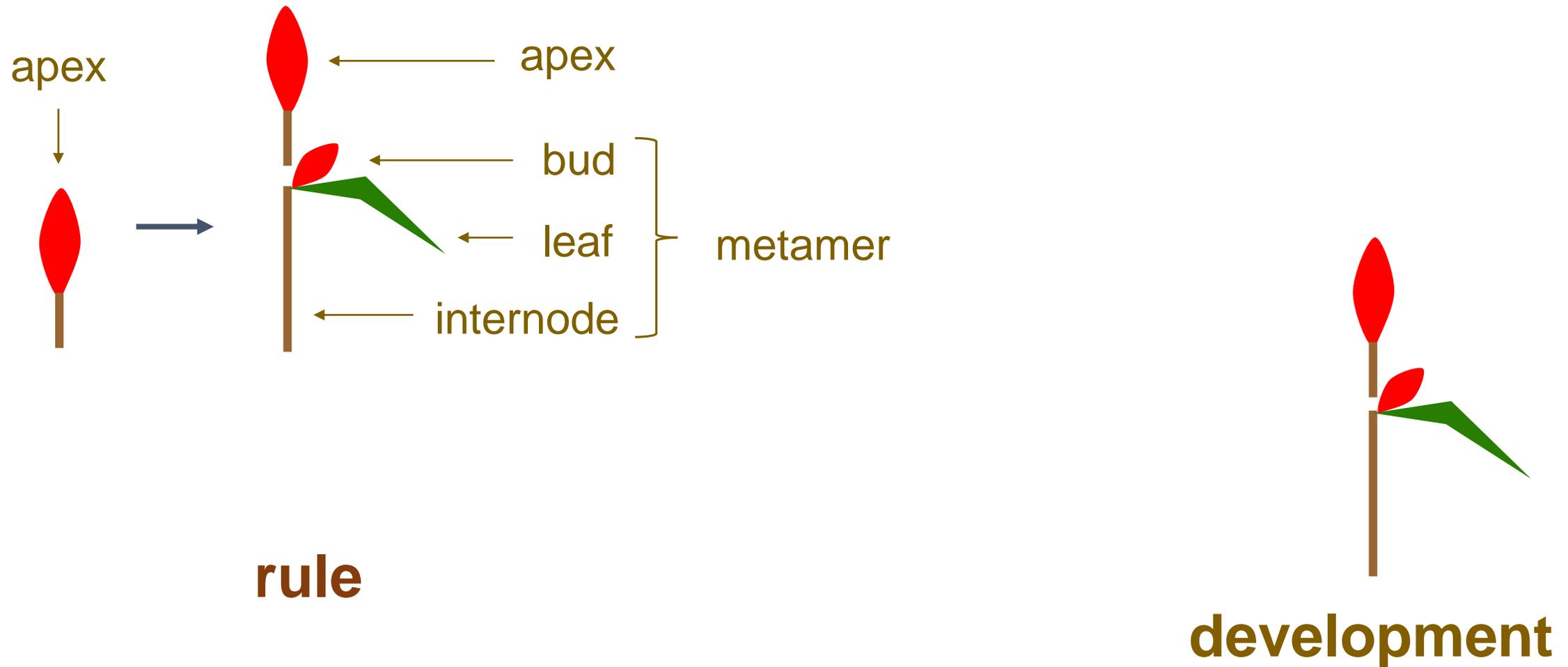
The fundamental developmental scheme



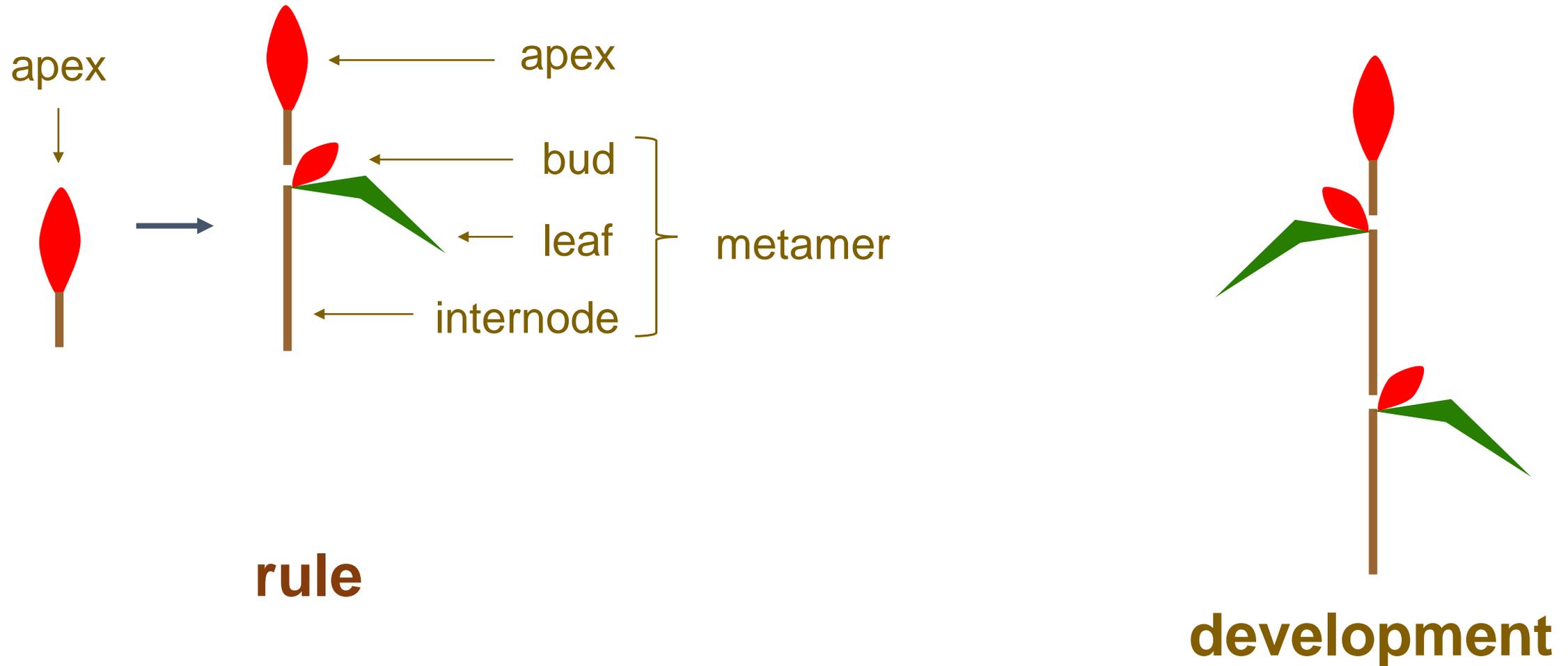
rule



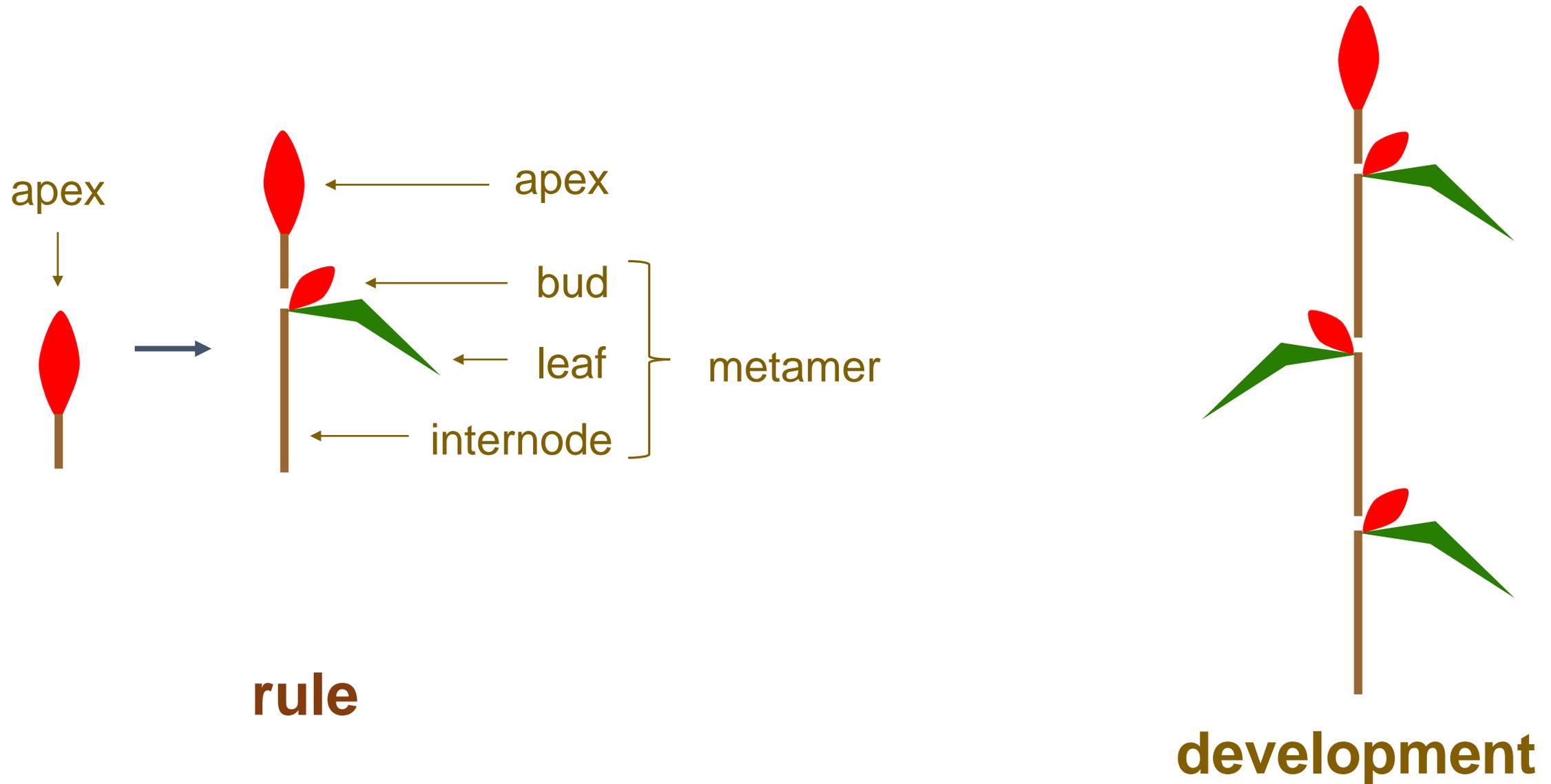
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The fundamental developmental scheme

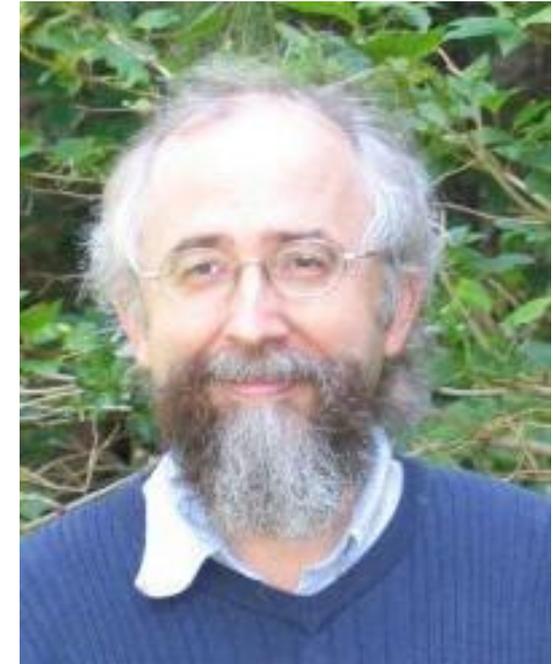


The fundamental developmental scheme



L-Systemy: język do modelowania wzrostu

- Później zastosowano do wzrostu roślin



Przemysław Prusinkiewicz

D0L-Systems

- Formal grammar $G = (V, \omega, P)$
- $V \rightarrow$ alphabet of symbols containing elements that can be replaced
- $\omega \rightarrow$ axiom (the initial state of the system)
- $P \rightarrow$ a set of production rules defining the way symbols can be replaced

- Example:
 - $V = (A, M)$
 - $\omega = A$
 - $P = [(A \rightarrow MA), (M \rightarrow M)]$

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n=1: MA

n=2: MMA

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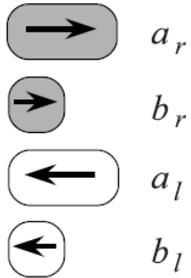
n=0: A

n=1: MA

n=2: MMA

n=3: MMMA

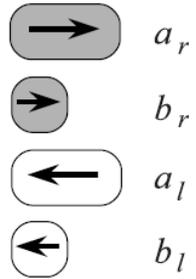
Anabaena Catenula: Model 1



Anabaena Catenula: Model 1

$$p_1 : a_r \rightarrow a_l b_r$$

$$p_2 : a_l \rightarrow b_l a_r$$



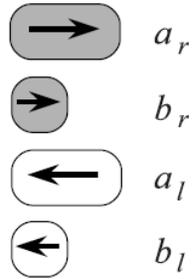
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$$p_3 : b_r \rightarrow a_r$$

$$p_4 : b_l \rightarrow a_l$$



Anabaena Catenula: Model 1

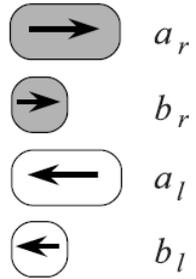
$\emptyset : a_r$

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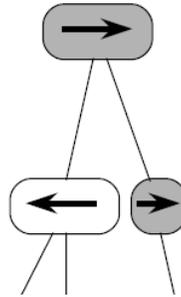
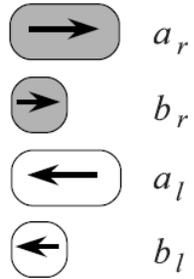
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a_r

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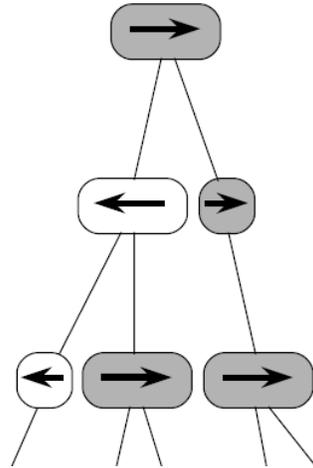
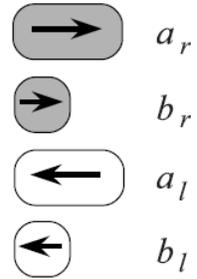
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a_r

$a_l b_r$

$b_l a_r a_r$

Anabaena Catenula: Model 1

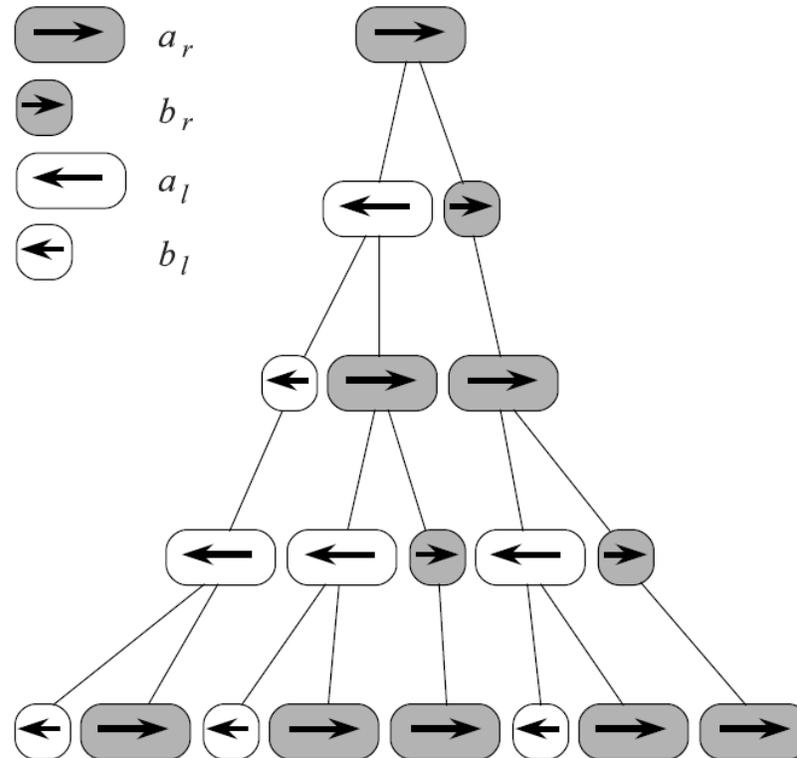
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a_r

$a_l b_r$

$b_l a_r a_r$

$a_l a_l b_r a_l b_r$

$b_l a_r b_l a_r a_r b_l a_r a_r$

L-Py

The screenshot shows the L-Py software interface. The main window is titled "L-Py - New" and contains a menu bar (File, L-systems, Edit, View, Help), a toolbar with icons for New, Open, Save, Save All, Close, Run, Animate, Stop, Rewind, Step, Debug, Profile, and View 3D, and a tabbed editor area with tabs for test, ANAModel2, New, bush, stochBush, bush3D, and Equisetum. The code editor displays the following L-system code:

```
1 Axiom:  
2  
3 derivation length: 1  
4 production:  
5  
6  
7 interpretation:  
8  
9  
10 endlsystem  
11 |
```

On the left side, there are panels for Description, Information, References, and Authors. At the bottom, there is a Help Card with the following content:

-	IncWidth	Increase the current line width or set it if a parameter is given. Params : line width (optional).	▲
!	DecWidth	Decrease the current line width or set it if a parameter is given. Params : line width (optional).	▼
SetWidth		Set current line width. Params : line width.	↕
Color			
;	IncColor	Increase the current material index or set it if a parameter is given. Params : color index (optional, positive int).	▼

At the bottom right, the status bar shows "Line 11, Column 0 (73)".

L-Py

```
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2  
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5  
6  
7  interpretation:  
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10 endlsystem  
11
```

L-Py (production)

- Reguła produkcji: $A \rightarrow AB$

A: produce A B

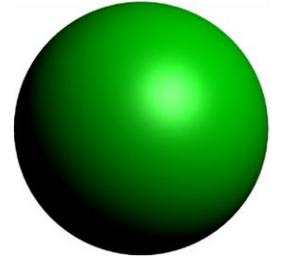
- Przejście w przestrzeni w bok:

f(długosc)

L-Py (interpretation)

- Rysowanie kuli:

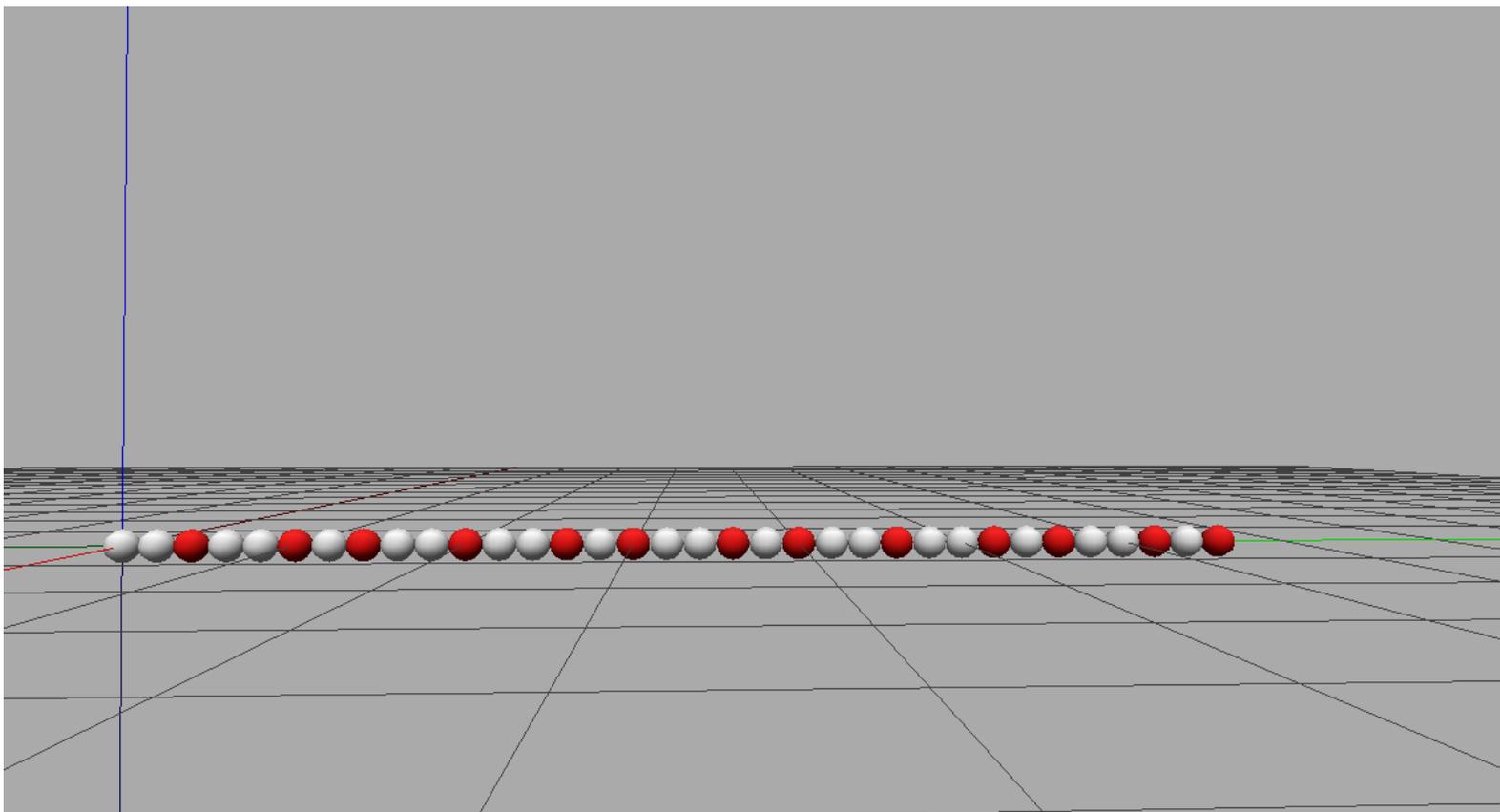
```
@0(rozmiar)
```



- Ustawianie koloru:

```
SetColor(liczba od 0 255)
```

Zaimplementuj model 1 w L-Py



Anabaena Catenula: Model 2

- Poprzedni model nie bierze pod uwagę zmiany wielkości komórek
- W eksperymentach w laboratorium zauważono że bakterie A się dzielą co 15 godzin a mniejsze bakterie typu B zmieniają się na bakterie A co 3 godziny
- Jednostka czasu dzieląca kroki symulacji może więc być 3 godziny

Parametric L-Systems

$$A(x) \xrightarrow{x>0} A(x-1)$$

$$A(x) \xrightarrow{x=0} A(x+3)B(x+3)$$

$$A(3) \rightarrow A(2) \rightarrow A(1) \rightarrow A(0) \rightarrow A(3)B(3)$$

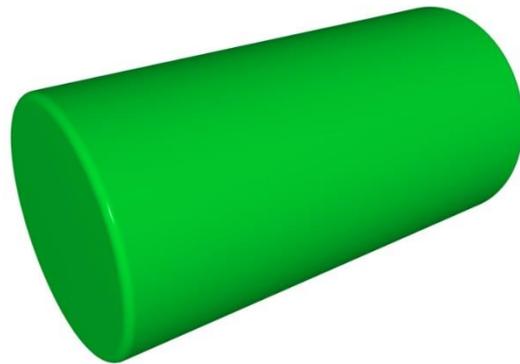
L-Py

```
A(x): if x > 0: produce A(x-1)
```

L-Py (interpretation)

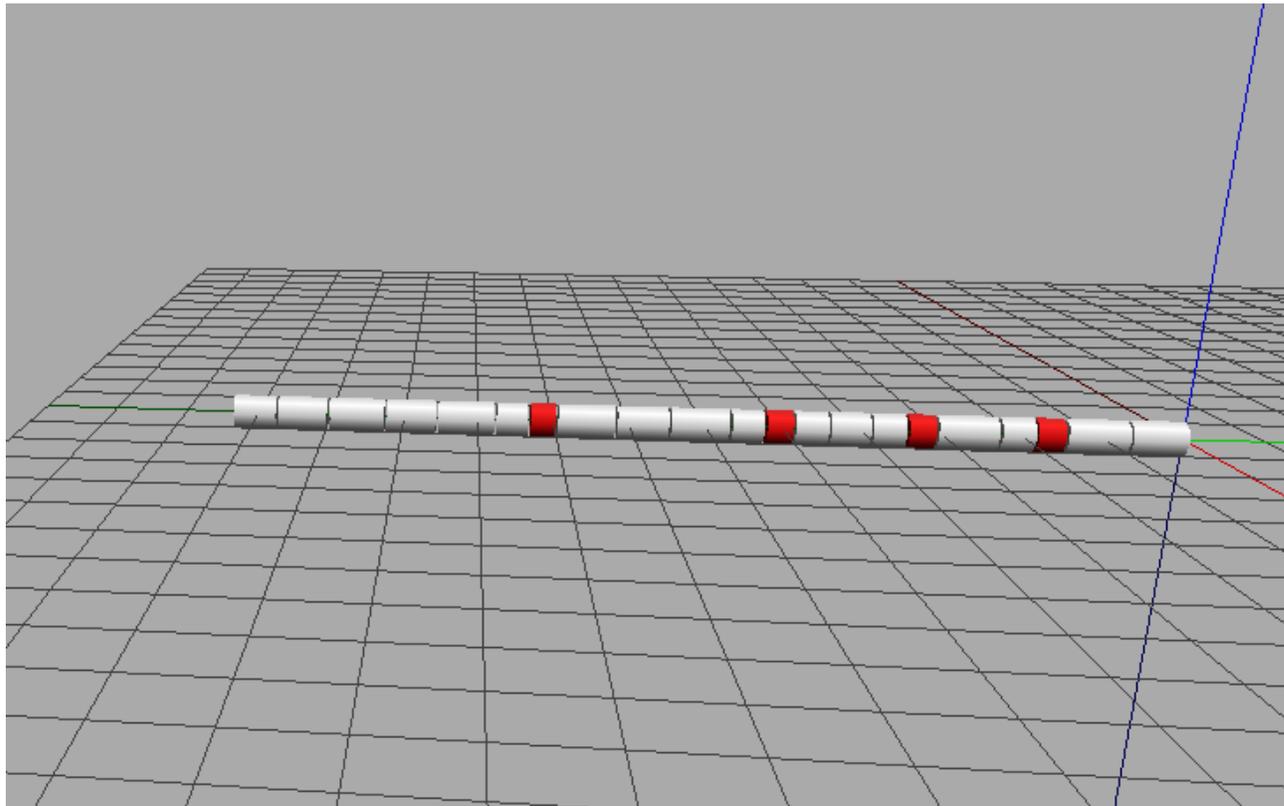
- Rysowanie cylindra:

F(długość)



Zaimplementuj model 2 w L-Py

- Załóż że jeden krok symulacji to 3 godziny czasu w rozwoju *Anabaena Catenula*

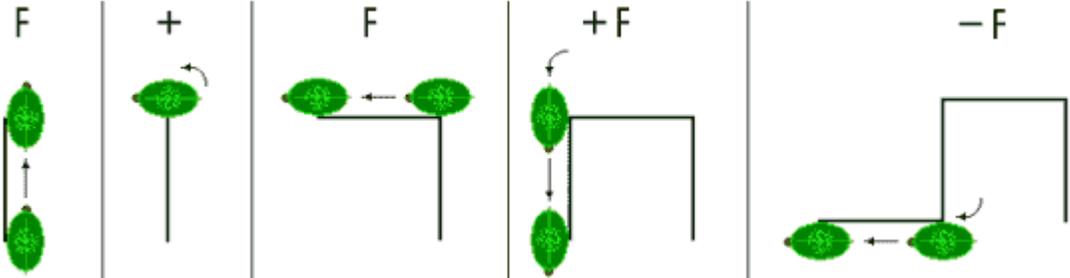


Struktury drzewiaste?



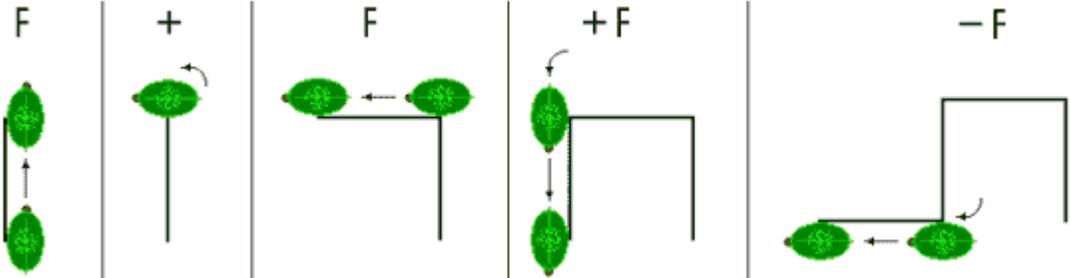
Turtle graphics

F+F+F-F

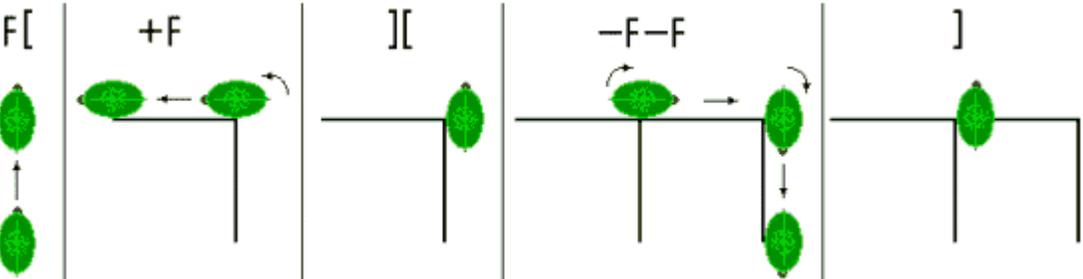


Turtle graphics

F+F+F-F

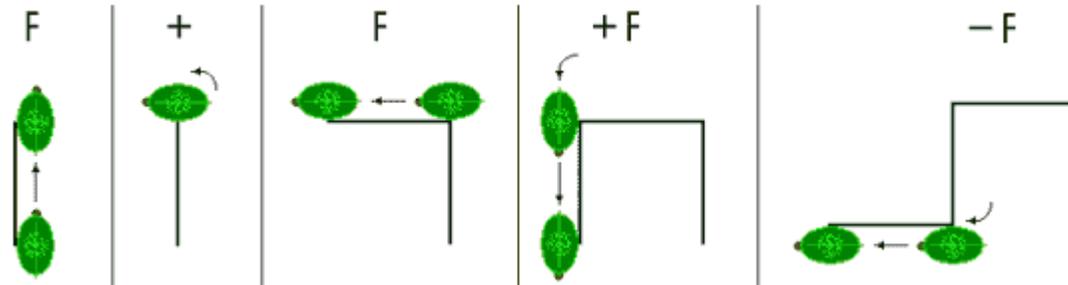


F[+F][-F-F]F



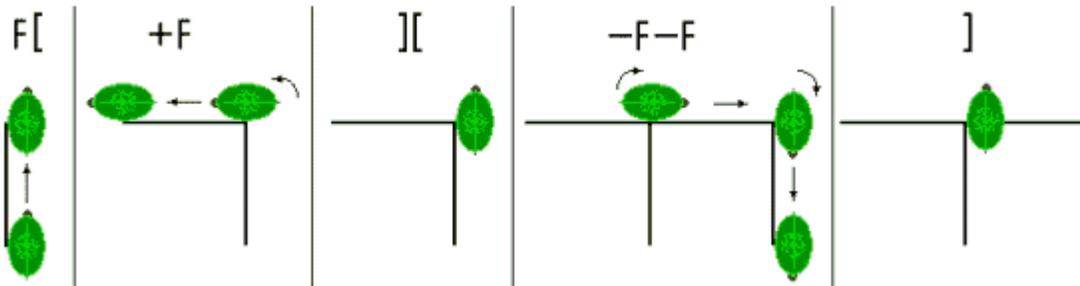
Turtle graphics

F+F+F-F



L-Py: F(1)+(90)...

F[+F][-F-F]F

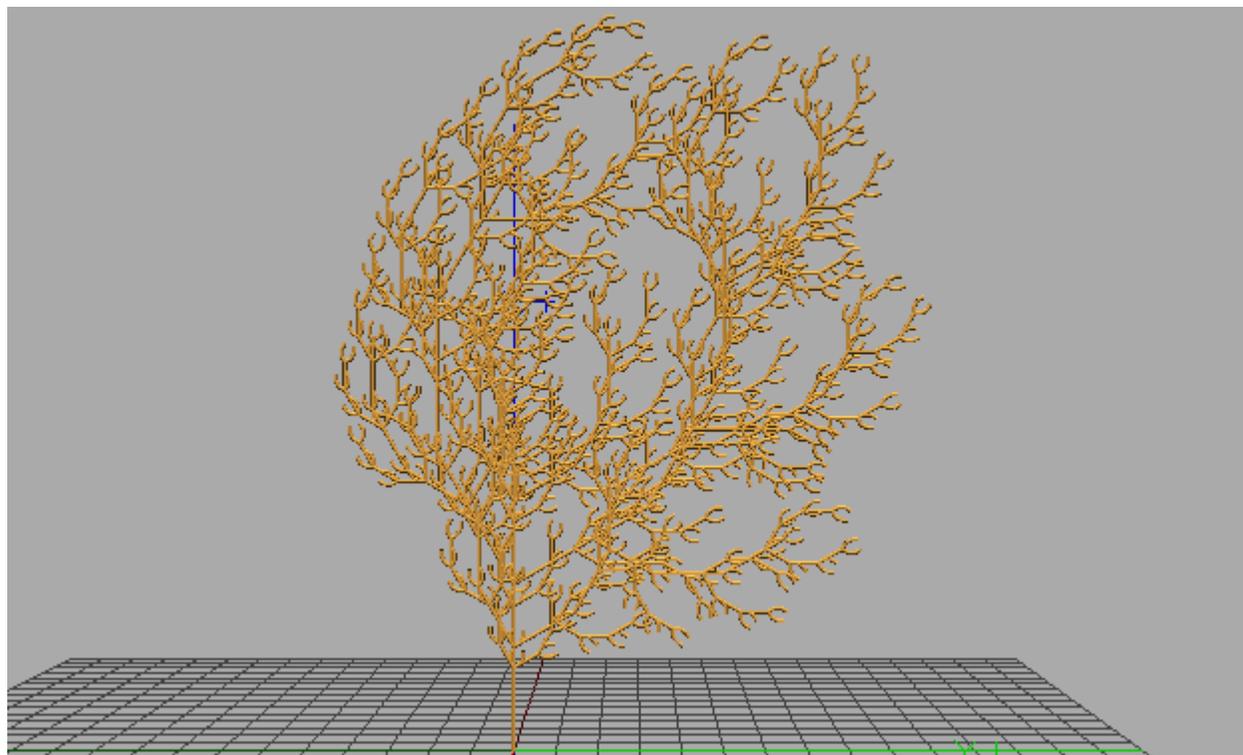


Struktury drzewiaste 2D

$\omega: F$

$p_1: F \rightarrow FF[- - F + F + F][+ F - F - F]$

$\delta = 30^\circ$



Stochastyczny L-System

L-Py:

```
import random
```

```
random.random() zwraca liczbe [0,1]
```

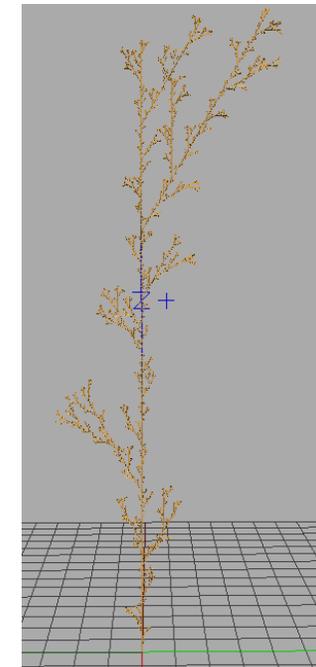
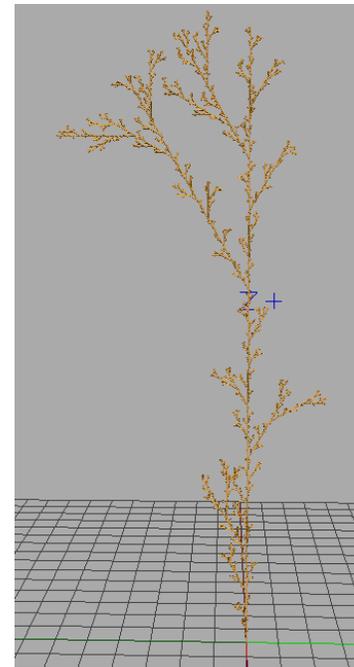
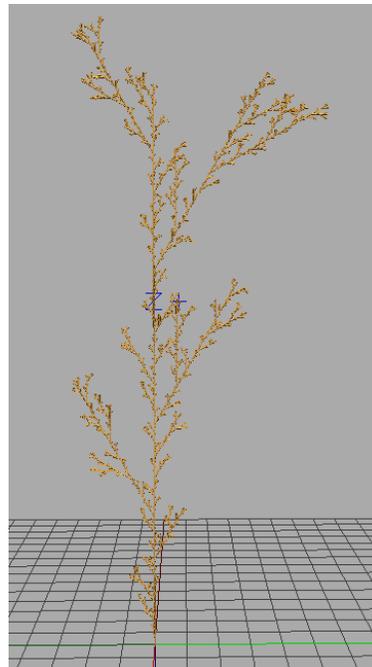
$\omega: F$

$p_1: F \xrightarrow{0.33} F[+F]F[-F]F$

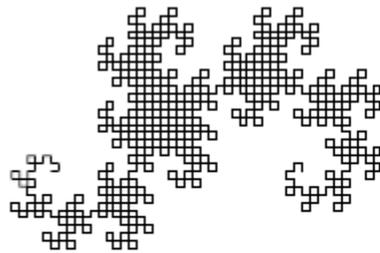
$p_2: F \xrightarrow{0.33} F[+F]F$

$p_3: F \xrightarrow{0.33} F[-F]F$

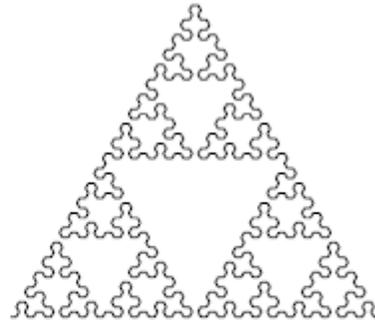
$\delta = 30^\circ$



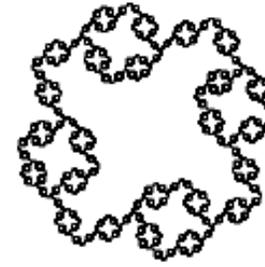
Fraktale za pomocy L-Systems



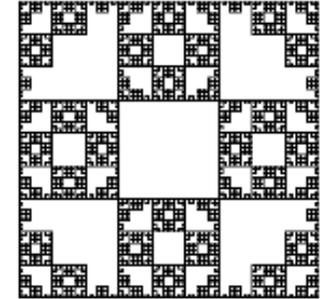
a $n=10, \delta=90^\circ$
 F_1
 $F_1 \rightarrow F_1 + F_2 +$
 $F_2 \rightarrow -F_1 - F_2$



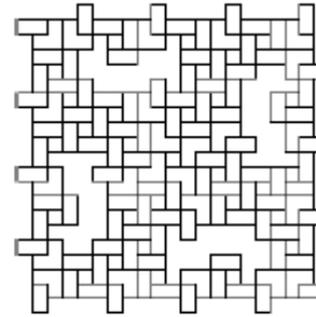
b $n=6, \delta=60^\circ$
 F_r
 $F_1 \rightarrow F_r + F_1 + F_r$
 $F_r \rightarrow F_1 - F_r - F_1$



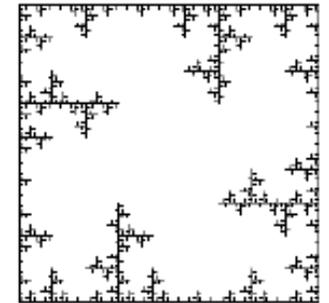
a $n=4, \delta=90^\circ$
 $F-F-F-F$
 $F \rightarrow FF-F-F-F-F+F$



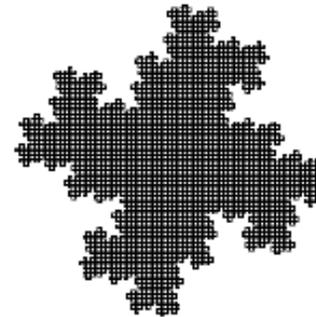
b $n=4, \delta=90^\circ$
 $F-F-F-F$
 $F \rightarrow FF-F-F-F-F$



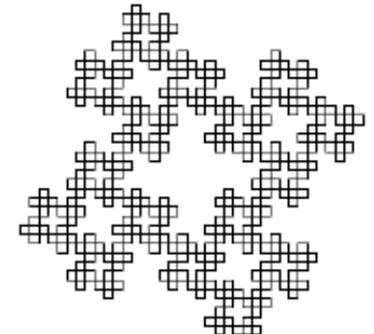
c $n=3, \delta=90^\circ$
 $F-F-F-F$
 $F \rightarrow FF-F+F-F-F$



d $n=4, \delta=90^\circ$
 $F-F-F-F$
 $F \rightarrow FF-F-F-F$

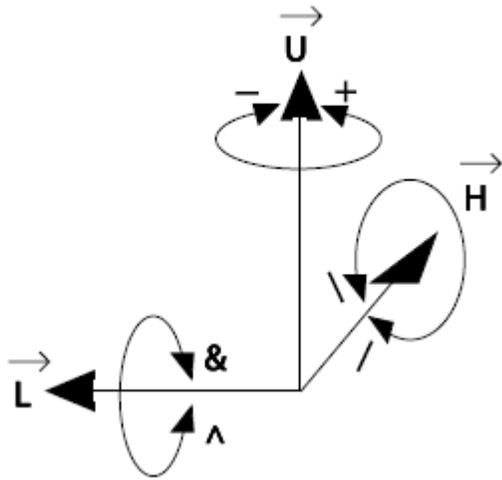


e $n=5, \delta=90^\circ$
 $F-F-F-F$
 $F \rightarrow F-FF-F-F$



f $n=4, \delta=90^\circ$
 $F-F-F-F$
 $F \rightarrow F-F+F-F-F$

L-System 3D



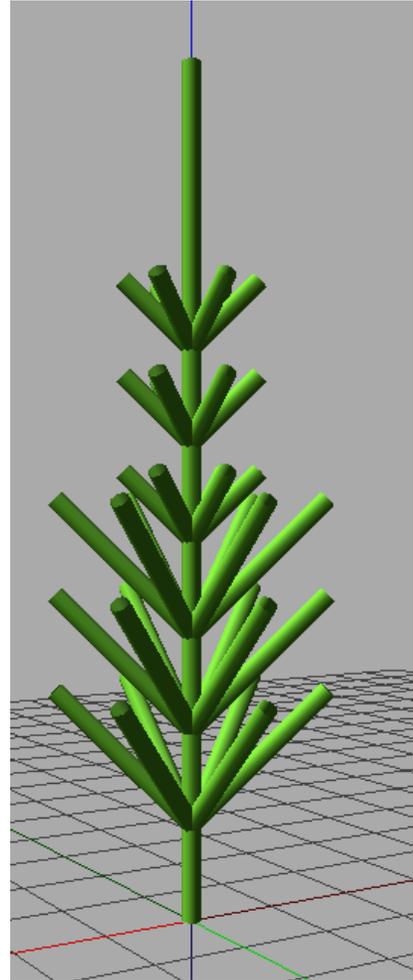
$$\mathbf{R}_U(\alpha) = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_L(\alpha) = \begin{bmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix}$$

$$\mathbf{R}_H(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$

$$[\vec{H}' \ \vec{L}' \ \vec{U}'] = [\vec{H} \ \vec{L} \ \vec{U}] \mathbf{R}$$

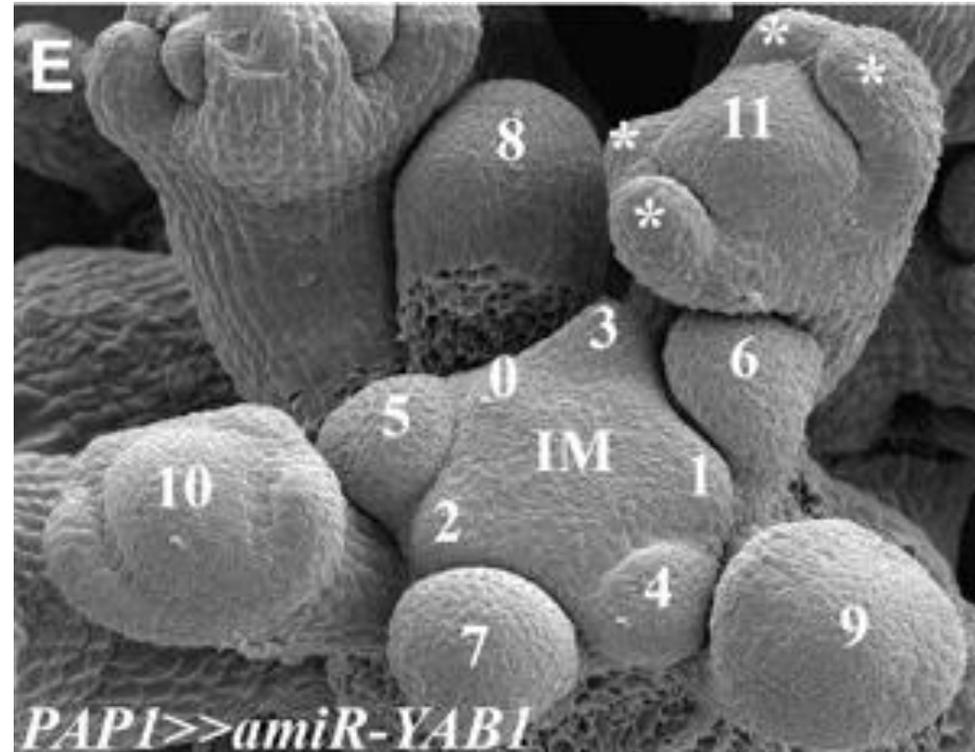
Equisetum Arvense



Filotaksja



Merystem wierzchołkowy



<http://www.plantcell.org/content/20/5/1217/F5.expansion>



Monostichous (1,7)
 $j = 1, p = 7, \theta = 47.3^\circ$



Lucas (3,4) Spiral
 $j = 1, p = 3, \theta = 99.5^\circ$



Fibonacci (3,5) Spiral
 $j = 1, p = 2, \theta = 137.5^\circ$



Anomalous (5,7) Spiral
 $j = 1, q = 2, \theta = 151.1^\circ$



Spiro-distichous (2,13)
 $j = 1, q = 6, \theta = 167.4^\circ$



Distichous
 $j = 1, \theta = 180^\circ$



Bijugate 2x(2,3)
 $j = 2, p = 2, \theta = 68.8^\circ$



Spiro-decussate 2x(2,13)
 $j = 2, q = 6, \theta = 83.7^\circ$



Decussate
 $j = 2, \theta = 90^\circ$



Multijugate 3x(1,2)
 $j = 3, p = 2, \theta = 45.8^\circ$



Tricussate
 $j = 3, \theta = 60^\circ$



Whorled
 $j = 4, \theta = 45^\circ$



Monostichous (1,7)
 $j = 1, p = 7, \theta = 47.3^\circ$



Lucas (3,4) Spiral
 $j = 1, p = 3, \theta = 99.5^\circ$



Fibonacci (3,5) Spiral
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 $j = 1, q = 6, \theta = 167.4^\circ$



Distichous
 $j = 1, \theta = 180^\circ$



Bijugate 2x(2,3)
 $j = 2, p = 2, \theta = 68.8^\circ$



Spiro-decussate 2x(2,13)
 $j = 2, q = 6, \theta = 83.7^\circ$



Decussate
 $j = 2, \theta = 90^\circ$



Multijugate 3x(1,2)
 $j = 3, p = 2, \theta = 45.8^\circ$



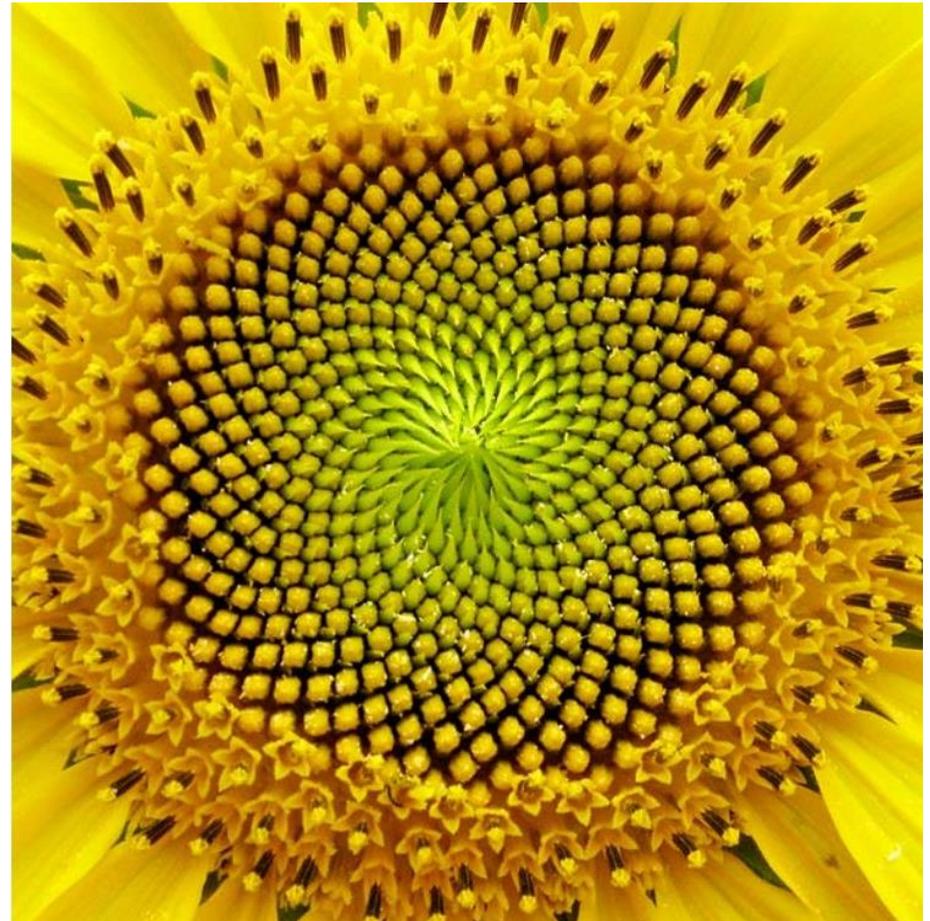
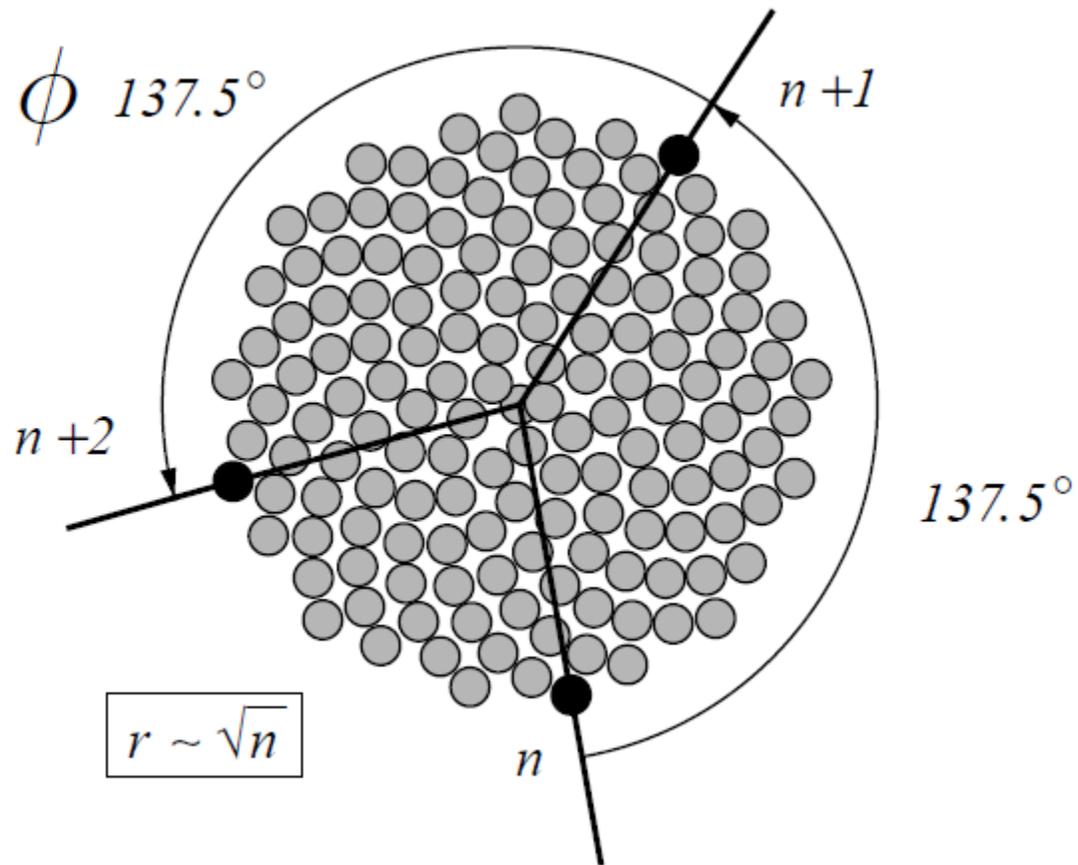
Tricussate
 $j = 3, \theta = 60^\circ$



Whorled
 $j = 4, \theta = 45^\circ$

Równanie Vogel'a

$$\phi = n * 137.5^\circ, \quad r = c\sqrt{n}$$



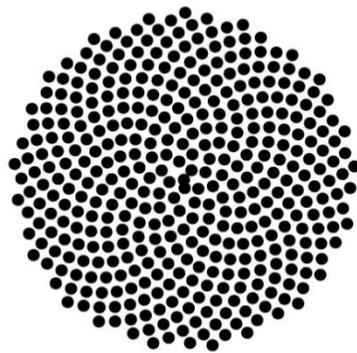
Różne kąty dywergencji

a



137.3

b



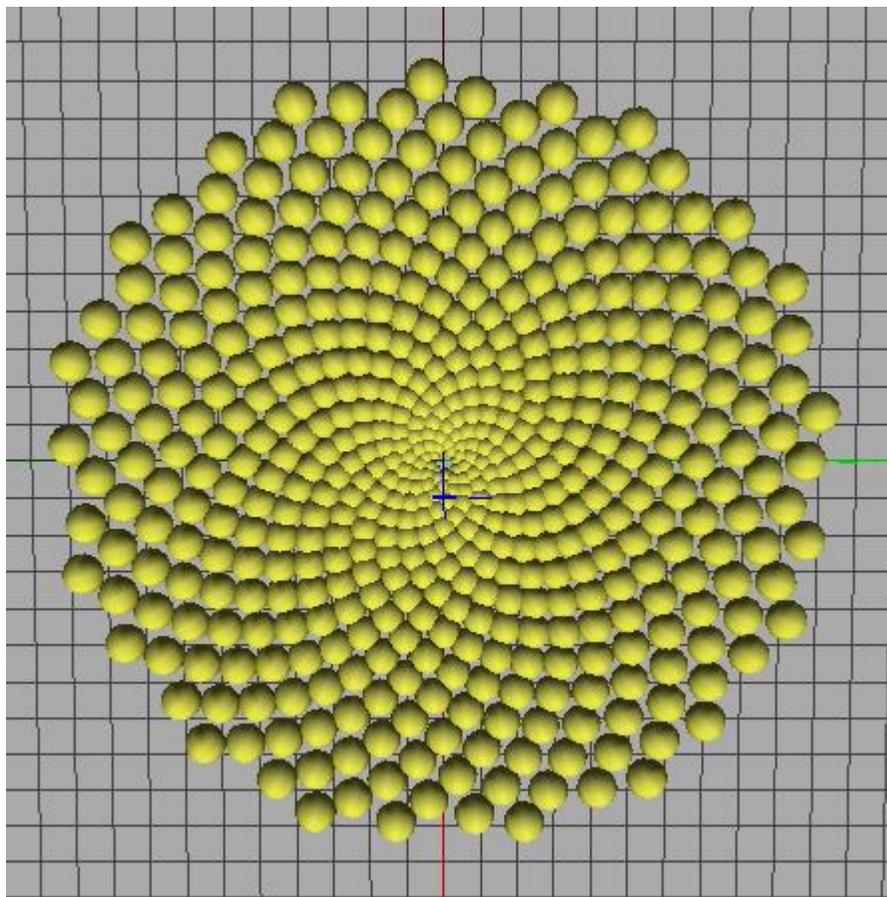
137.5

c

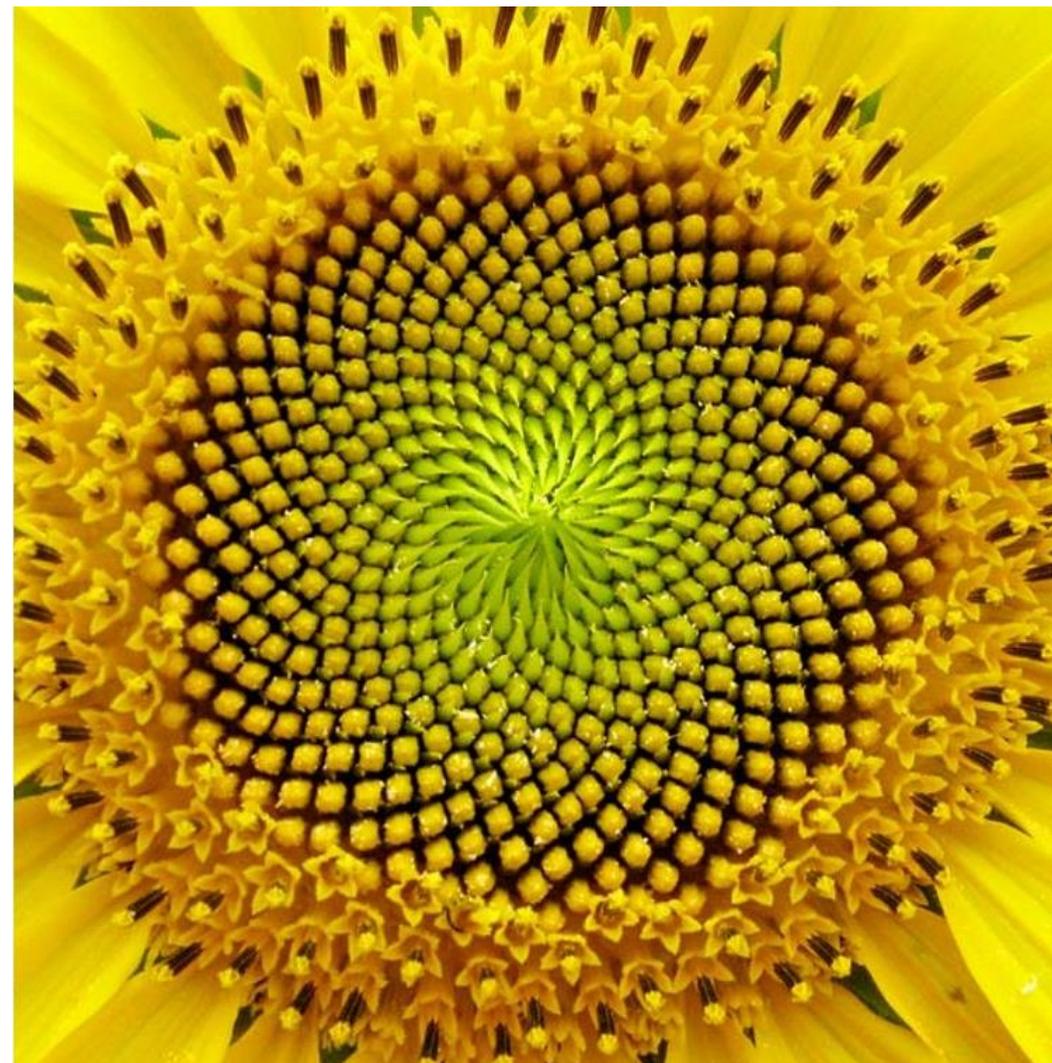


137.6

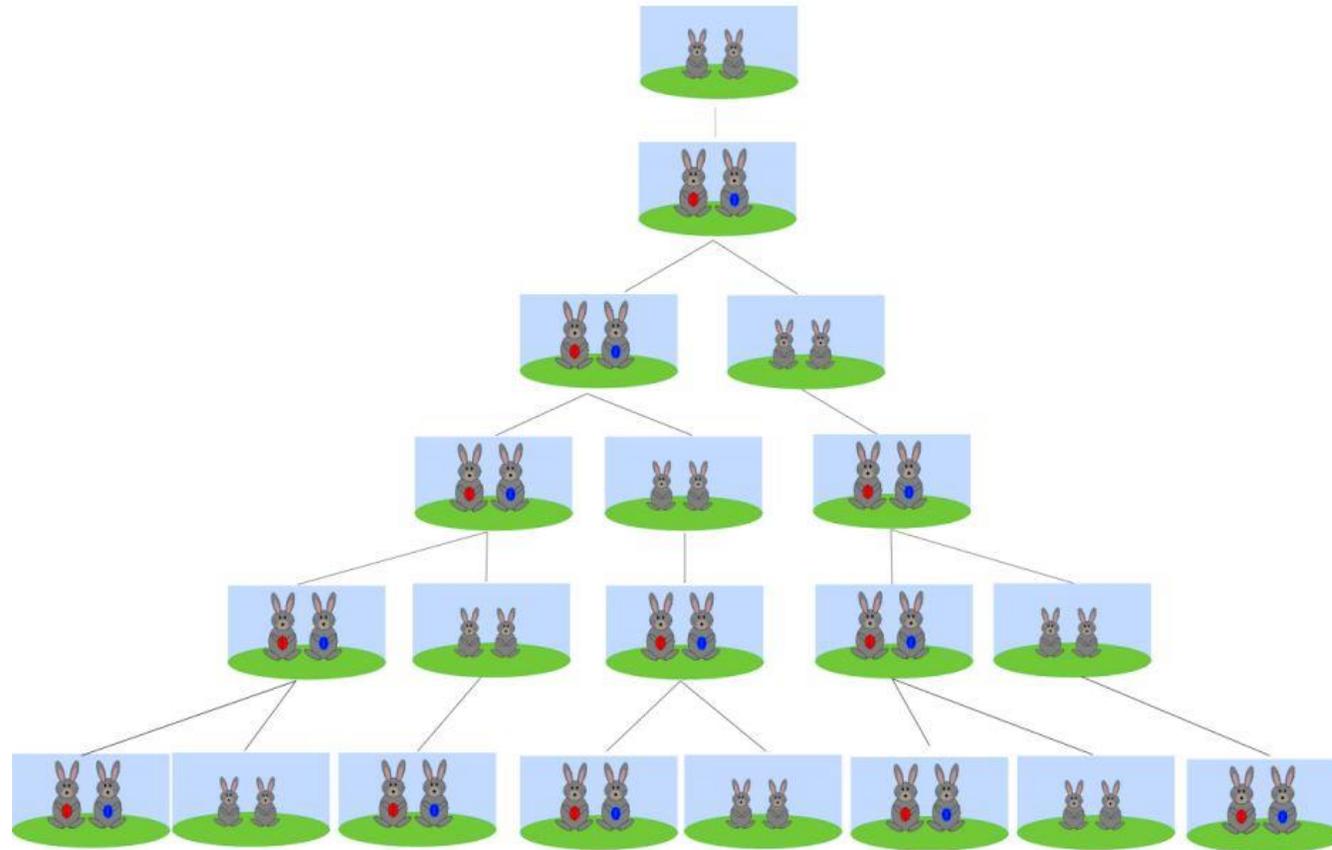
Model Słonecznika



Fibonacci, 137.5



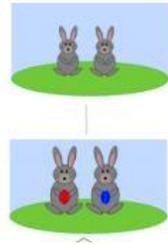
Fibonacci numbers



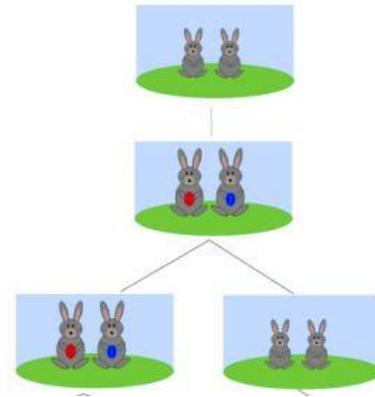
Fibonacci numbers



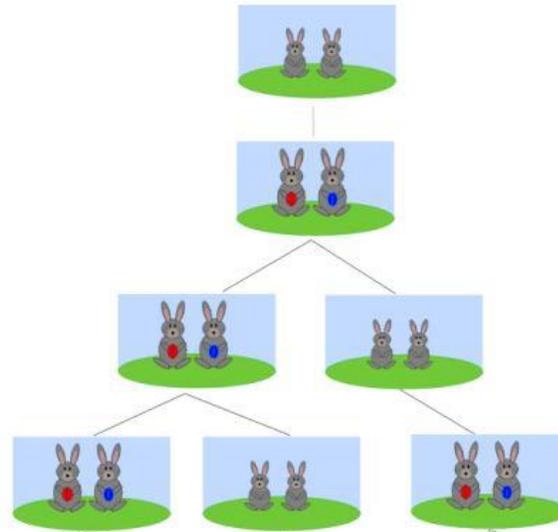
Fibonacci numbers



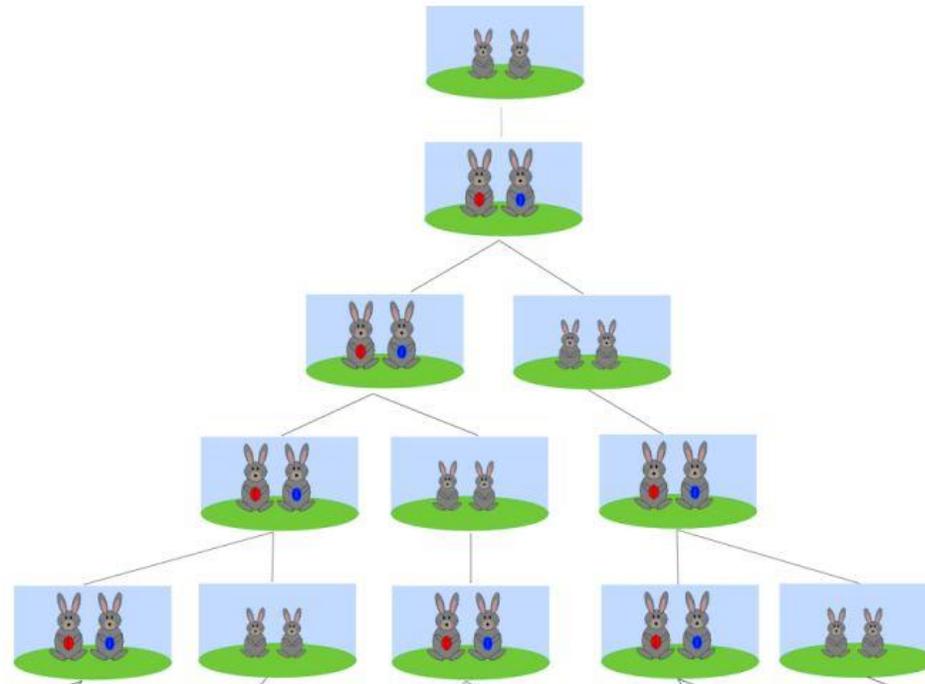
Fibonacci numbers



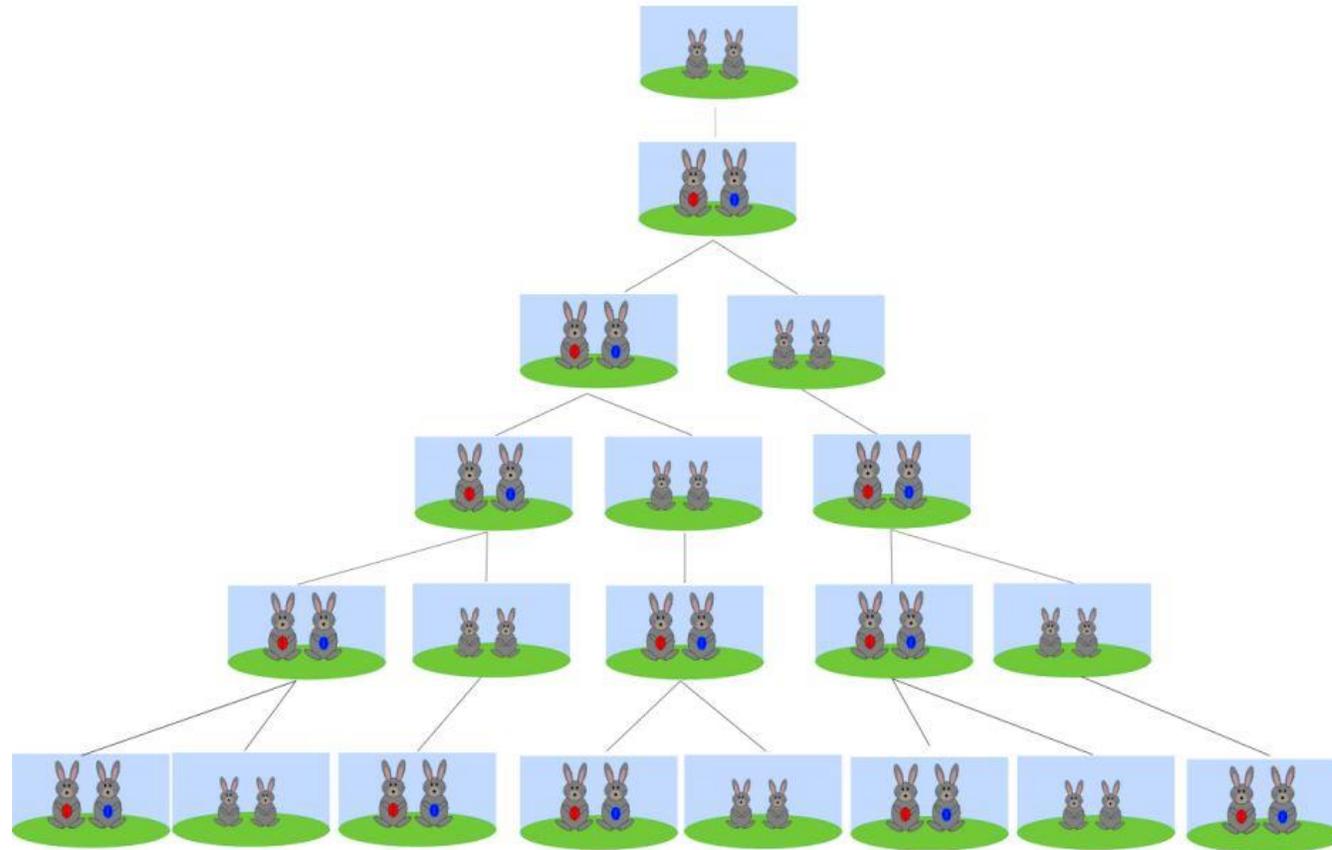
Fibonacci numbers



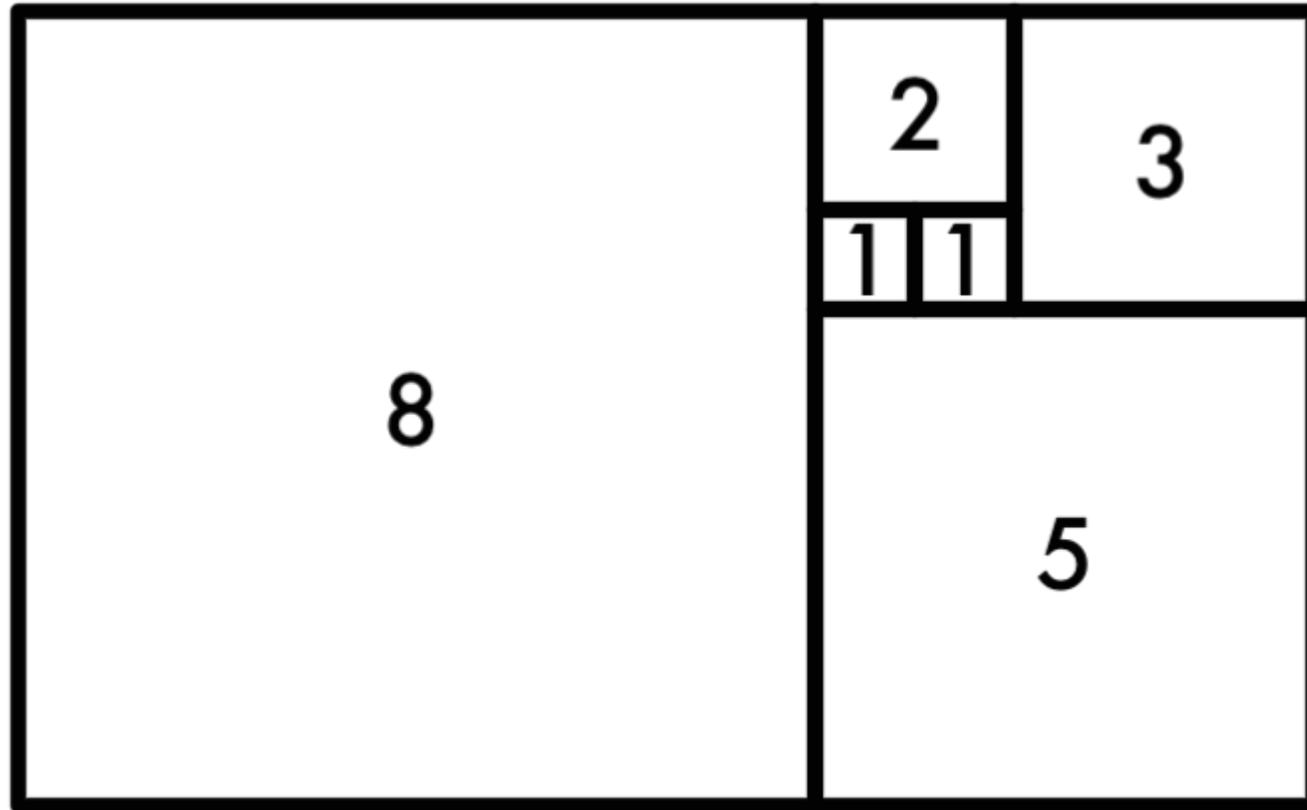
Fibonacci numbers



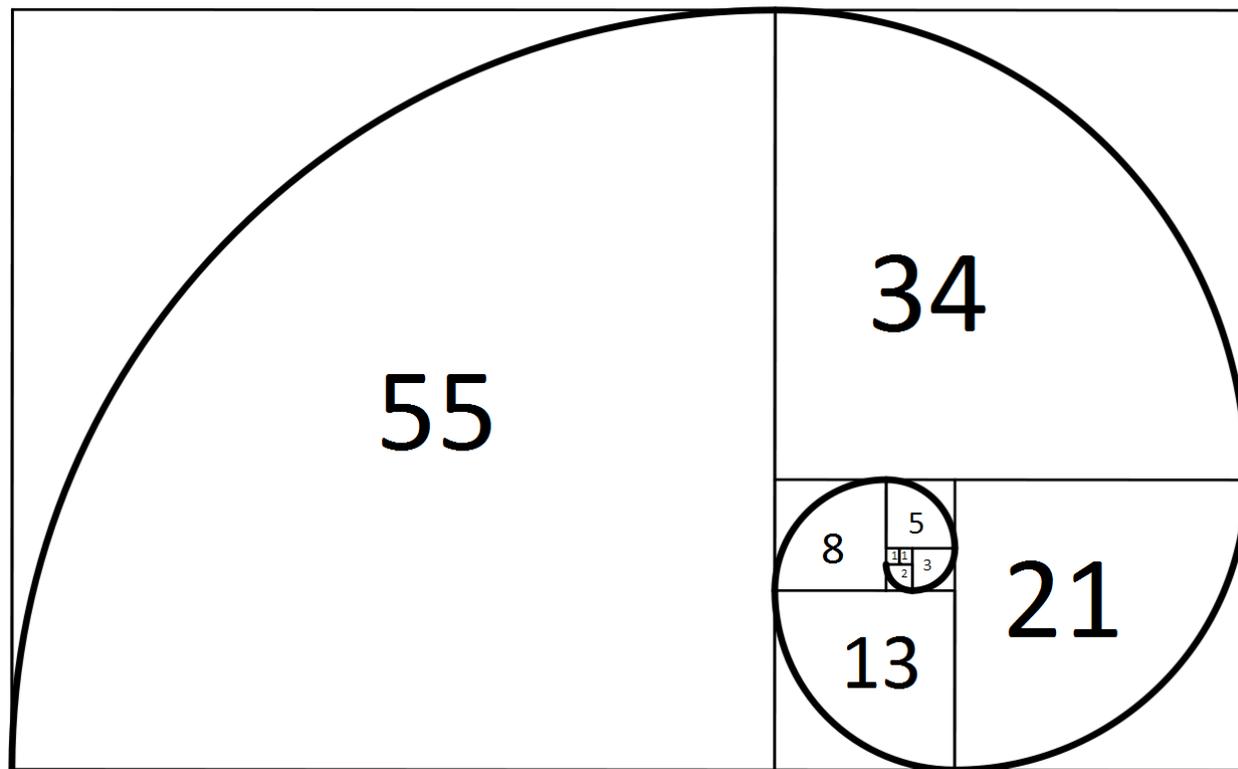
Fibonacci numbers



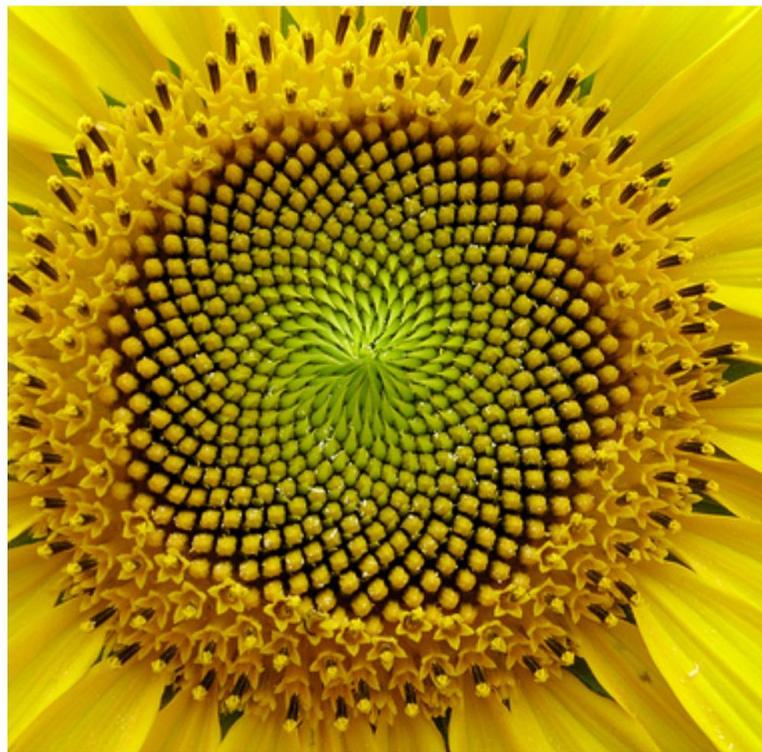
Geometric interpretation



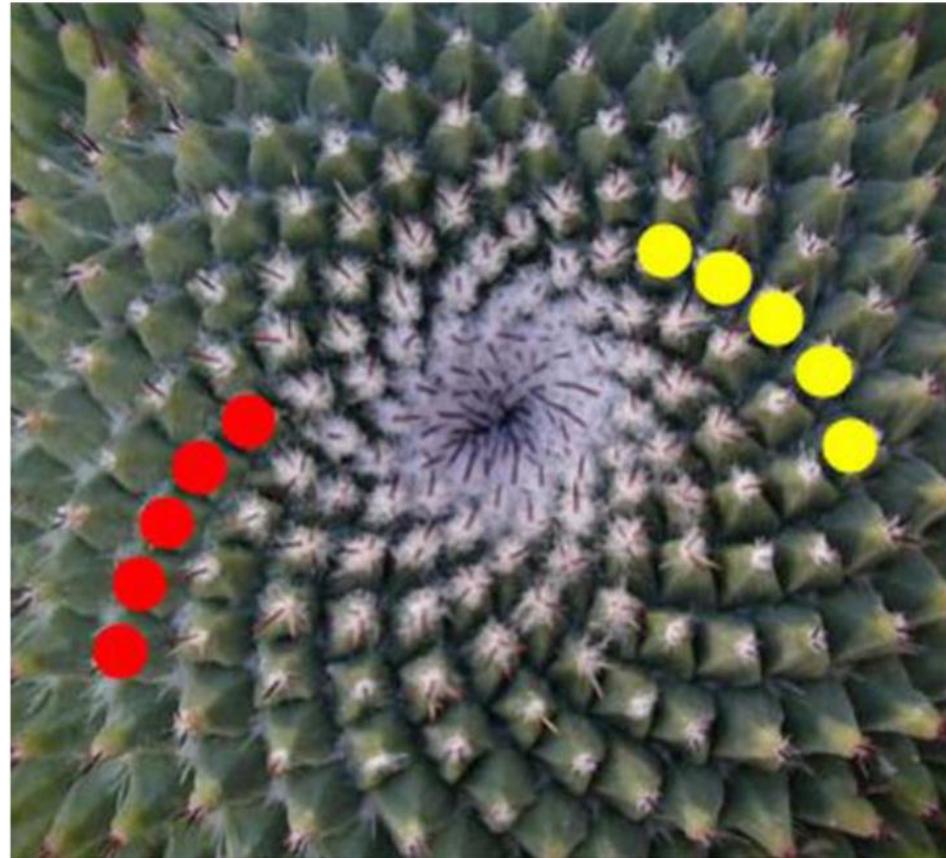
Fibonacci spirals



Przykłady

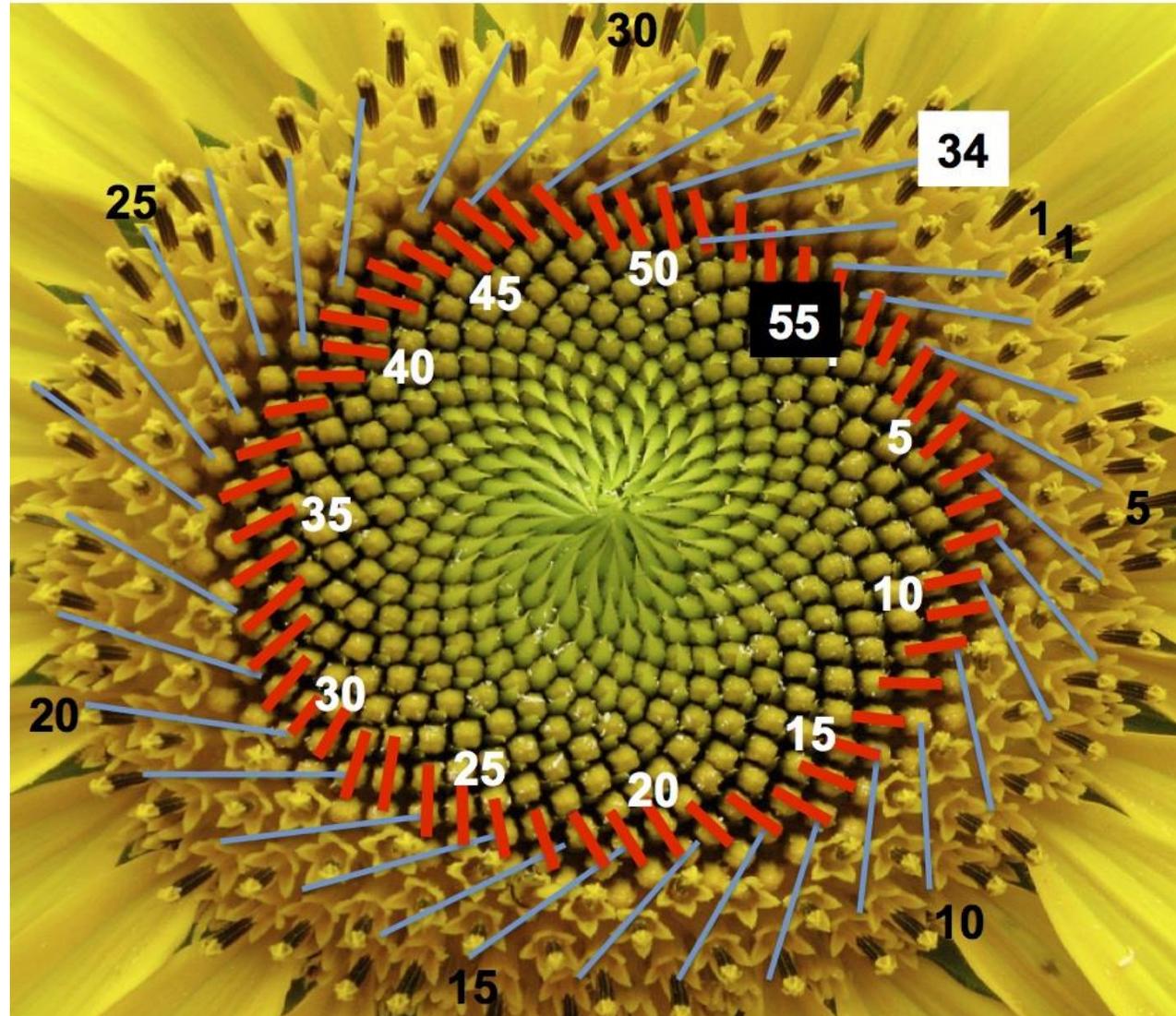


Parastichies

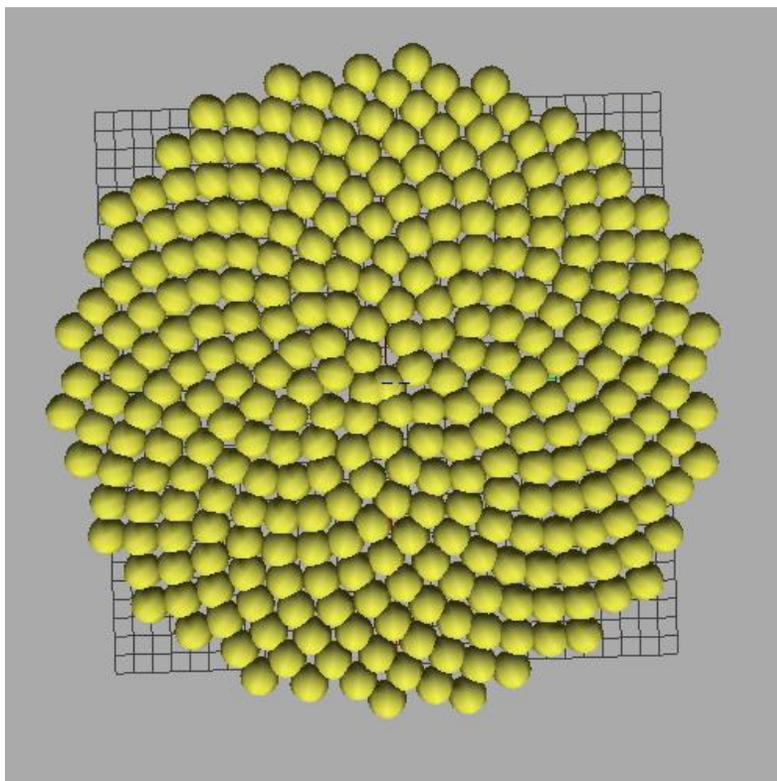


0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144,

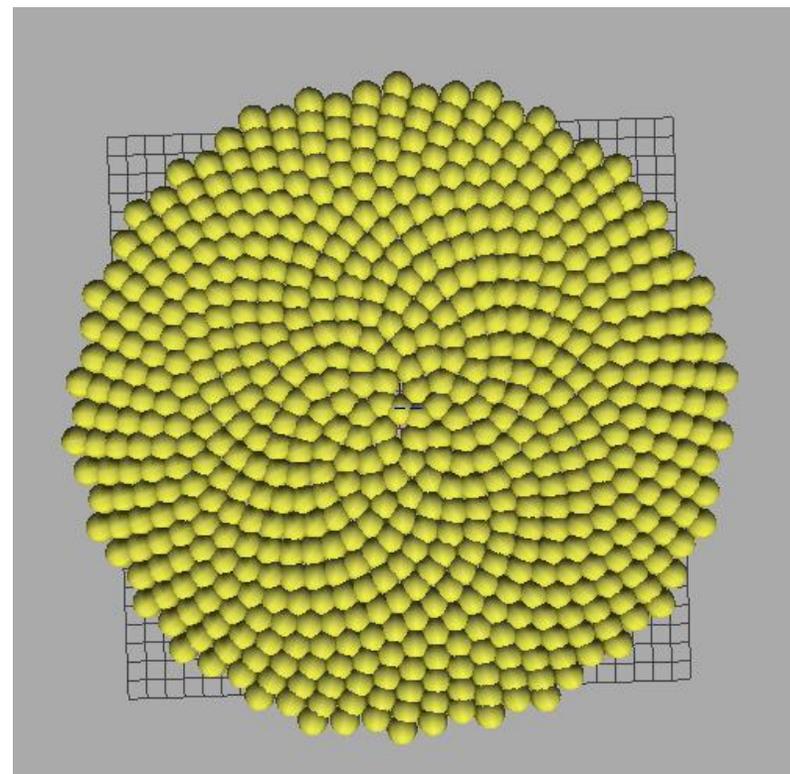
Słonecznik



Inne znane filotaksje

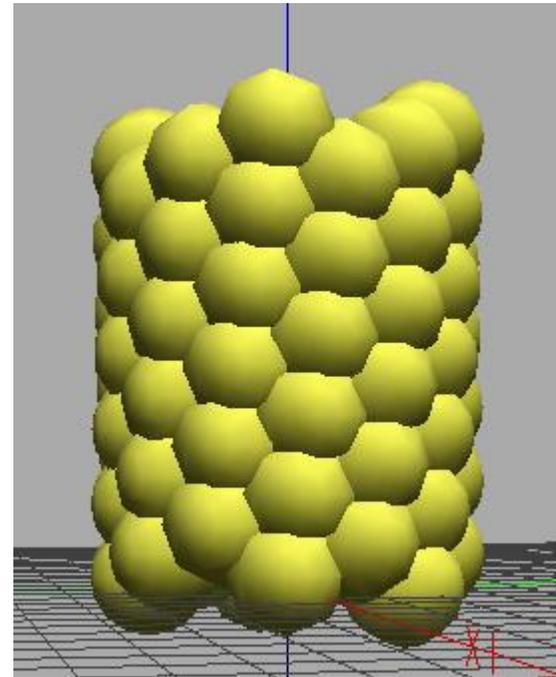


Lucas, 99.5

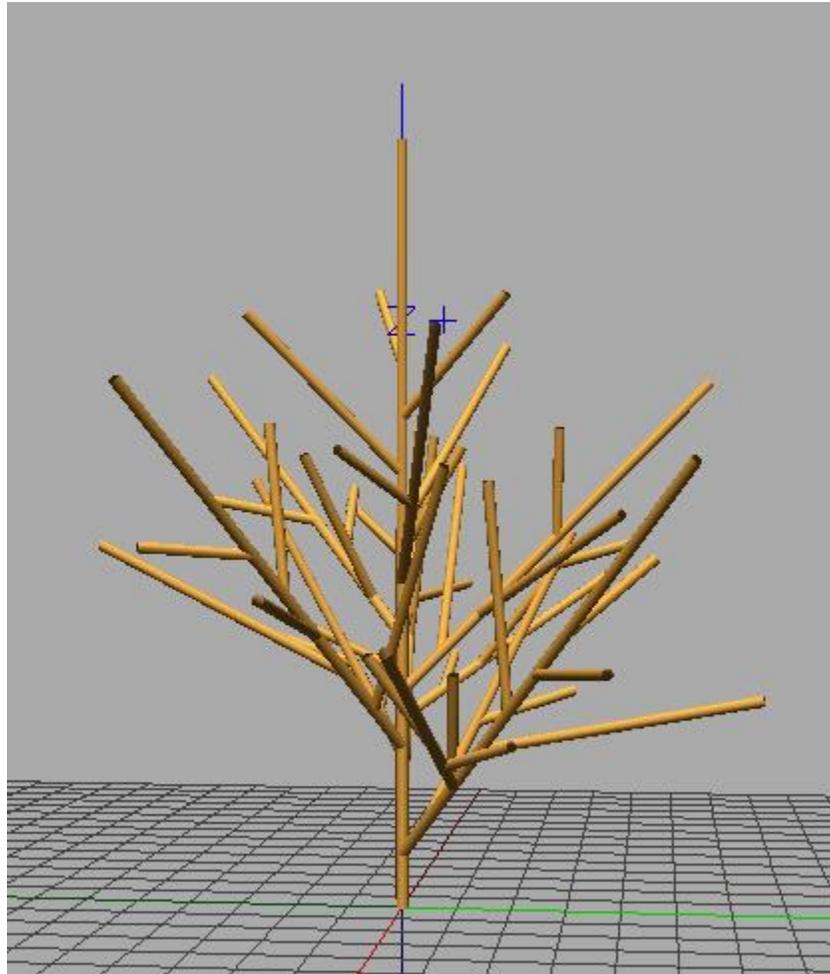


Bijugate, 68.8

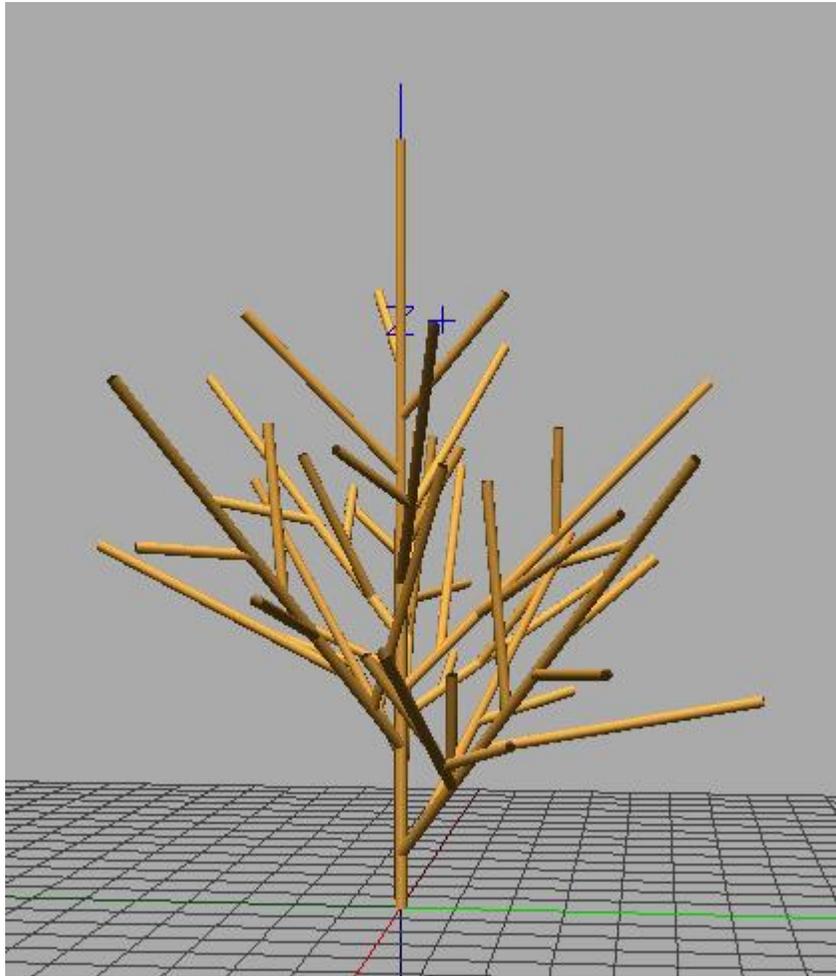
Model Ananasa (cylinder)



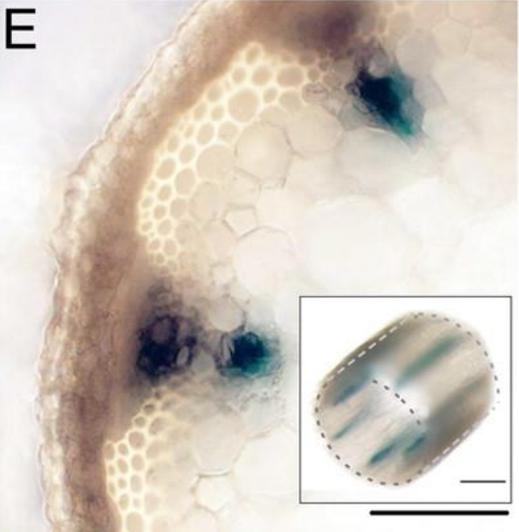
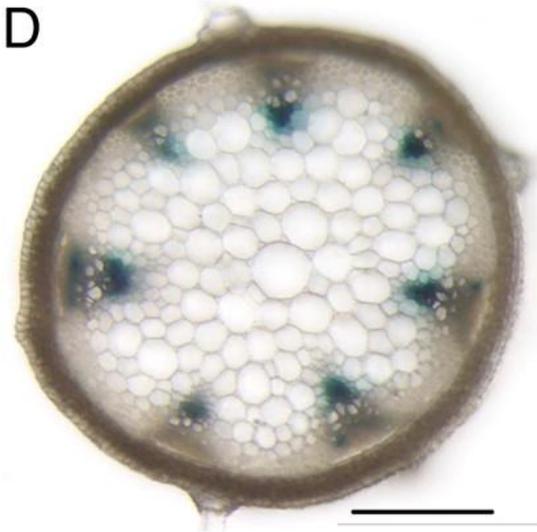
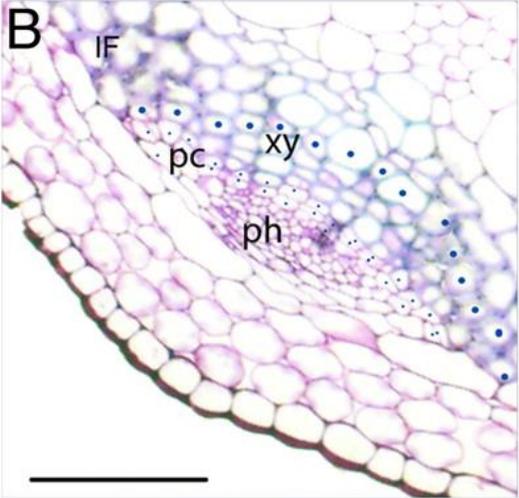
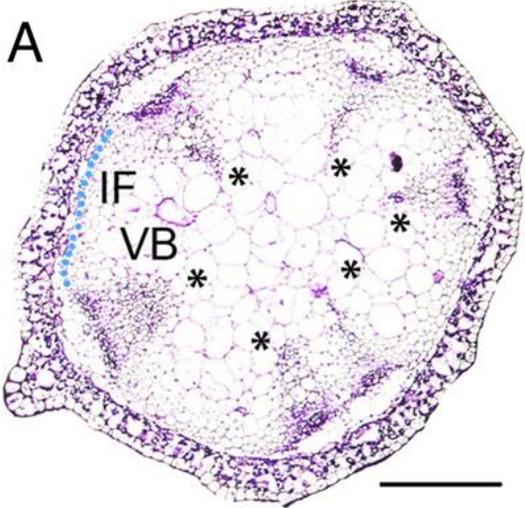
Model drzewa (Fibonacci spirala)



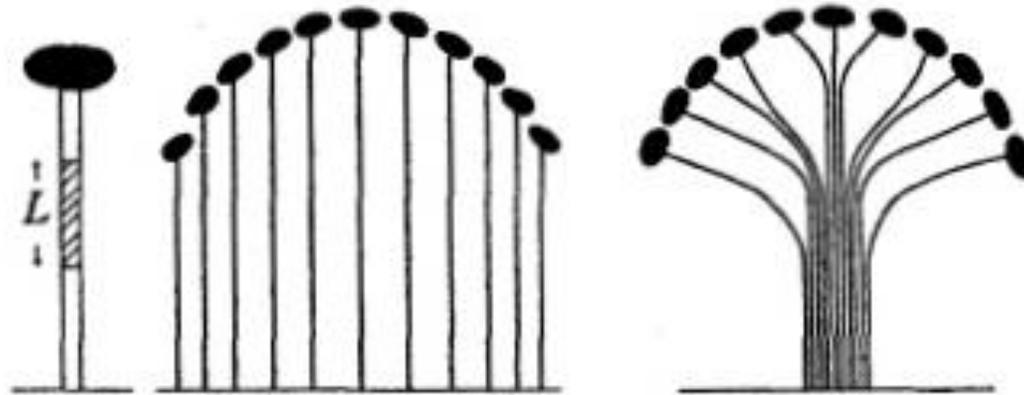
Różnorodna grubość gałęzi nie jest ujęta w modelu



Tkanka przewodząca

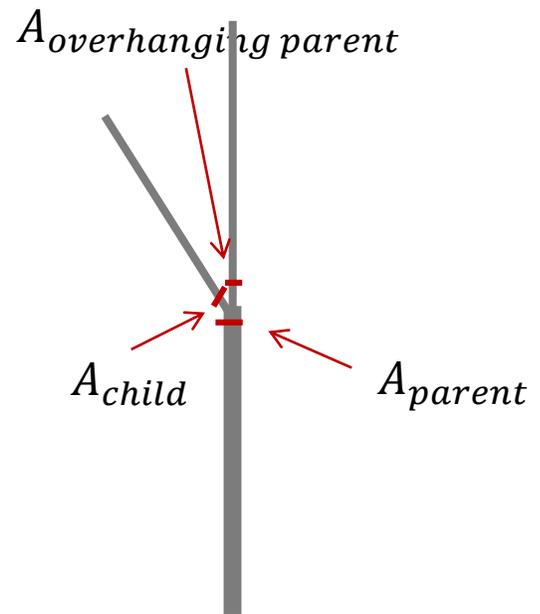


Pipe model

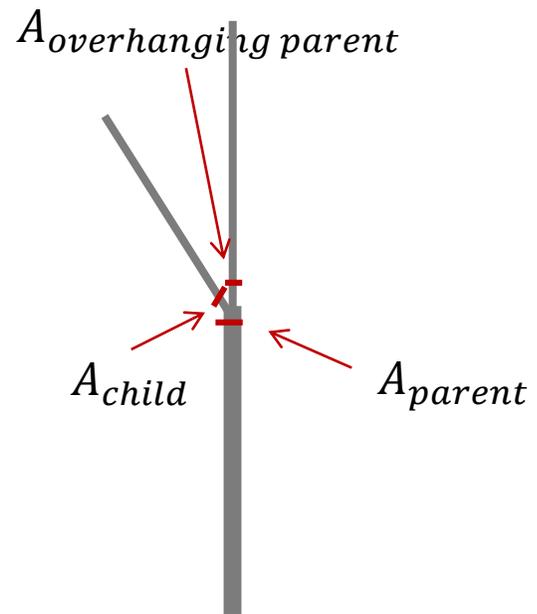


Shinozaki et al. 1964

Pipe model

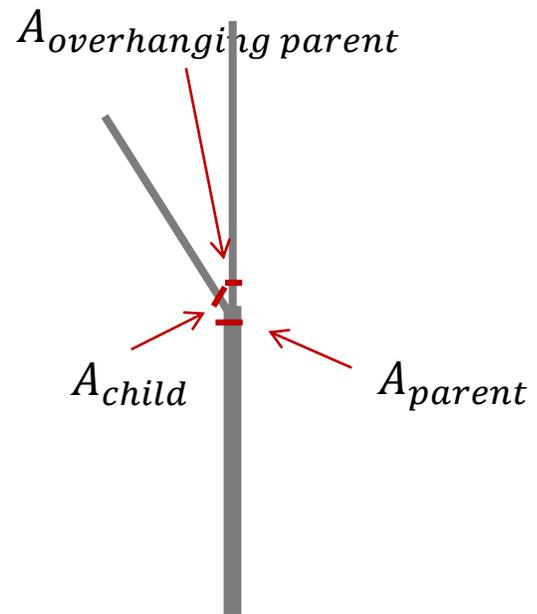


Pipe model



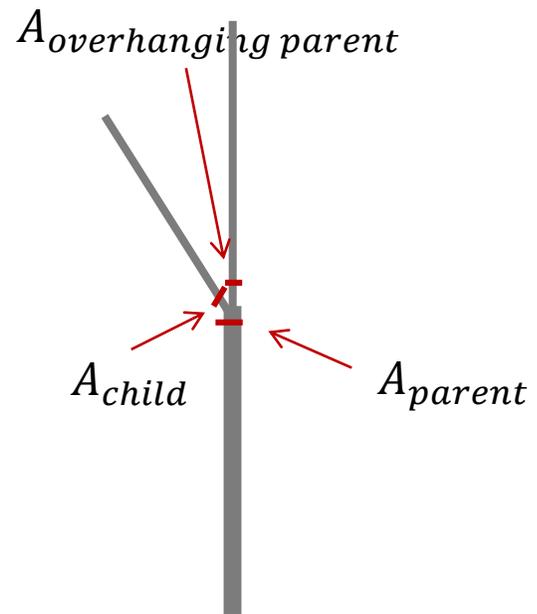
$$A_p = A_c + A_{op}$$

Pipe model



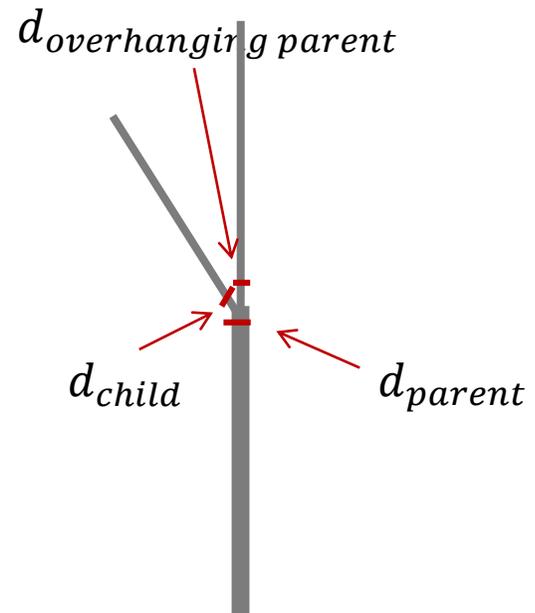
$$\pi r_p^2 = \pi r_c^2 + \pi r_{op}^2$$

Pipe model



$$r_p^2 = r_c^2 + r_{op}^2$$

Pipe model



$$d_p^n = d_c^n + d_{op}^n$$

$$n \in [1,3]$$

Context-sensitive L-Systems

$S < F \rightarrow E$ F stanie się E, jak stoi przed S

$F > S \rightarrow E$ F stanie się E, jak stoi za S

$T < F > S \rightarrow E$ F stanie się E, jak stoi przed T i za S

Context-sensitive L-Systems

$S < F \rightarrow E$ F stanie się E, jak stoi przed S

$F > S \rightarrow E$ F stanie się E, jak stoi za S

$T < F > S \rightarrow E$ F stanie się E, jak stoi przed T i za S

ω : *baaaaaaaaa*

p_1 : *b < a* \rightarrow *b*

p_2 : *b* \rightarrow *a*

Context-sensitive L-Systems

$S < F \rightarrow E$ F stanie się E, jak stoi przed S

$F > S \rightarrow E$ F stanie się E, jak stoi za S

$T < F > S \rightarrow E$ F stanie się E, jak stoi przed T i za S

baaaaaaaaa

ω : *baaaaaaaaa*

p_1 : *b < a* \rightarrow *b*

p_2 : *b* \rightarrow *a*

Context-sensitive L-Systems

$S < F \rightarrow E$ F stanie się E, jak stoi przed S

$F > S \rightarrow E$ F stanie się E, jak stoi za S

$T < F > S \rightarrow E$ F stanie się E, jak stoi przed T i za S

ω : *baaaaaaaaa*
 p_1 : *b < a* \rightarrow *b*
 p_2 : *b* \rightarrow *a*

baaaaaaaaa
abaaaaaaaa

Context-sensitive L-Systems

$S < F \rightarrow E$ F stanie się E, jak stoi przed S

$F > S \rightarrow E$ F stanie się E, jak stoi za S

$T < F > S \rightarrow E$ F stanie się E, jak stoi przed T i za S

ω : *baaaaaaaaa*
 p_1 : *b < a* \rightarrow *b*
 p_2 : *b* \rightarrow *a*

baaaaaaaaa
abaaaaaaaa
aabaaaaaaaa
aaabaaaaaaaa
aaaabaaaaaa
...

Context-sensitive L-Systems []

$S \rightarrow [H]M$

$S[HI[JK]L]MNO$

Context-sensitive L-Systems []

$S \rightarrow [H]M$

$S[HI[JK]L]MNO$

$L-Py \rightarrow \text{ignore: } +-$

Context-sensitive parametric L-Systems

$$A(x) < B(y) > C(z): x + y + z > 10 \rightarrow E\left(\frac{x + y}{2}\right) F\left(\frac{y + z}{2}\right)$$

Context-sensitive parametric L-Systems

$$A(x) < B(y) > C(z): x + y + z > 10 \rightarrow E\left(\frac{x + y}{2}\right) F\left(\frac{y + z}{2}\right)$$

$$\dots A(4)B(5)C(6) \dots$$

Context-sensitive parametric L-Systems

$$A(x) < B(y) > C(z): x + y + z > 10 \rightarrow E\left(\frac{x+y}{2}\right) F\left(\frac{y+z}{2}\right)$$

$$\dots A(4)B(5)C(6) \dots$$

$$4 + 5 + 6 > 10$$

Context-sensitive parametric L-Systems

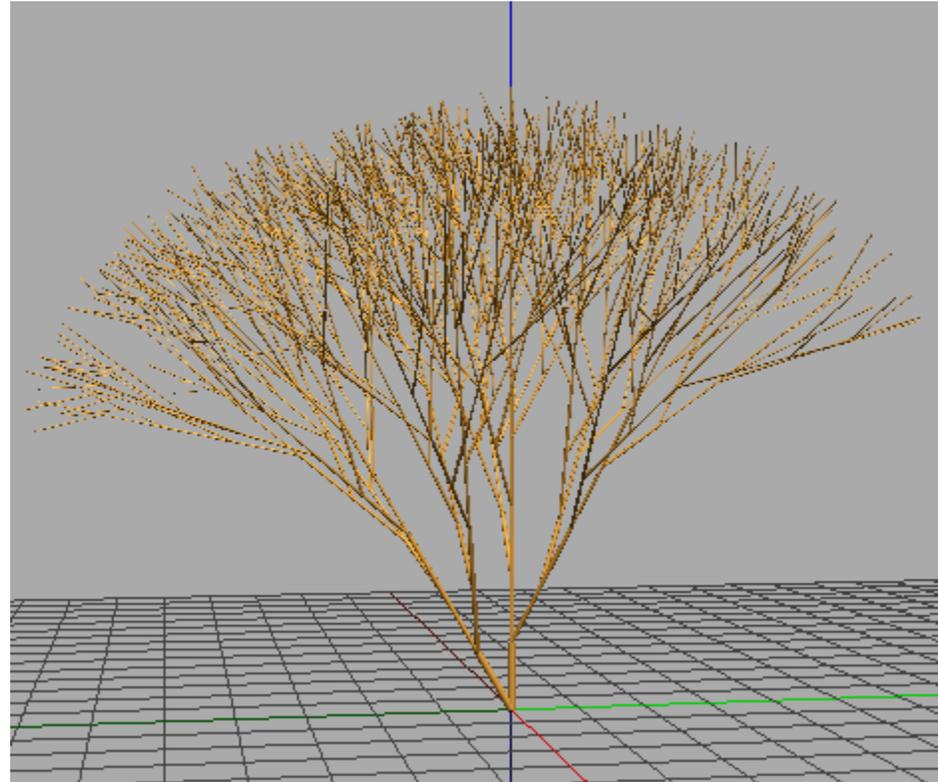
$$A(x) < B(y) > C(z): x + y + z > 10 \rightarrow E\left(\frac{x+y}{2}\right)F\left(\frac{y+z}{2}\right)$$

$$\dots A(4)B(5)C(6) \dots$$

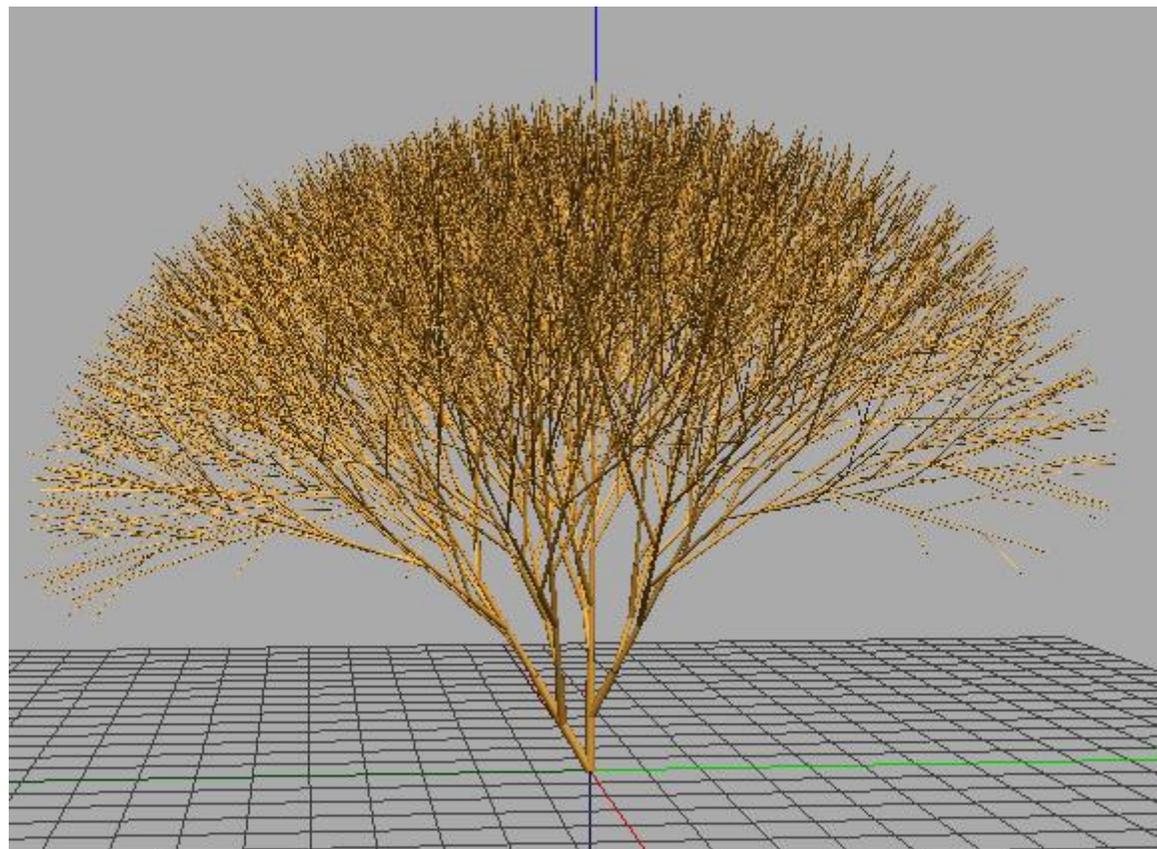
$$4 + 5 + 6 > 10$$

$$B(5) \rightarrow E(4.5)F(5.5)$$

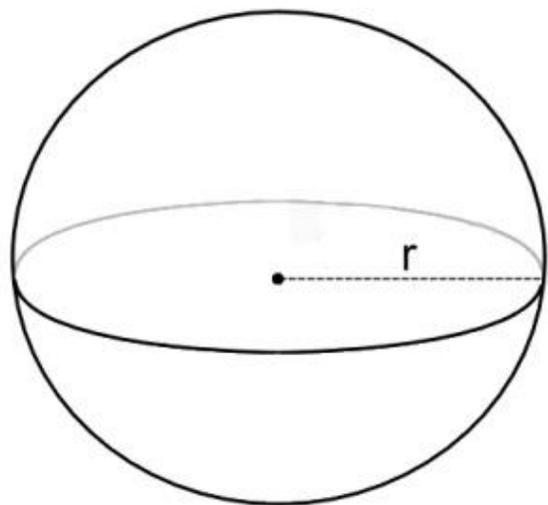
Context-sensitive pipe model



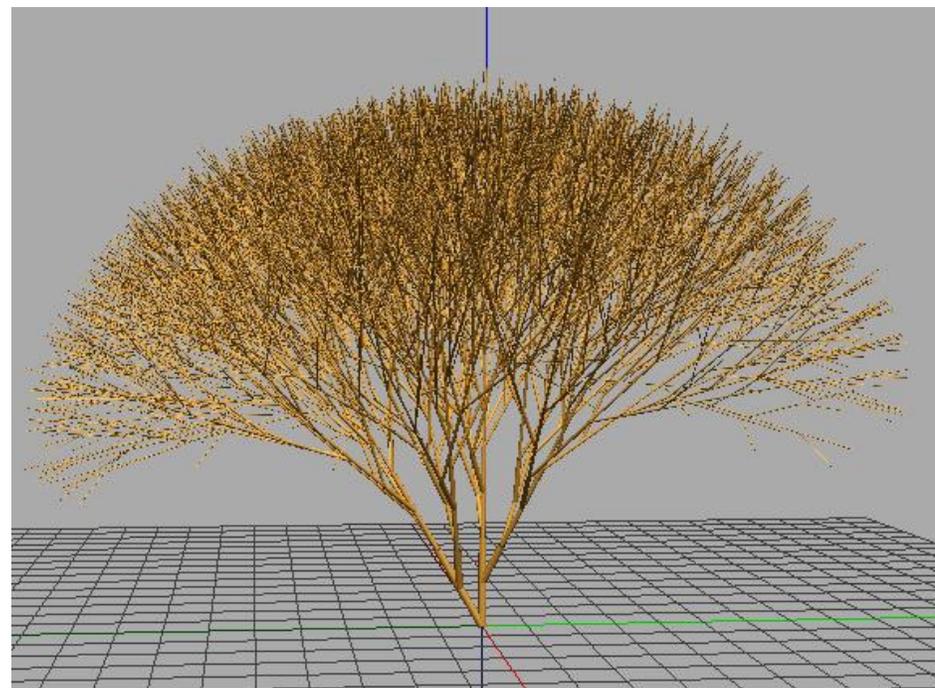
Za gęsta struktura drzewiasta



Przestrzeń i L-System



Objętość kuli $\sim r^3$

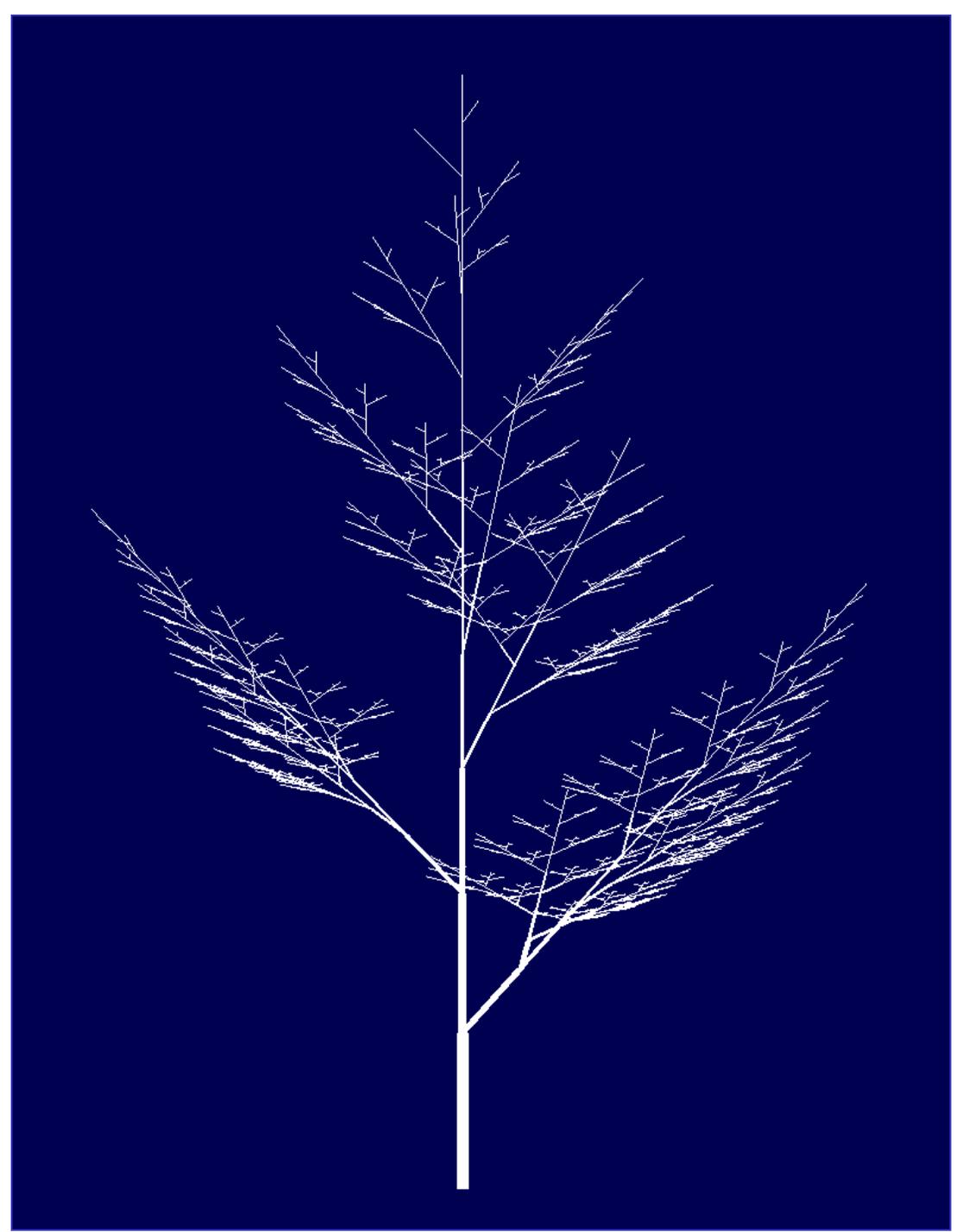


Liczba gałęzi $\sim 2^r$

Modelowanie drzew

First 3D tree model

Hisao Honda 1971



Recursive tree models

Aono and Kunii 1984

Reeves and Blau 1985

Bloomenthal 1985

Oppenheimer 1986

de Reffye et al. 1988

Weber and Penn 1995

Lintermann and Deussen 1999

Prusinkiewicz et al. 2001

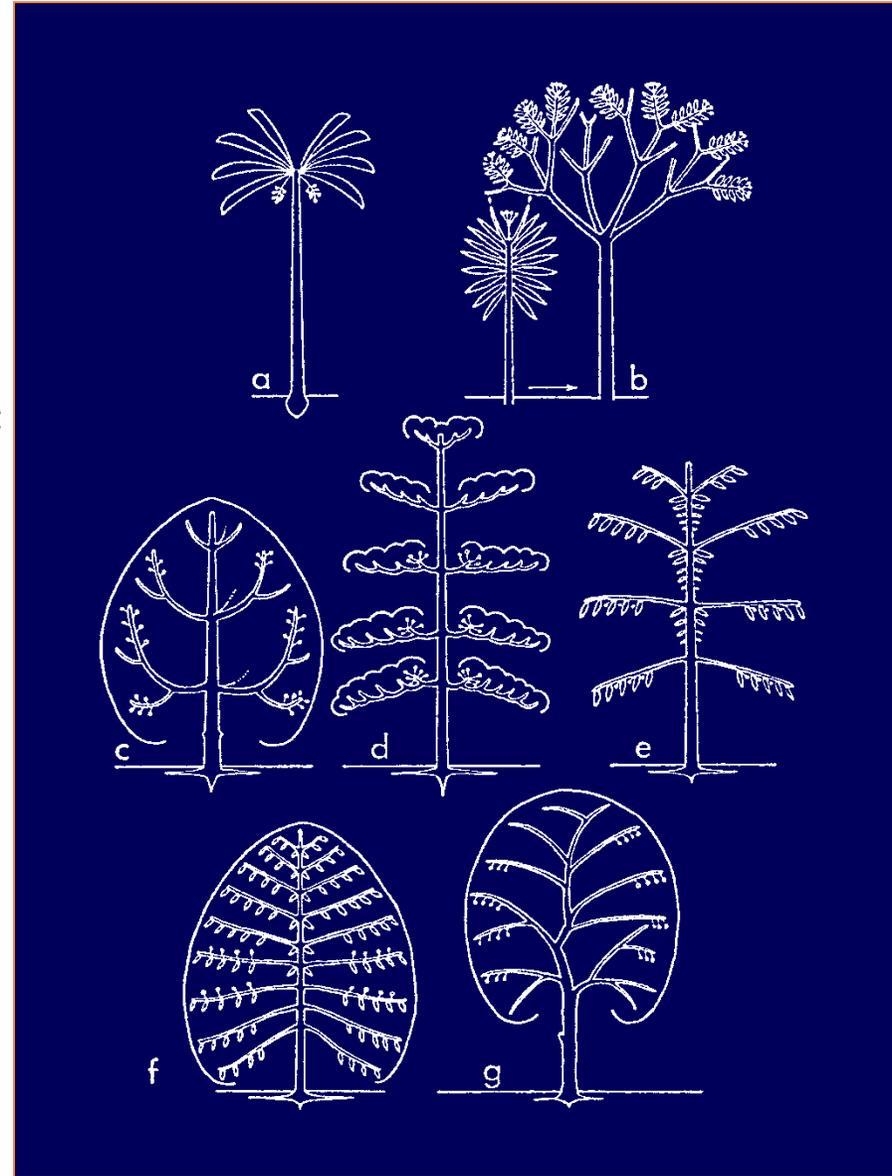


Architectural models

Hallé, Oldeman, Tomlinson 1978

“Organization of trees reflects the **precisely controlled** genetic program which determines their development. [...]

This program is **disrupted** by environmental factors.”



F. Hallé, R.A.A Oldeman, P.B. Tomlinson:
Tropical trees and forests: An architectural analysis. Springer, Heidelberg 1978.

Architectural models do not suffice

Sachs & Novoplansky 1995,
Sachs 2004

“The form of a tree is generated by self-organization in which **alternative branches compete** with one another, following **no strict plan** or pre-pattern.”



T. Sachs and A. Novoplansky. Tree form: Architectural models do not suffice. *Israel Journal of Plant Sciences*, 43:203-212, 1995.

Space-based models

Ulam 1962

Cohen 1967

Arvo and Kirk 1988

Greene 1989

Chiba et al. 1994

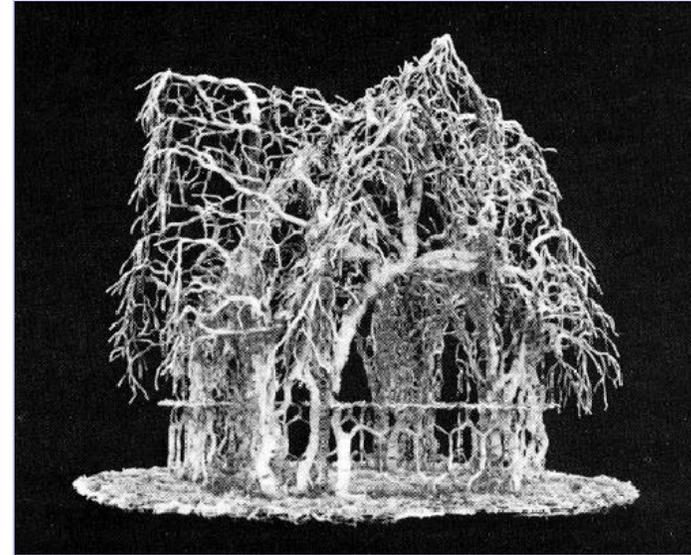
Prusinkiewicz et al. 1994

Mech and Prusinkiewicz 1996

Benes 2002

Rodkaew et al. 2003

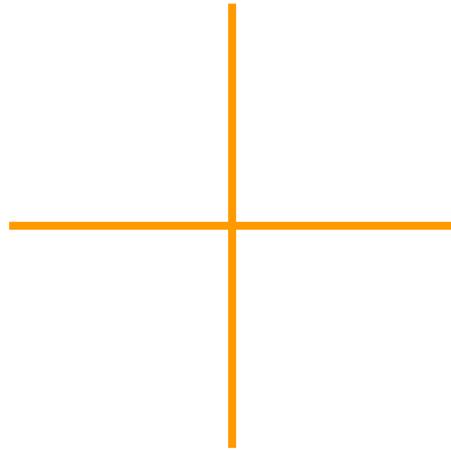
Runions et al. 2007



Self-organization of
branches in space

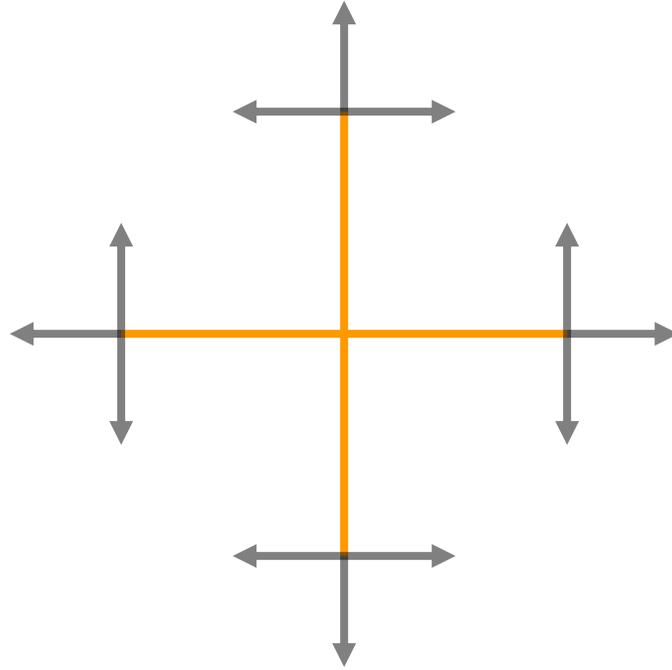
- Self-organization
 - Process in which global pattern and structure emerge from interactions among the lower-level components of the system.
- Database amplification
 - Simple mechanism (economically encoded in the genome) can generate complex patterns and structures
- Reason for modelling
 - The emergence of form through self-organization is difficult to comprehend without models

Example – Cellular Automaton



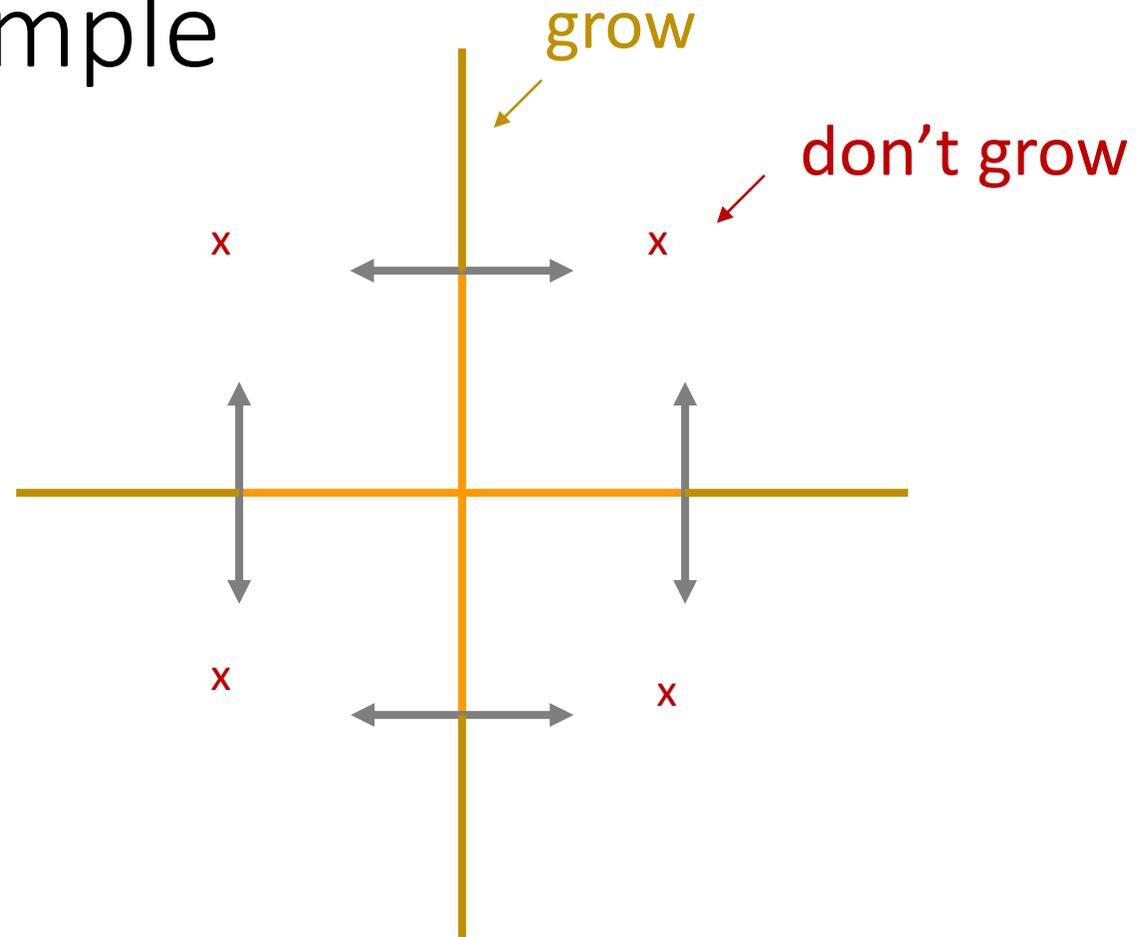
Consider a branching structure...

Example



At the end of each branch there are 3 buds.

Example



Rule 1: If there is enough space, grow.

Rule 2: If there isn't enough space, don't grow.

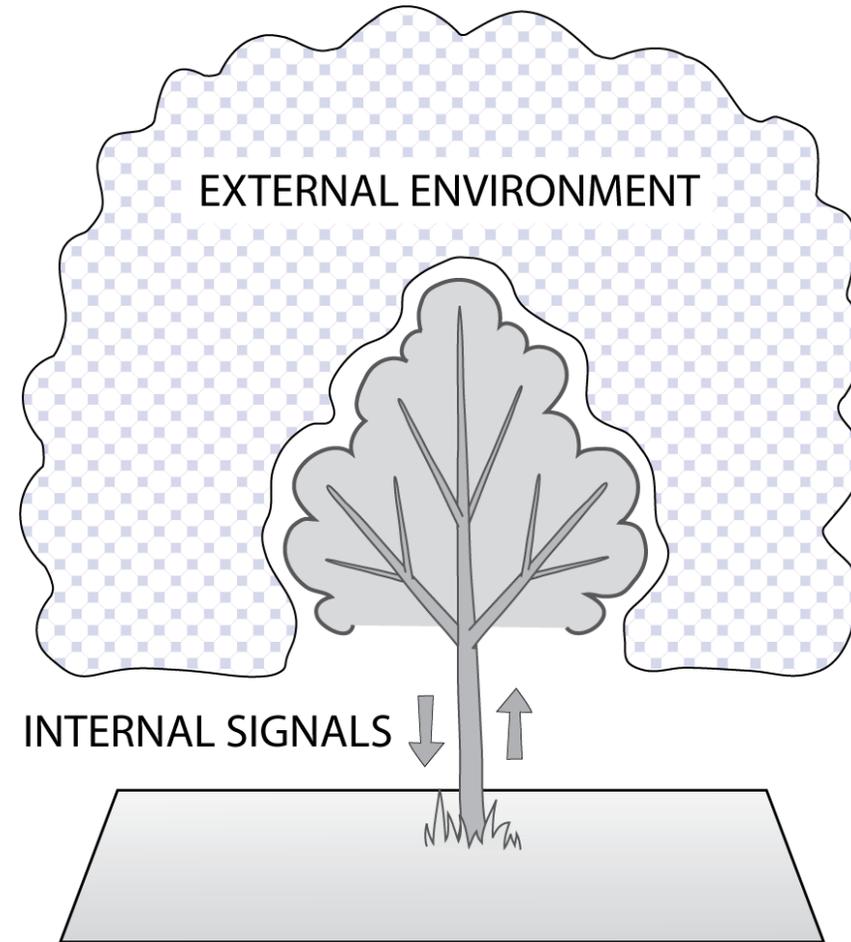
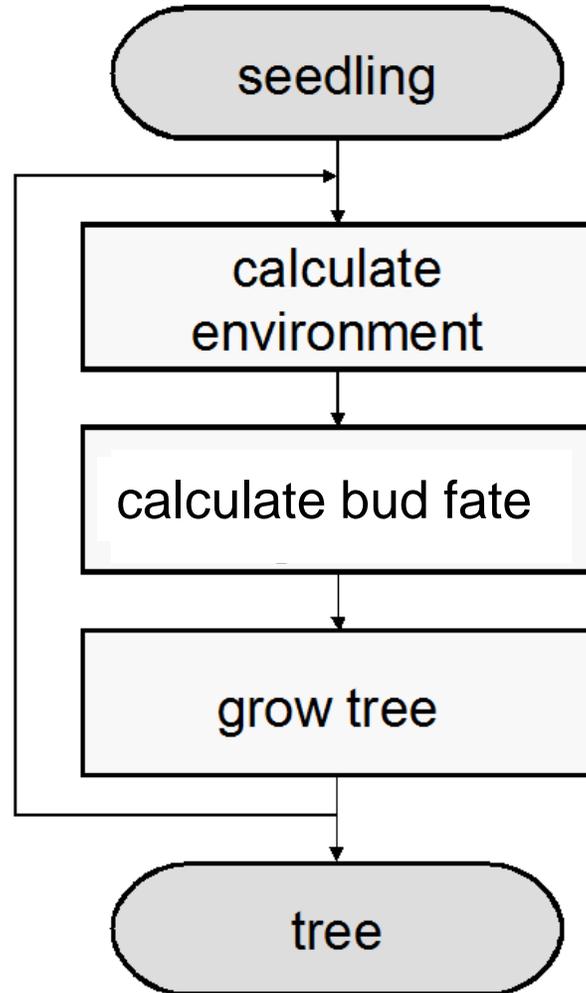
What structures will emerge?

Synchronous growth

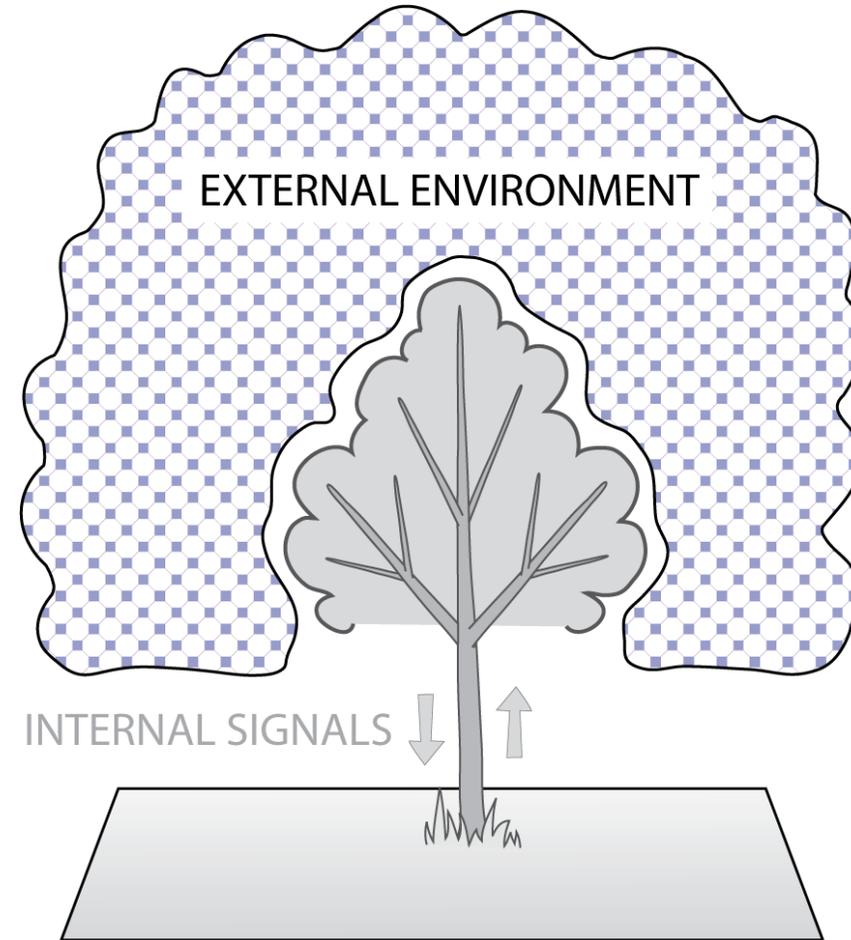
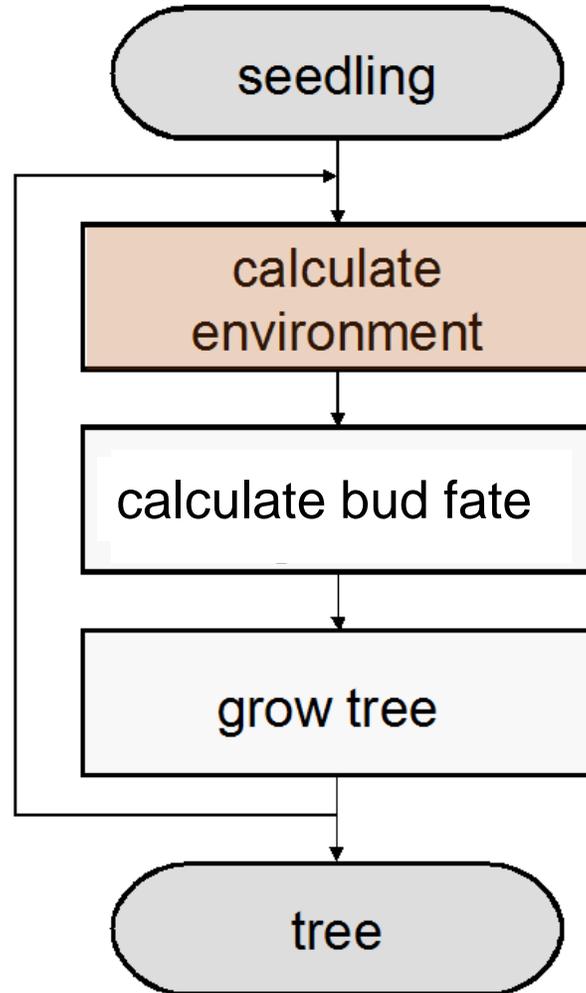
Asynchronous growth

Combining Architectural and Self-organizing Models

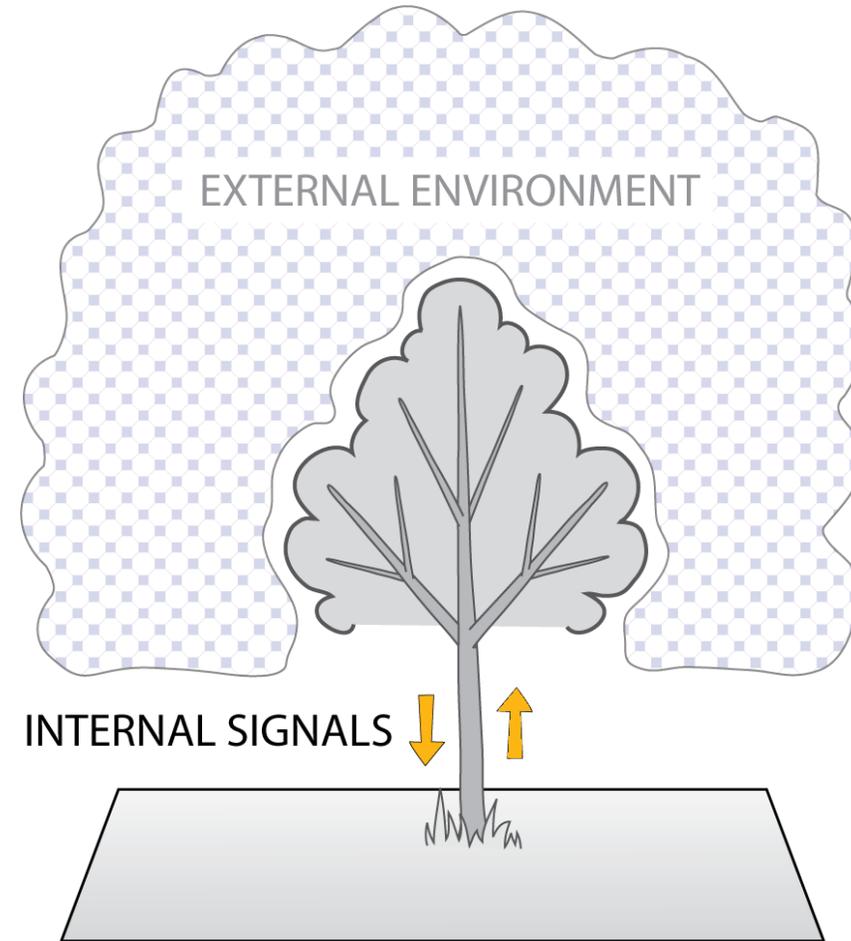
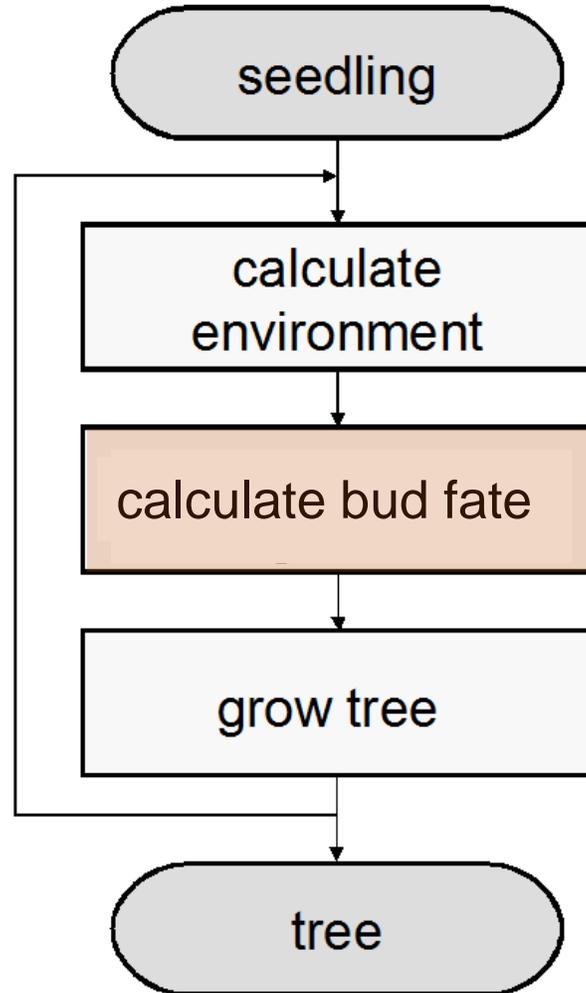
Simulation Overview



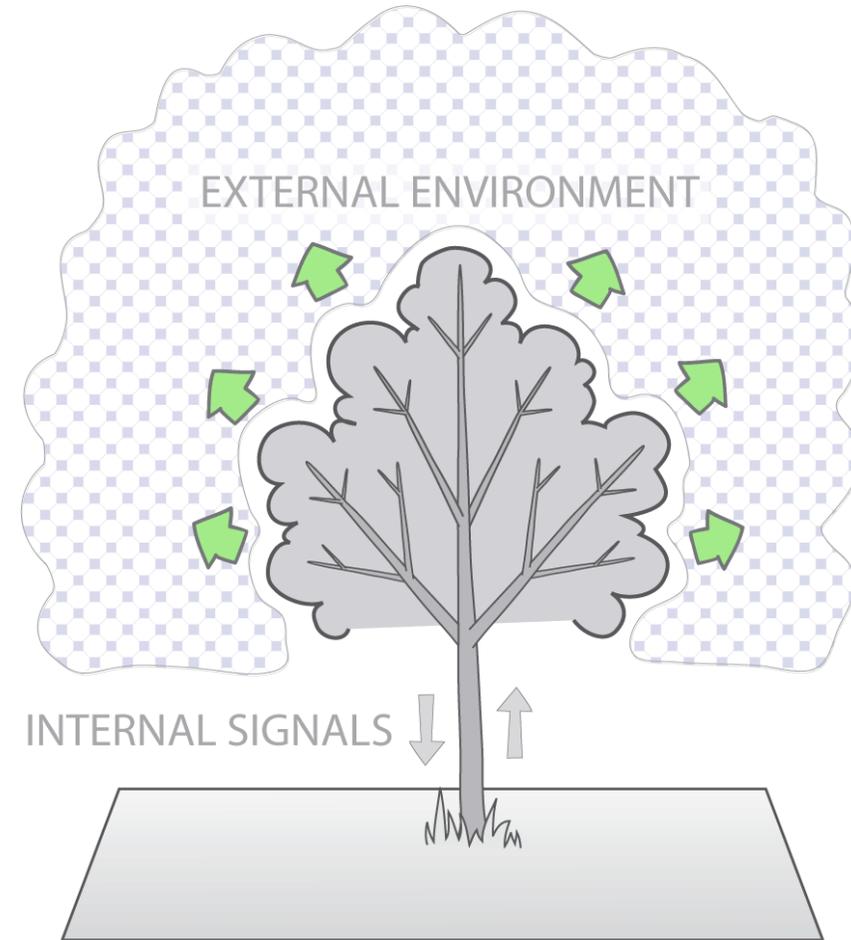
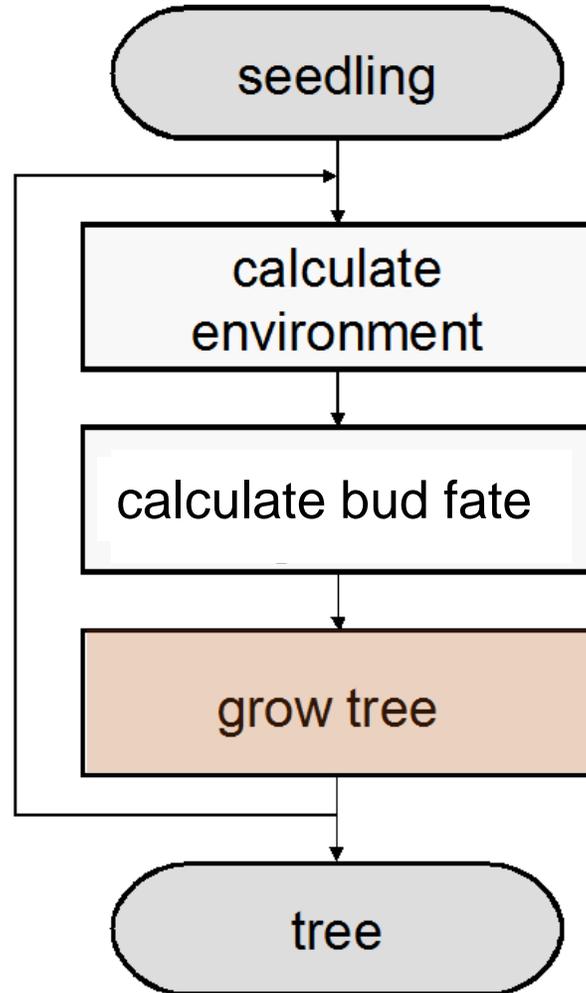
Simulation Overview



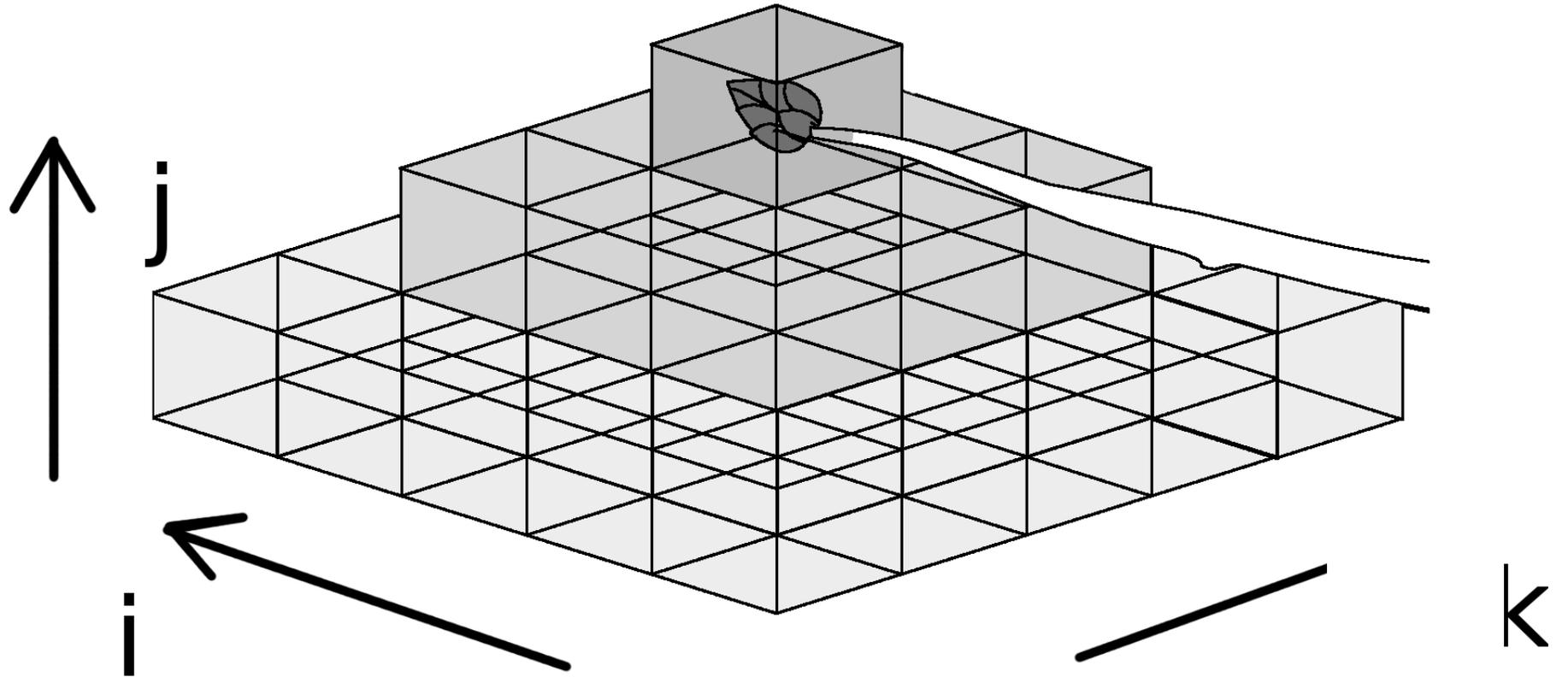
Simulation Overview



Simulation Overview

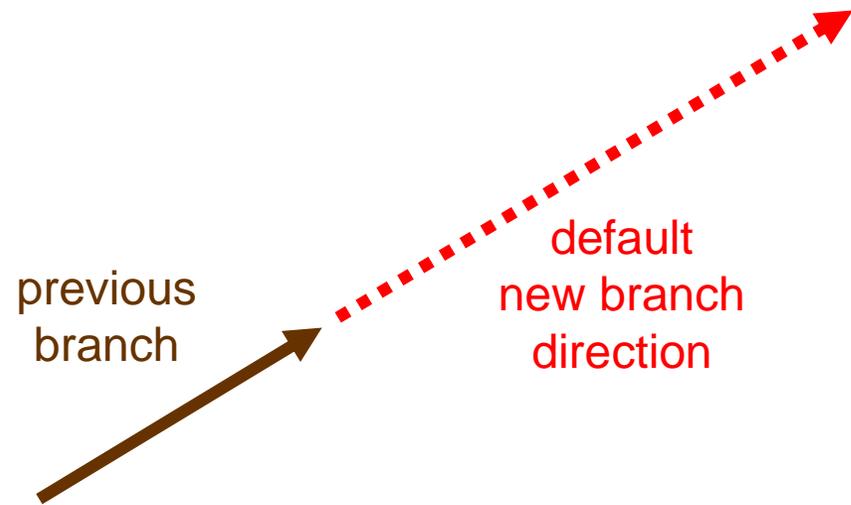


Calculating Environment

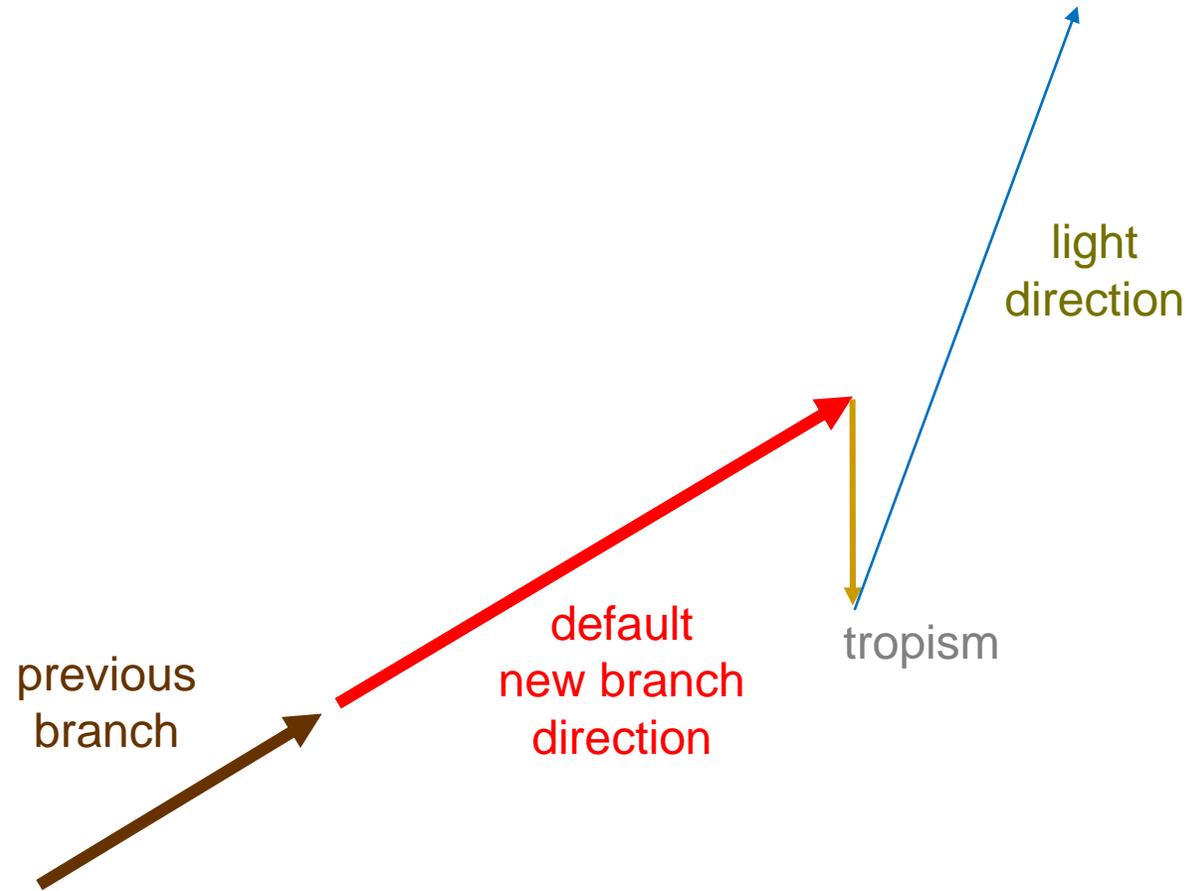


Shadow propagation

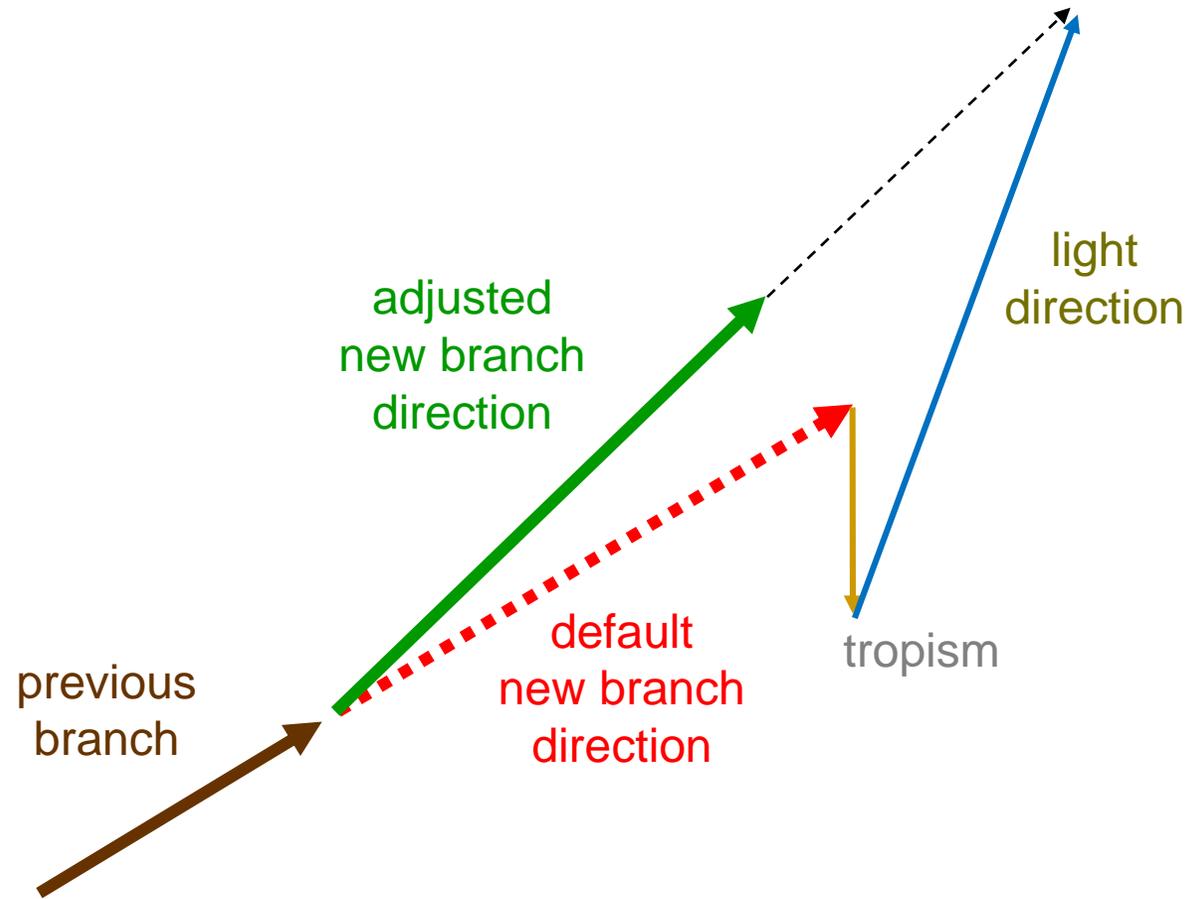
Branch orientation



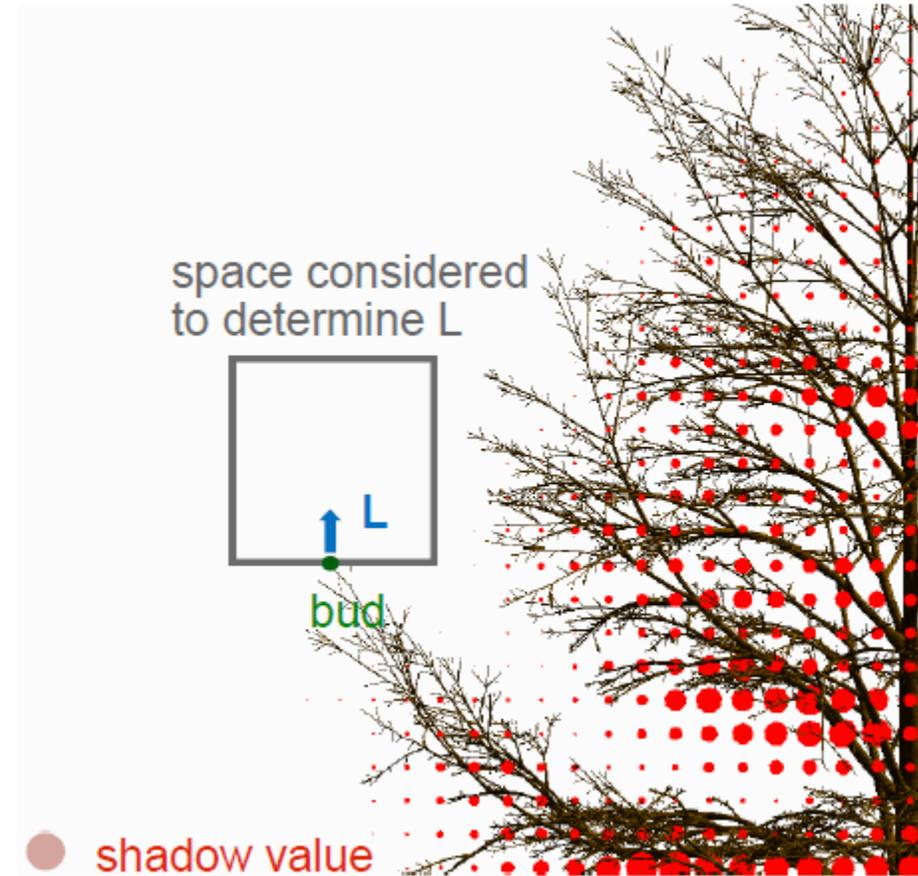
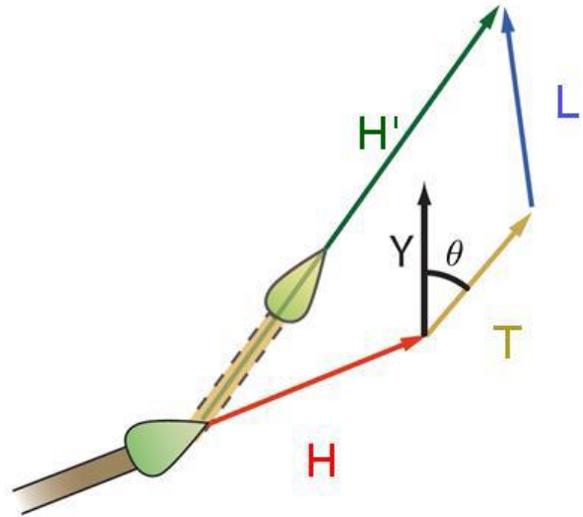
Branch orientation

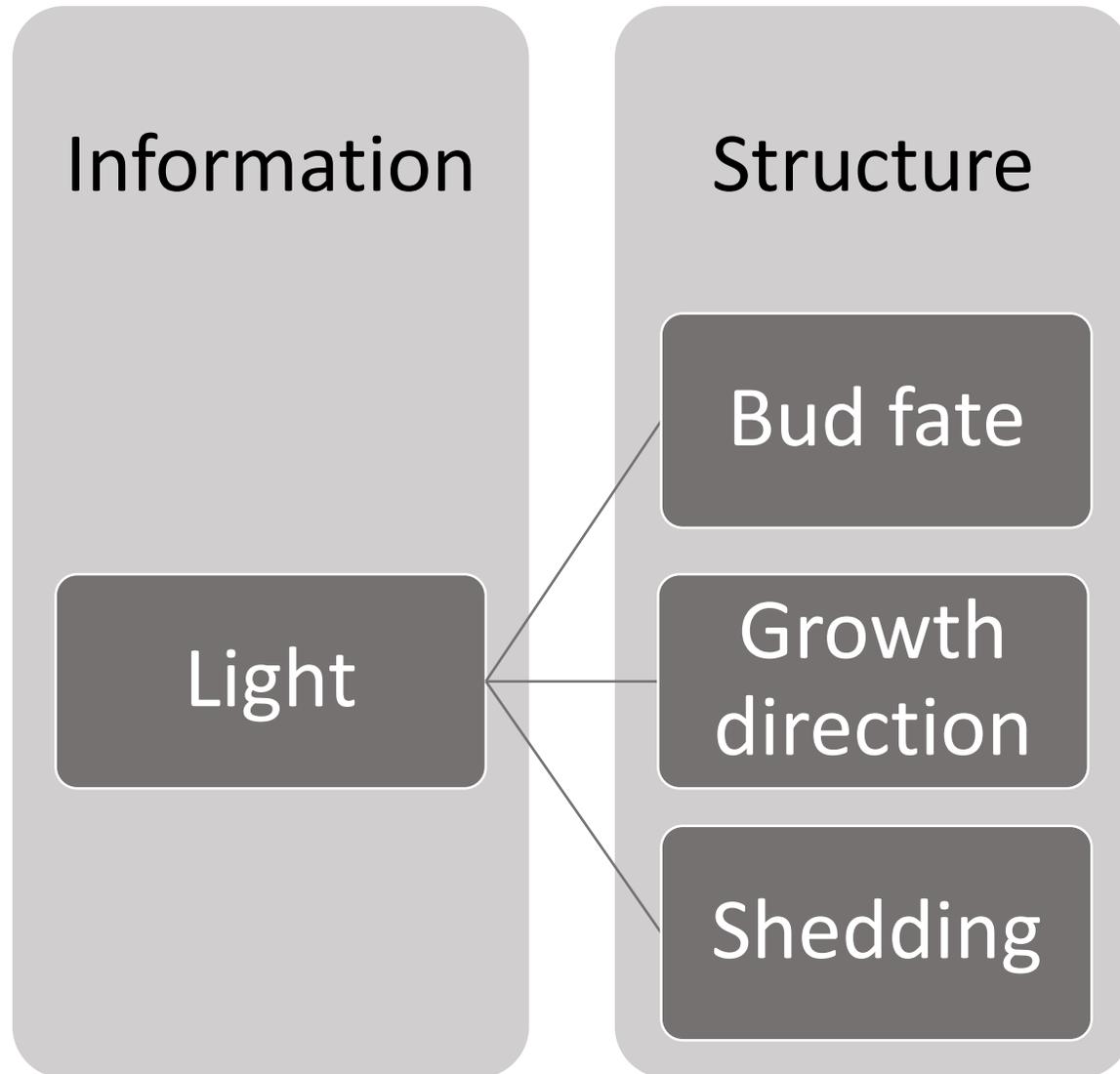


Branch orientation



Calculating Growth Direction





Information

Structure

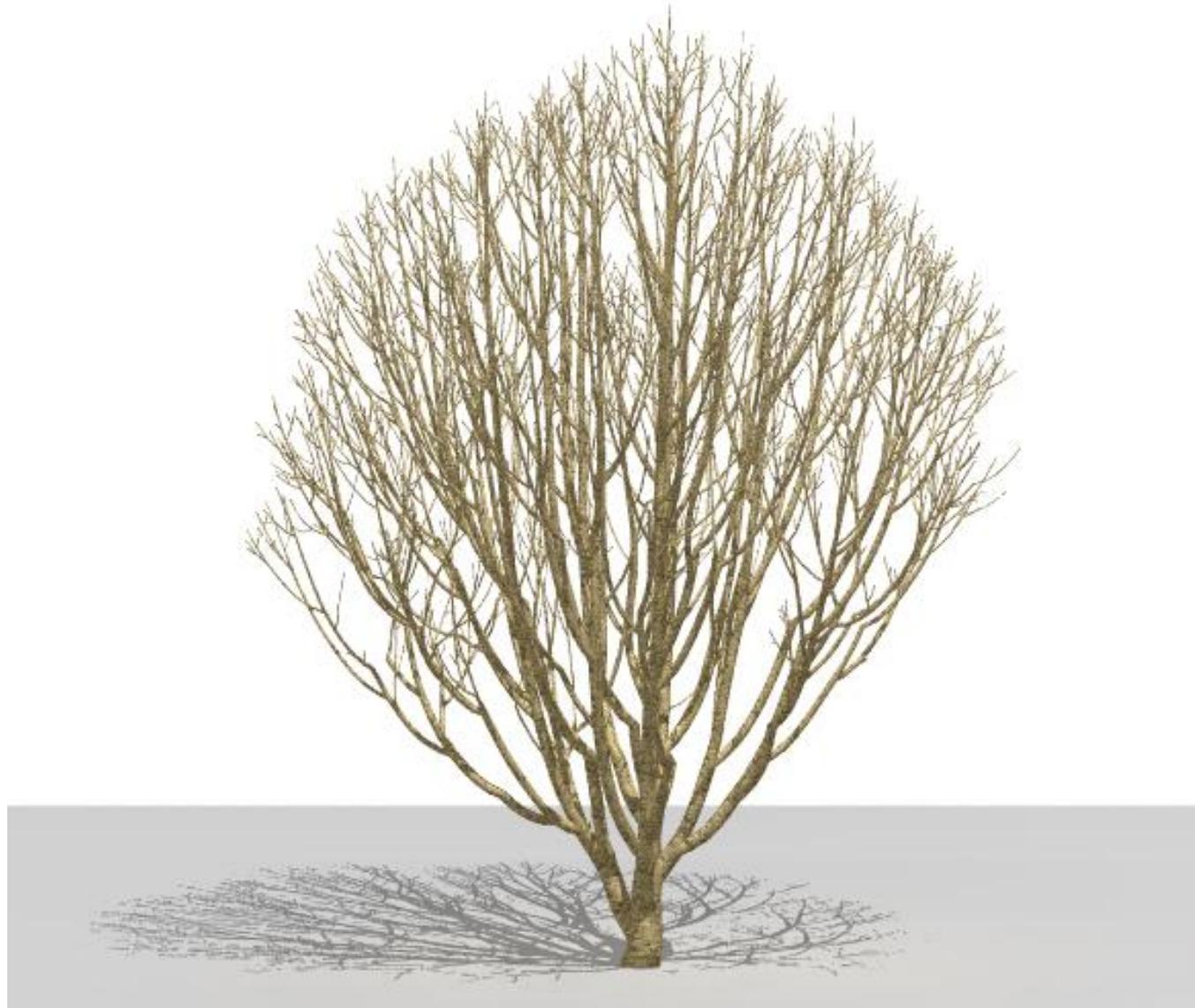
Light

Bud fate

Growth direction

Shedding

Model controlled by competition for light only

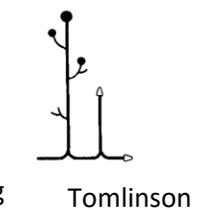
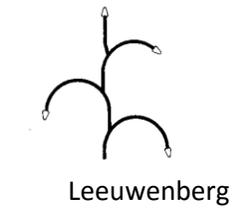
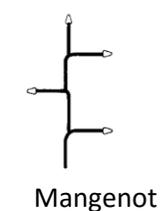
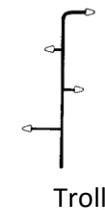
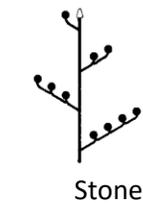
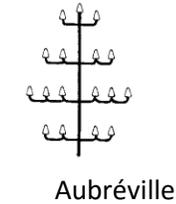
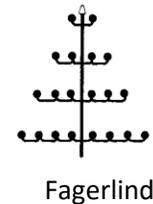
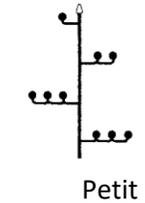
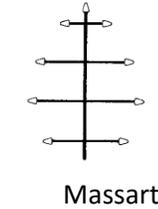
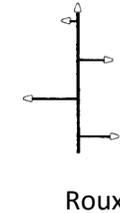
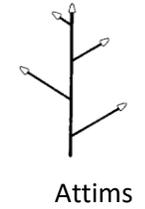


Architectural models

Hallé, Oldeman, Tomlinson 1978

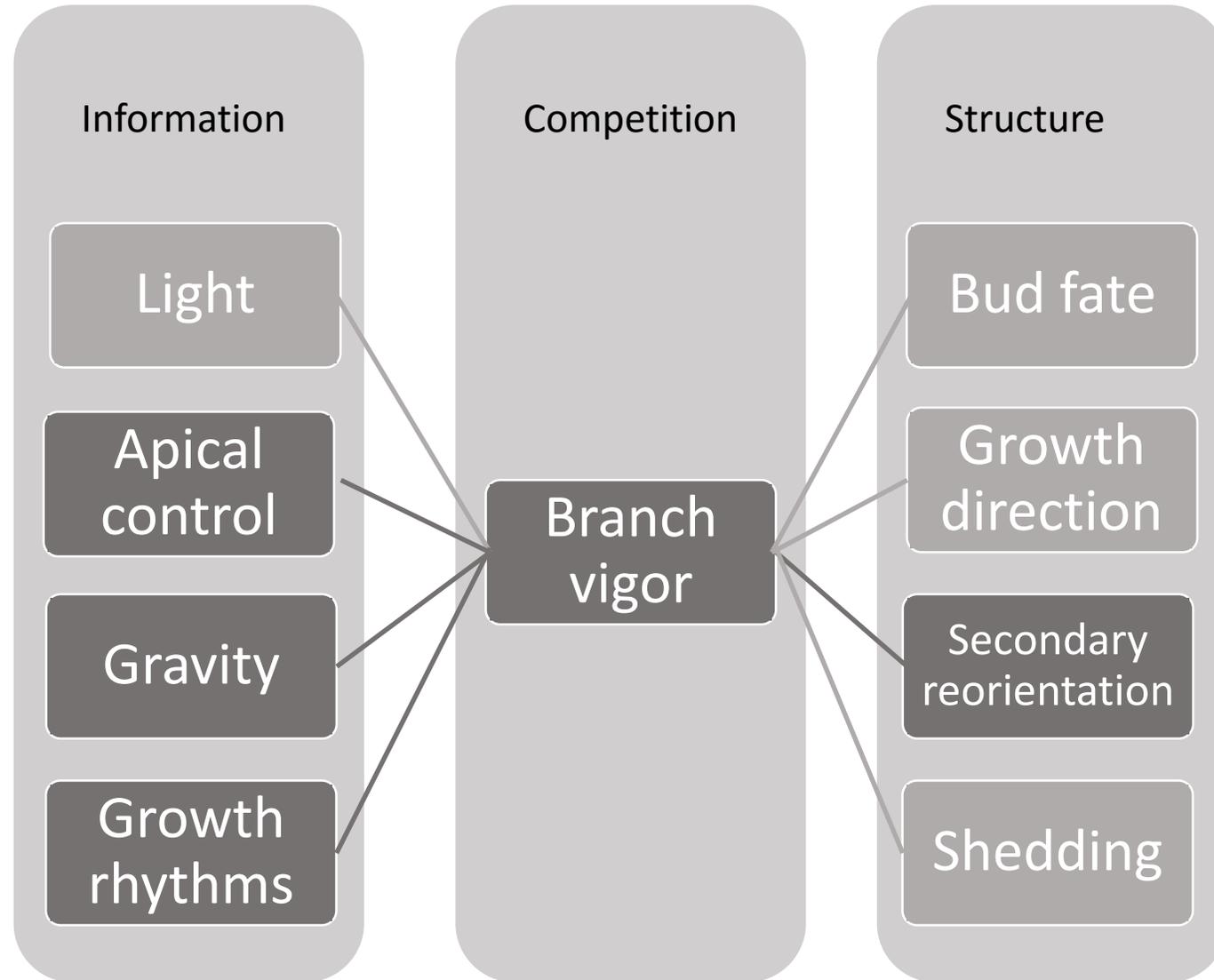
“Organization of trees reflects the **precisely controlled** genetic program which determines their development. [...]

This program is **disrupted** by environmental factors.”

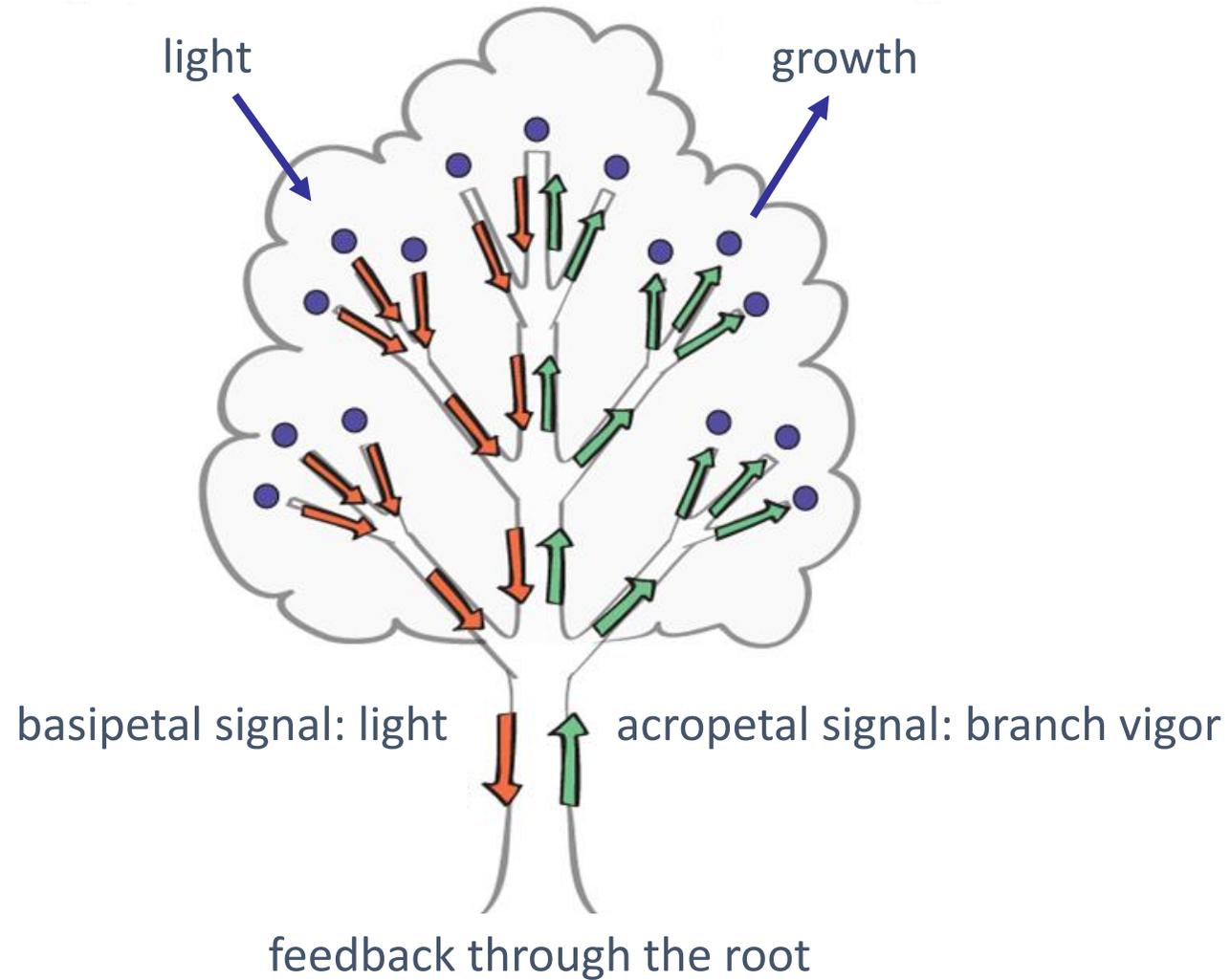


F. Hallé, R.A.A Oldeman, P.B. Tomlinson:

Tropical trees and forests: An architectural analysis. Springer, Heidelberg 1978.

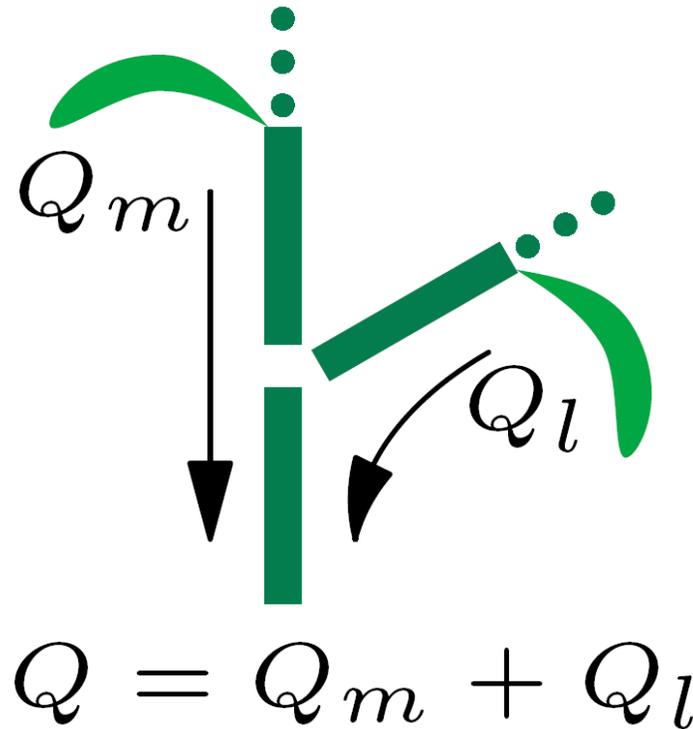


Overview: How to compute branch vigor

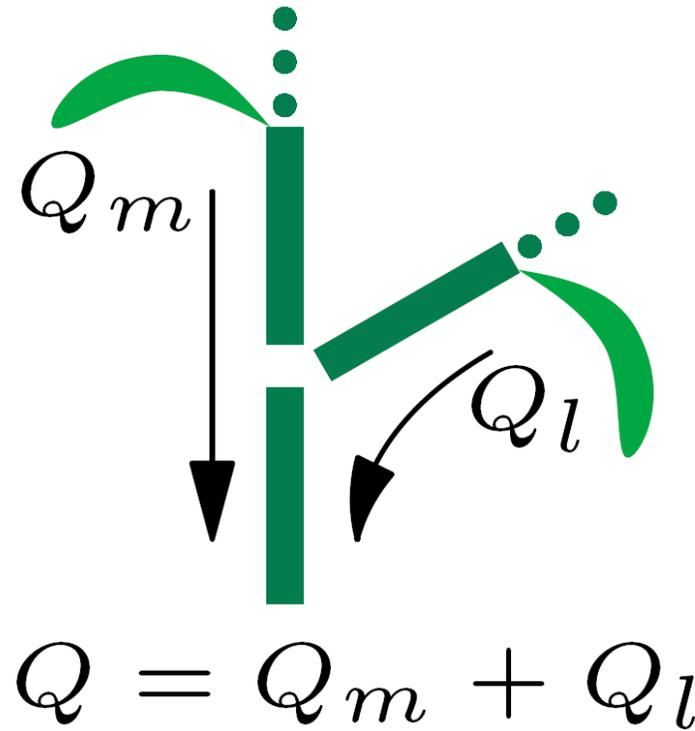


Internal Signals as Flux

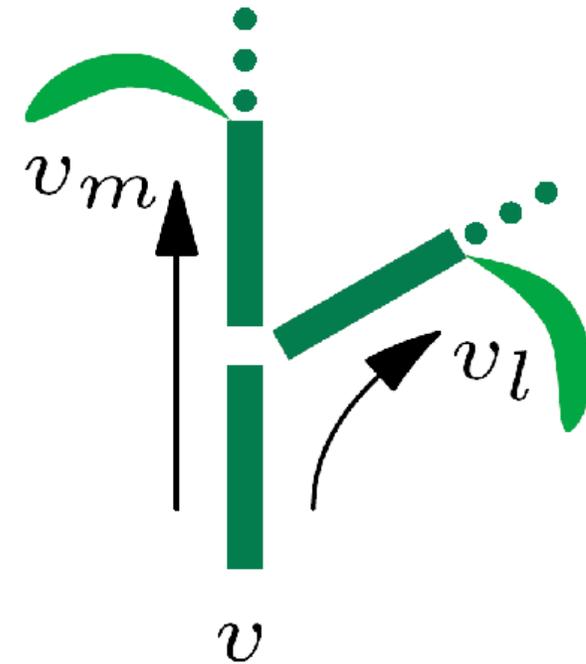
Light flux Q



Internal Signals as Flux



Light flux Q

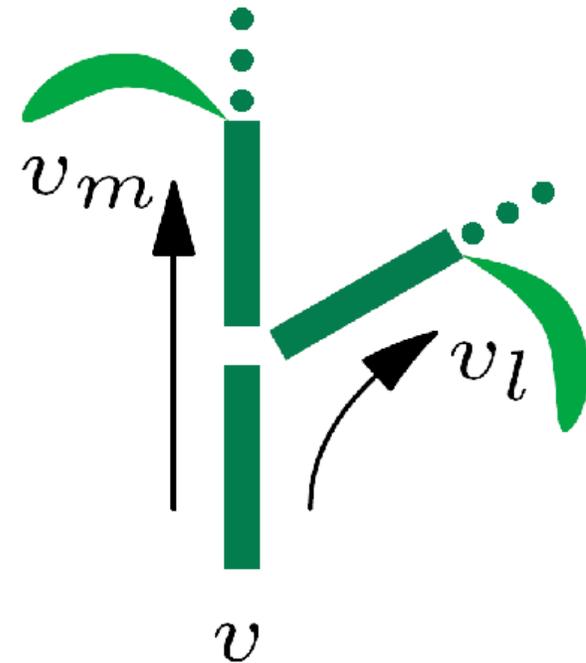


Vigor flux v

Vigor Flux Function

$$(1) v_m = v \frac{\Lambda Q_m}{\Lambda Q_m + (1 - \Lambda) Q_l}$$

$$(2) v_l = v \frac{\Lambda Q_l}{\Lambda Q_m + (1 - \Lambda) Q_l}$$



Vigor flux v

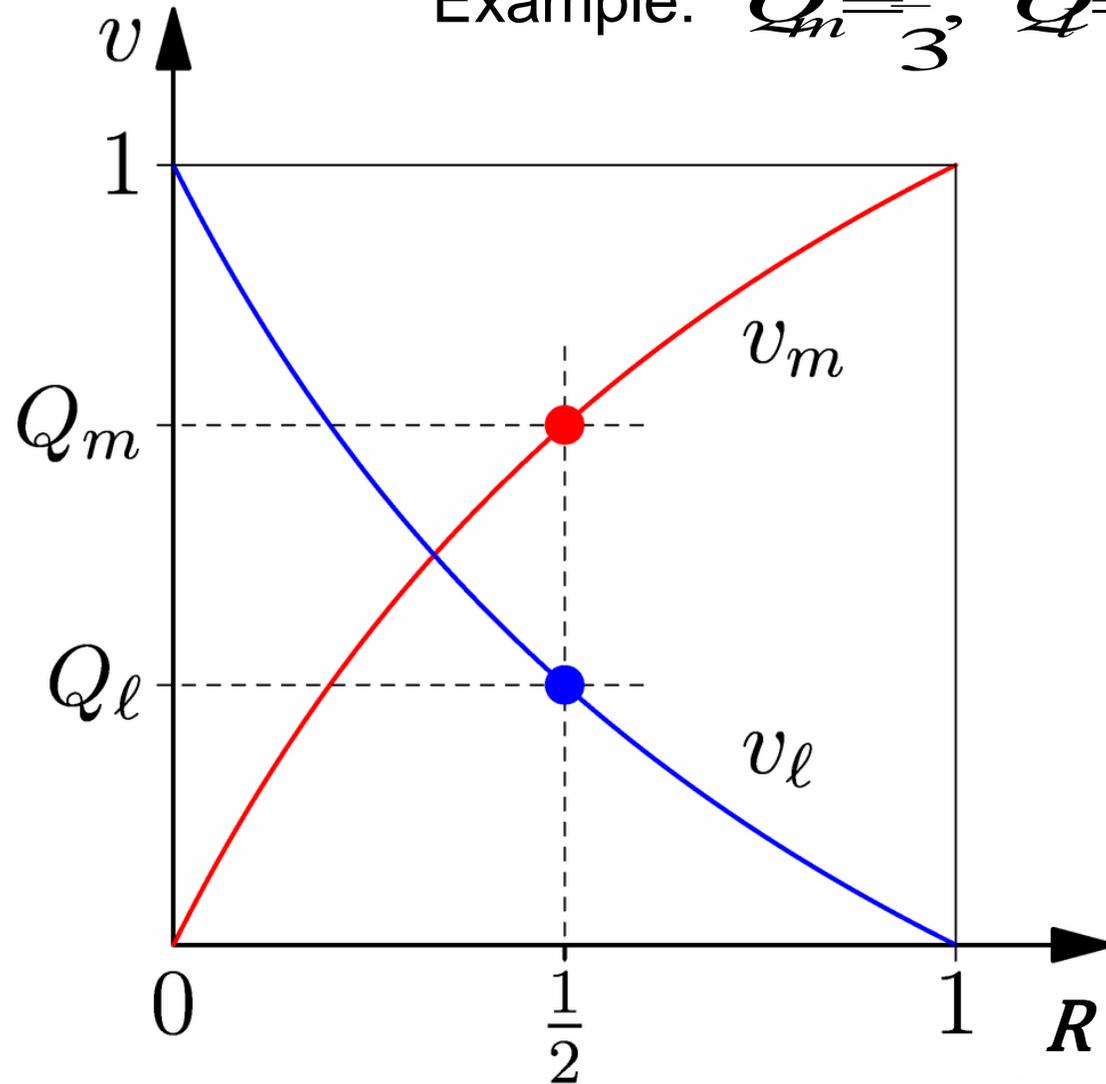
λ – Branch lineage

$$\lambda = \mathbf{R} [ax^2 + b((y + c)^2)] \quad a, b \in [0,1]; c \in [-1,1]$$

- Parameter **R** conceptualizes the relation between parent branch and child branch.
- A high value for parameter **R** favors parent branches, a low value child branches.

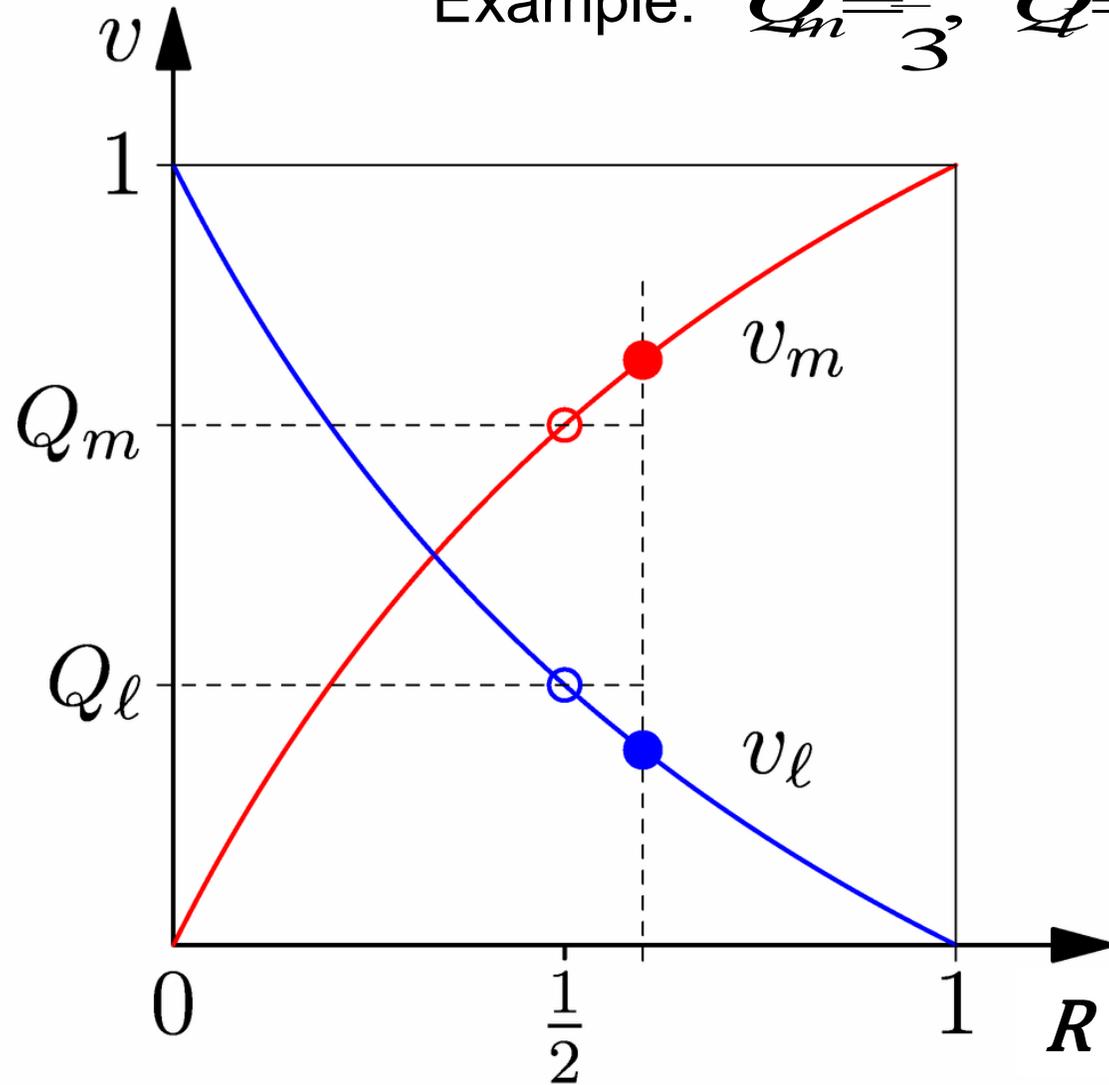
The meaning of R

Example: $Q_m = \frac{2}{3}$, $Q_l = \frac{1}{3}$



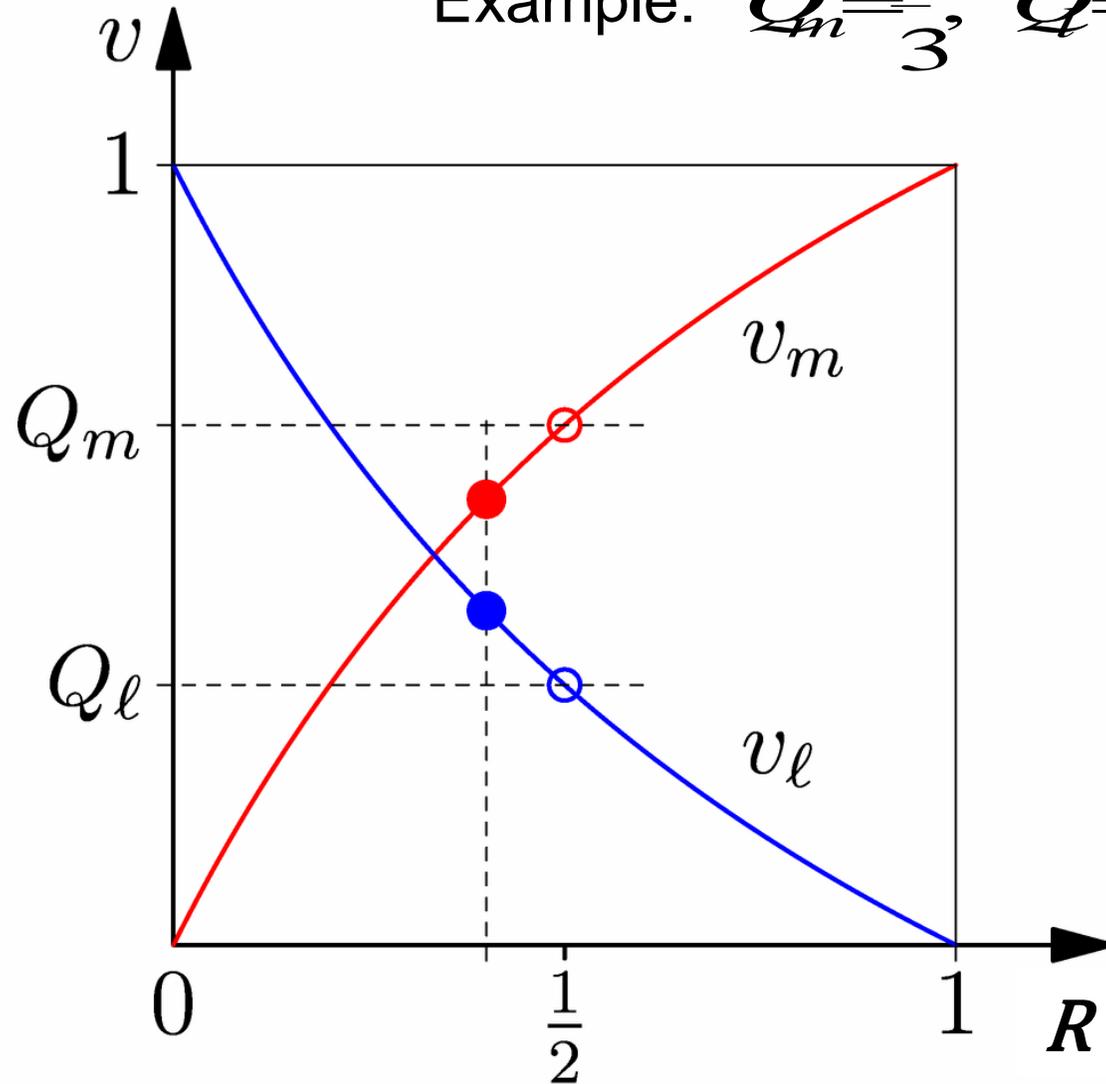
The meaning of R

Example: $Q_m = \frac{2}{3}$, $Q_l = \frac{1}{3}$

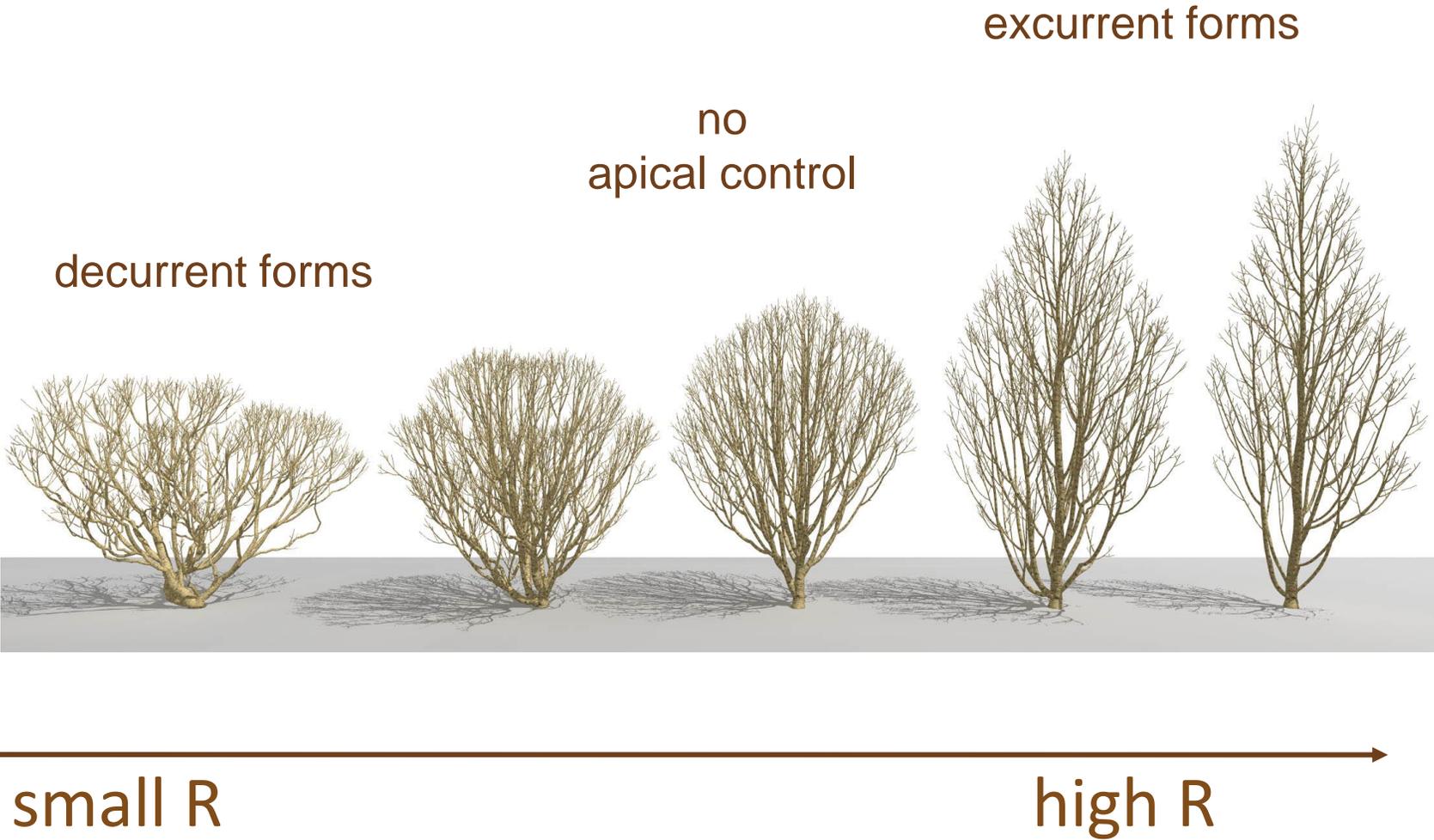


The meaning of R

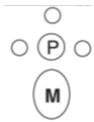
Example: $Q_m = \frac{2}{3}$, $Q_l = \frac{1}{3}$



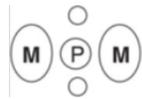
Apical control



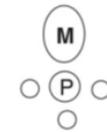
Gravimorphism



Hypotony (sympodial)



Amphitony



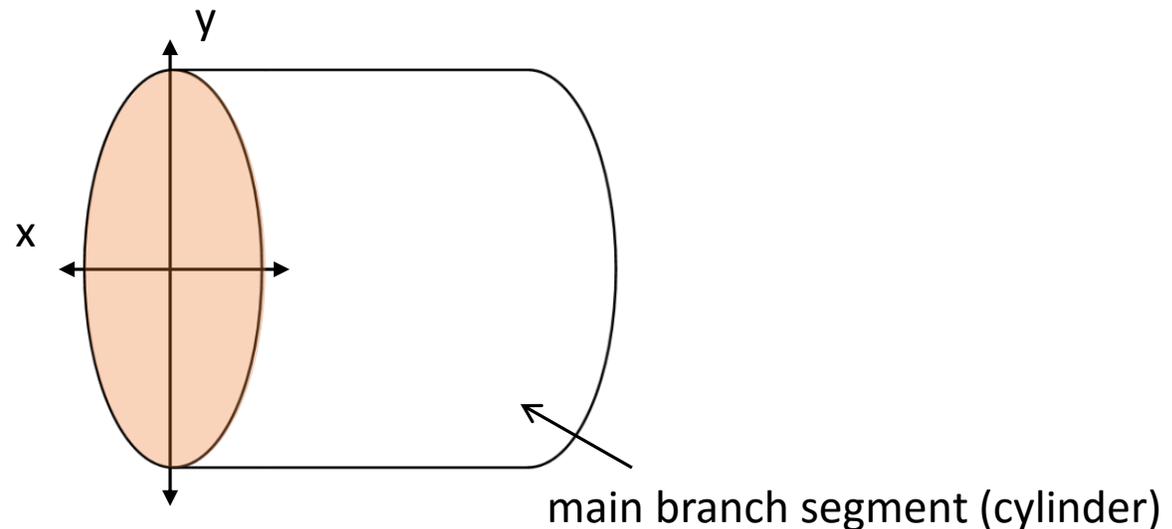
Epitony



λ – Preferential development of lateral axes
(Gravimorphism)

$$\lambda = R [ax^2 + b((y + c)^2)] \quad a, b \in [0,1]; c \in [-1,1]$$

- x and y denote the location of a lateral branch

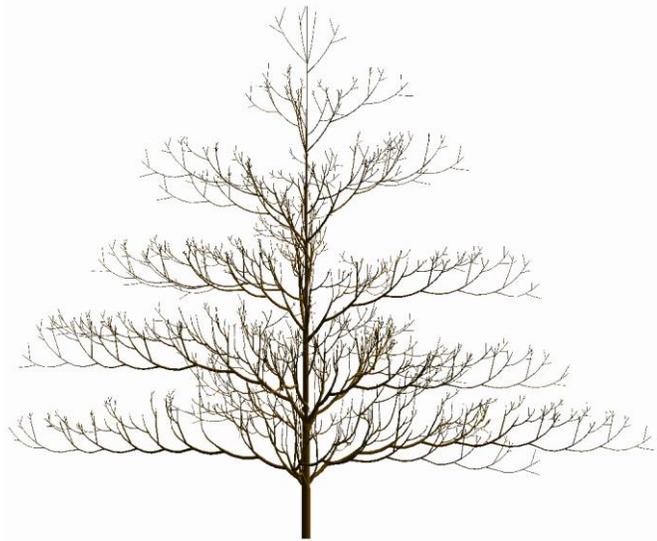


λ – Preferential development of lateral axes
(Gravimorphism)

$$\lambda = R [ax^2 + b((y + c)^2)] \quad a, b \in [0,1]; c \in [-1,1]$$

- x and y denote the location of a lateral branch
- Parameter a defines preference for buds located at the sides of a branch (**Amphitony**)
- Parameter b defines preference for buds located at the upper and lower surface of a branch
- Parameter c defines a preference for buds located at either upper or lower surface of a branch (**Epitony/Hypotony**)

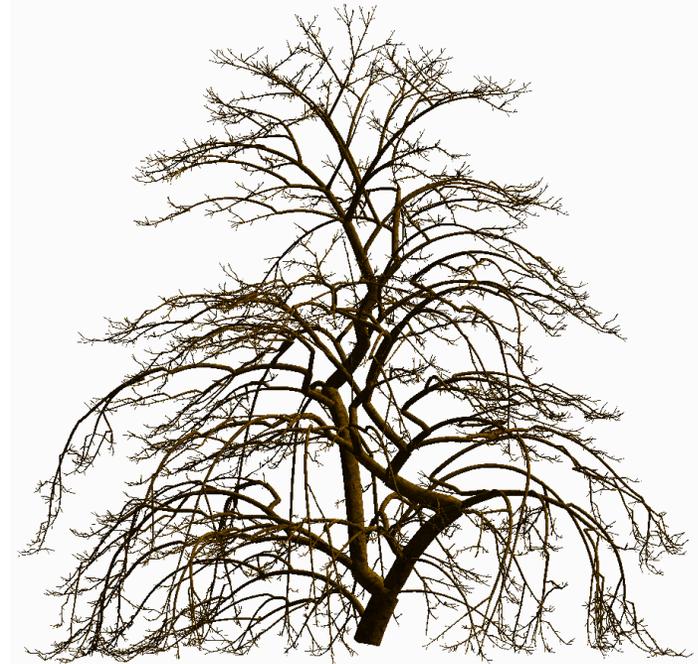
Gravimorphism - Examples



orthotropism ↑
+
hypotony (high c)



plagiotropism ↔
+
amphitony (high a)

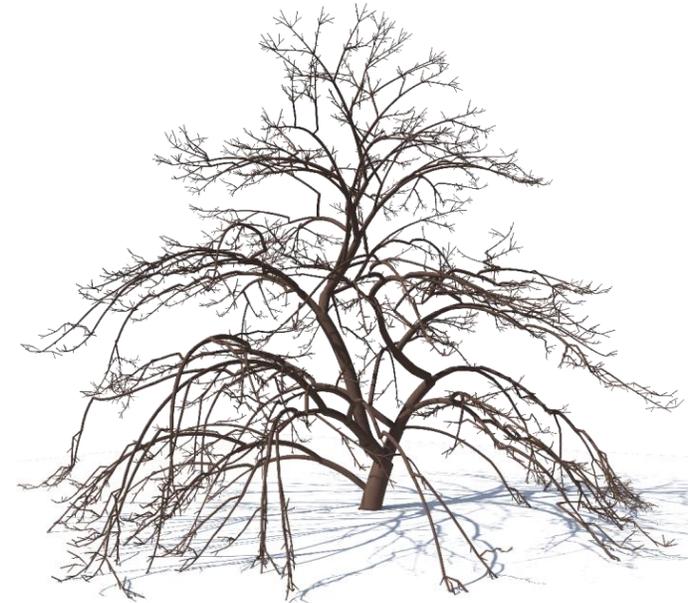


orthotropism ↓
+
epitony (low c)

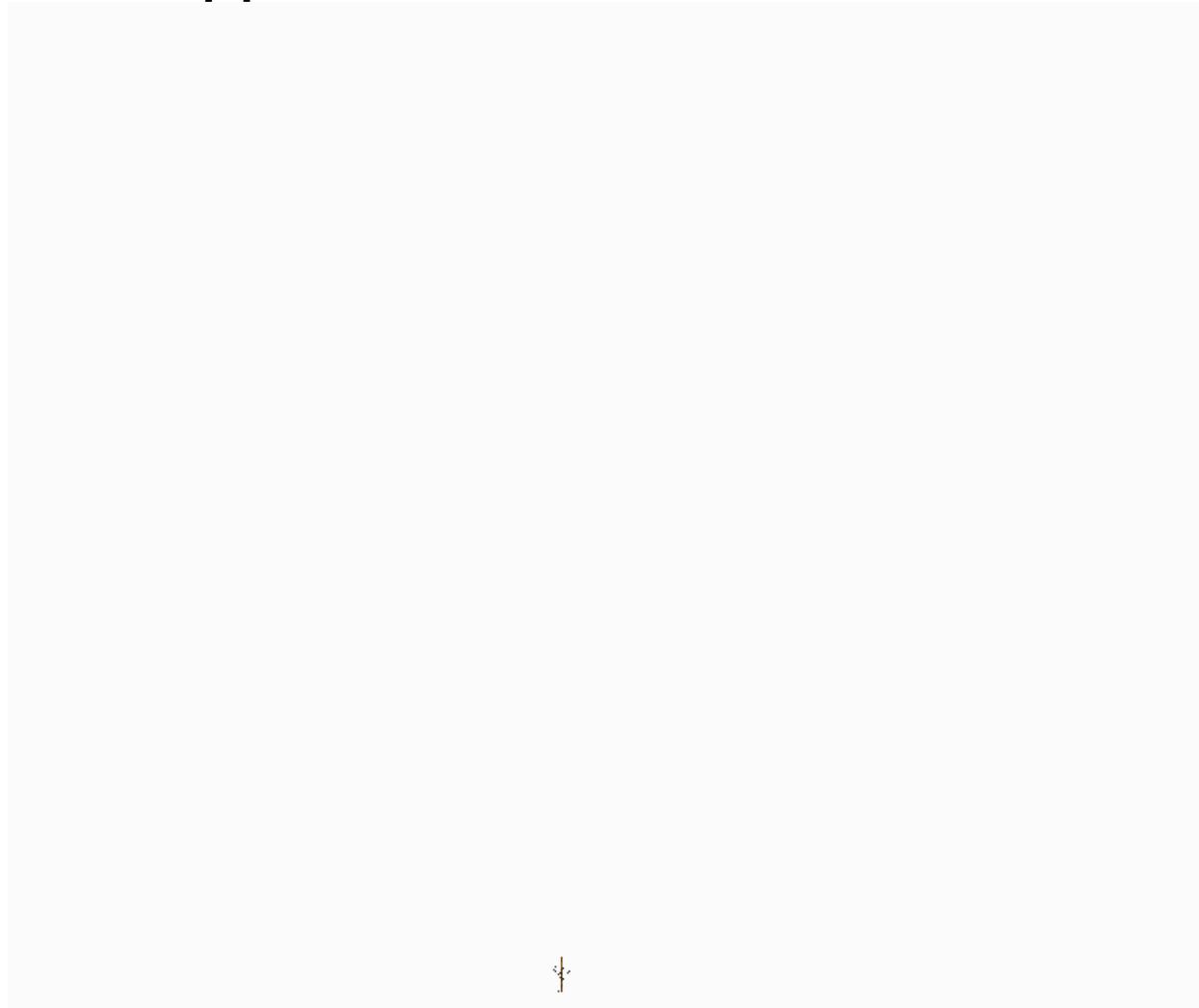


Branch Bending

- **Elasticity theory** – branches as elastic circular rods composed of isotropic and homogenous material (no stretching)
- Solve static equilibrium of gravity and forces resulting from growth
- Torque–based model



Branch Bending - Animation



Branch lineage

Gravity

Growth rhythms

Bud fate

Growth Rhythms



Branch lineage

Gravity

Growth rhythms

Bud fate

Growth Rhythms



Branch lineage

Gravity

Growth rhythms

Bud fate

Growth Rhythms



Branch lineage

Gravity

Growth rhythms

Bud fate

Growth Rhythms



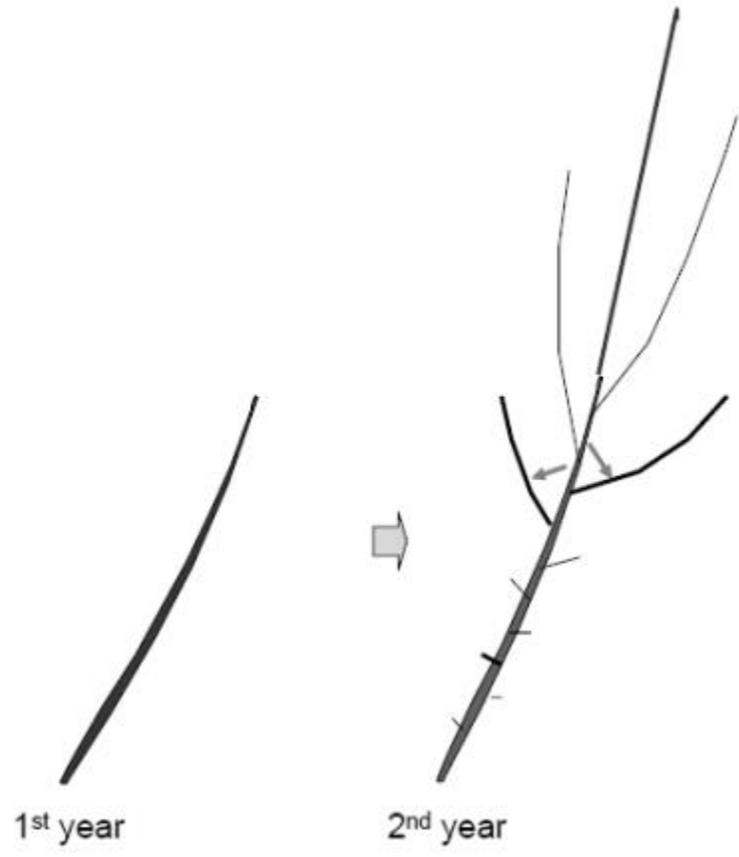
Branch lineage

Gravity

Growth rhythms

Bud fate

Proleptic growth



Growth Rhythms

- Shoot growth determined by shoot to root relation (*Borchert 1973*)



Growth Rhythms

- Shoot growth determined by shoot to root relation (*Borchert 1973*)
- Root flux R_{total} computed as a function of time

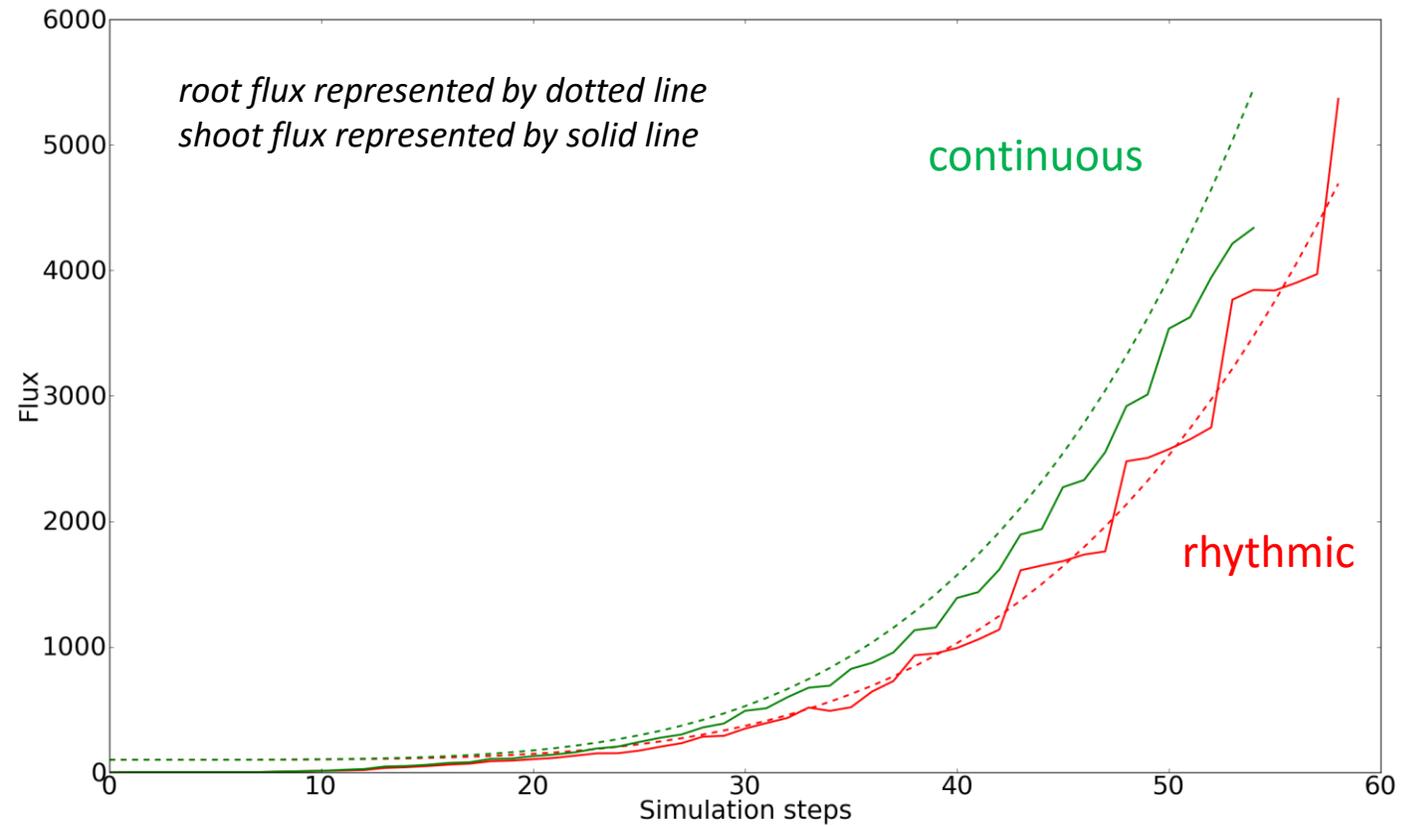
Growth Rhythms

- Shoot growth determined by shoot to root relation (*Borchert 1973*)
- Root flux R_{total} computed as a function of time
- If $Q_{total} > R_{total} \rightarrow stop\ growth$ (gradually)
 - Decrease length of branch segments
 - Increase bud activation threshold
 - Increase branch shedding threshold

Growth Rhythms

- Shoot growth determined by shoot to root relation (*Borchert 1973*)
- Root flux R_{total} computed as a function of time
- If $Q_{total} > R_{total} \rightarrow stop\ growth$ (gradually)
 - Decrease length of branch segments
 - Increase bud activation threshold
 - Increase branch shedding threshold
- If $Q_{total} \leq R_{total} \rightarrow grow$
 - Increase length of branch segments until maximum
 - Decrease bud activation until minimum
 - Decrease branch shedding threshold

Growth Rhythms - Example

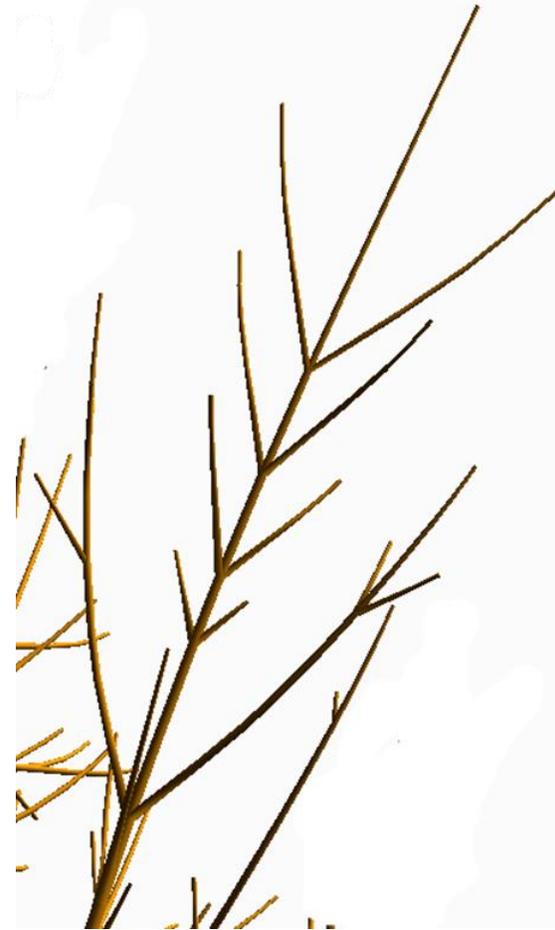


Root and shoot flux development in continuous example and rhythmic example

Growth Rhythms - Close up



Continuous development



Rhythmic development

Bud Fate – Dormant, Flowering and Active



- Buds which satisfy the inequality **flux < flowering threshold** become a flower and are removed from the simulation
- Threshold affects only **active buds**



Before calculating flux

Bud Fate – Dormant, Flowering and Active

- Buds which satisfy the inequality **flux < flowering threshold** become a flower and are removed from the simulation
- Threshold affects only **active buds**



Active bud



Dormant bud



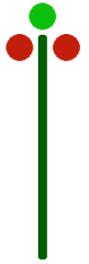
Before calculating flux



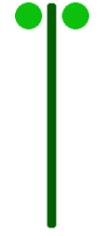
After calculating flux

Bud Fate – Dormant, Flowering and Active

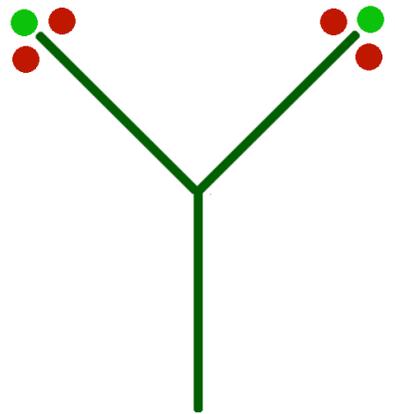
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Before calculating flux

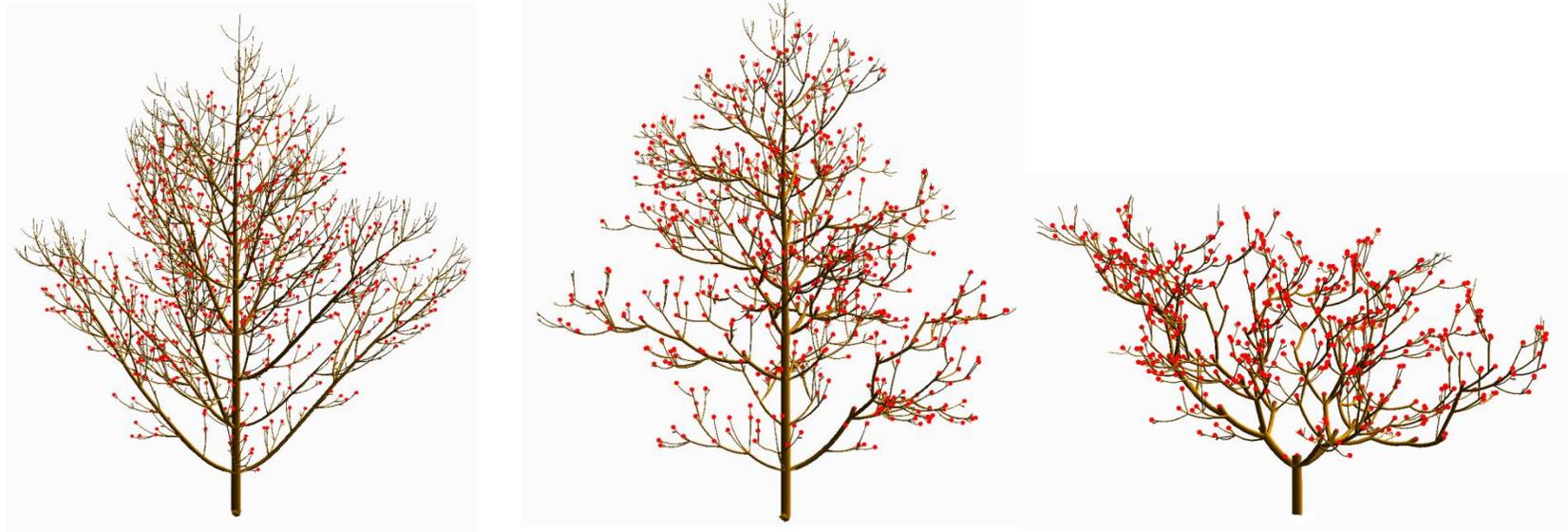


After calculating flux



After growth phase

Flowering and Architectural Models



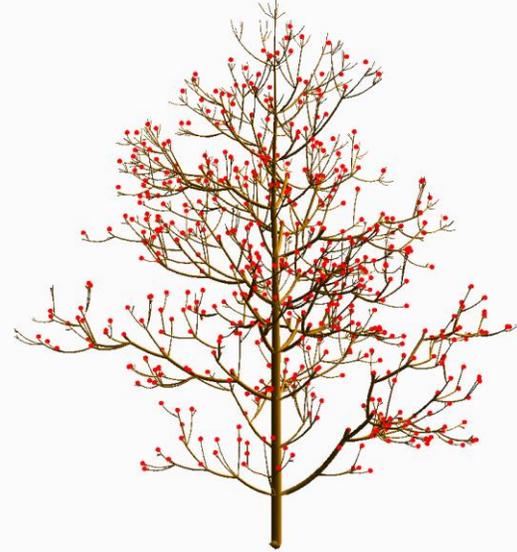
Flowering threshold

(pronounced parent child bias)

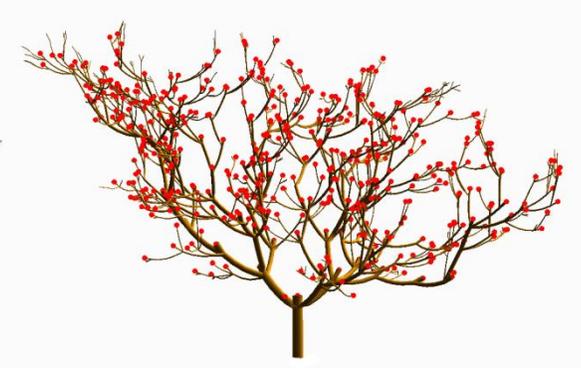
Flowering and Architectural Models



Rauh



Scarrone



Leeuwenberg



Flowering threshold

(pronounced parent child bias)

Plagiotropy and Architectural Models



Rauh

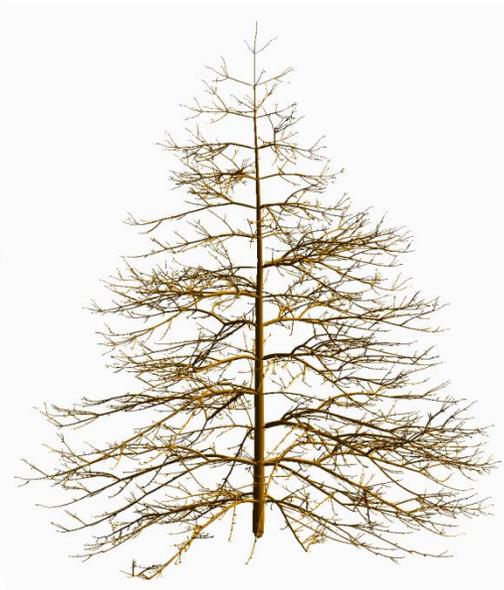


Plagiotropy threshold
(pronounced parent child bias)

Plagiotropny and Architectural Models



Rauh



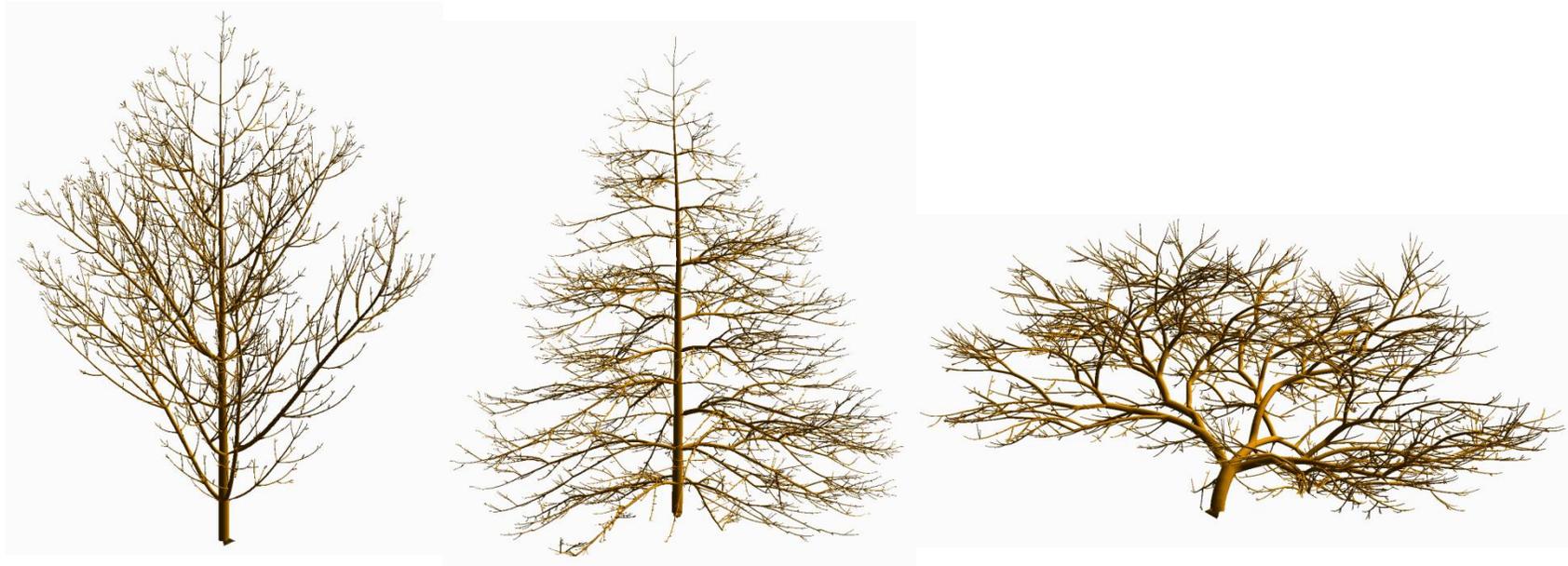
Massart



Plagiotropny threshold

(pronounced parent child bias)

Plagiotropy and Architectural Models



Rauh

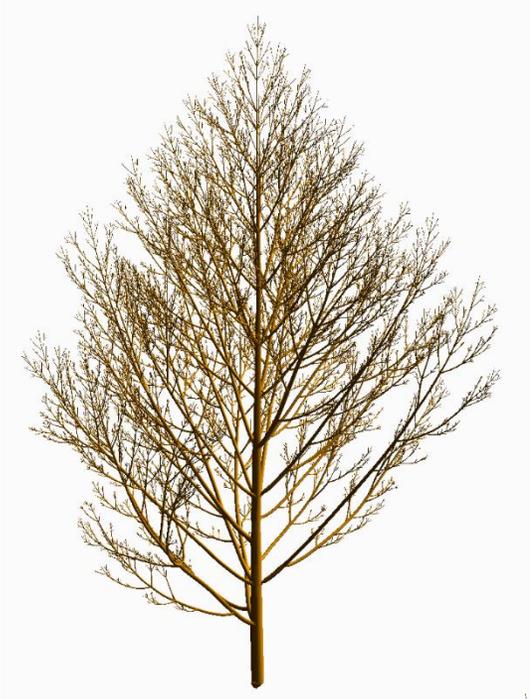
Massart

Troll

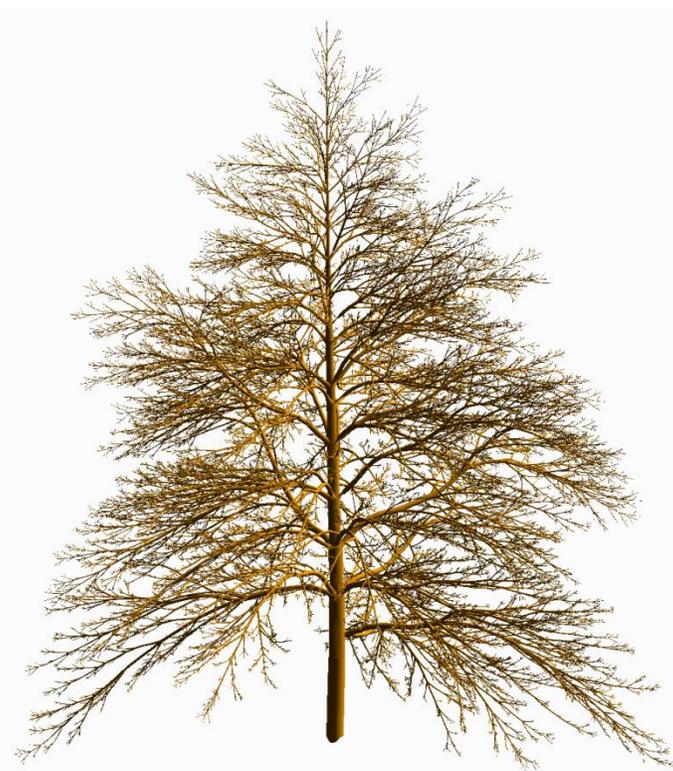


Plagiotropy threshold

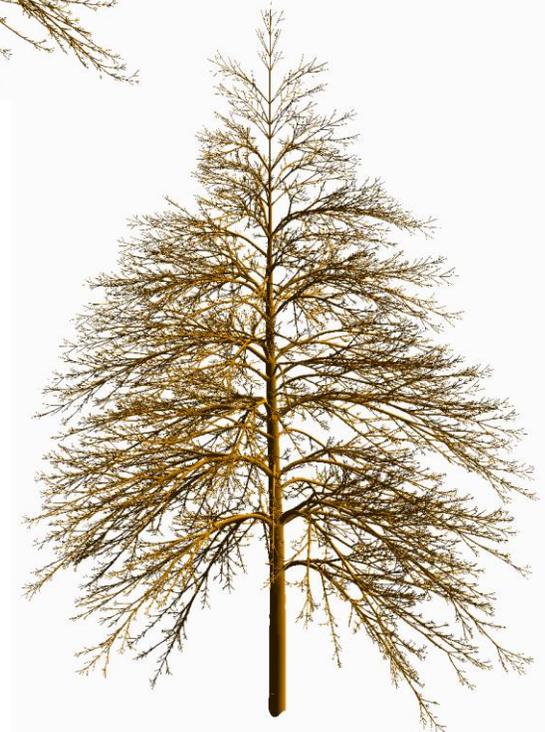
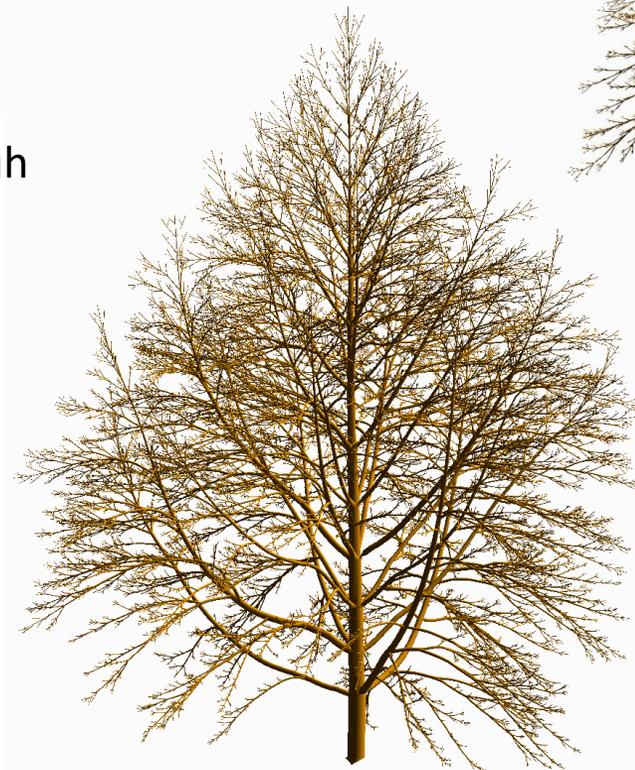
(pronounced parent child bias)



Rauh



Massart



Morphospace containing
the Architectural Models

Plagiotropy x Flowering

Troll

Massart

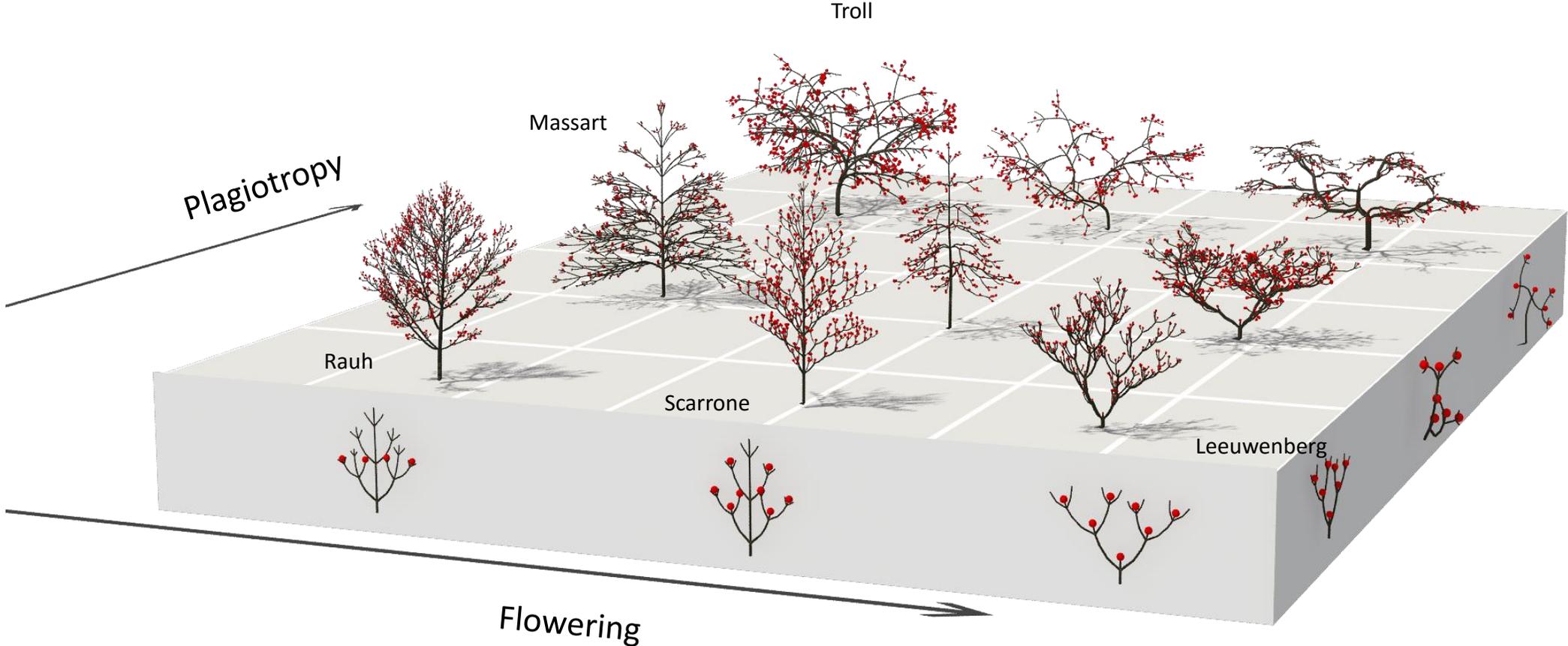
Rauh

Scarrone

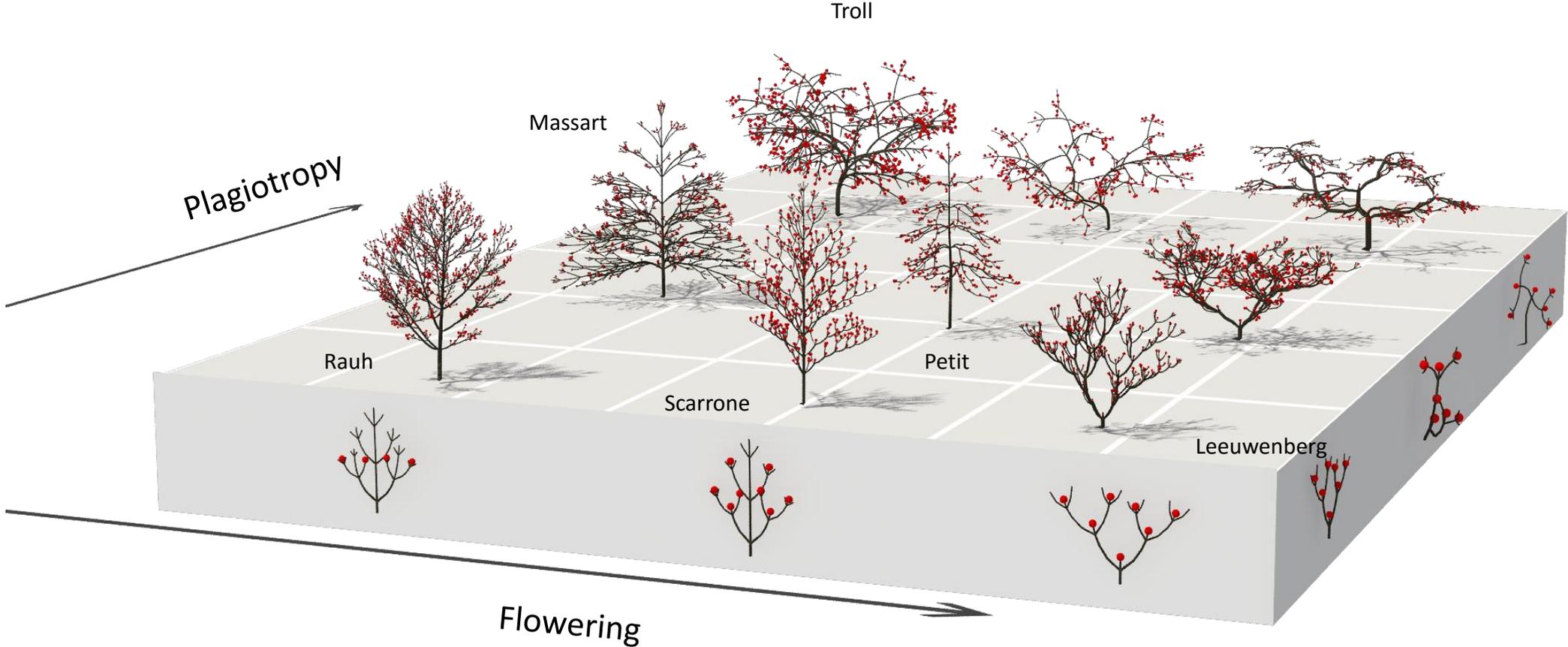
Leeuwenberg

Plagiotropy

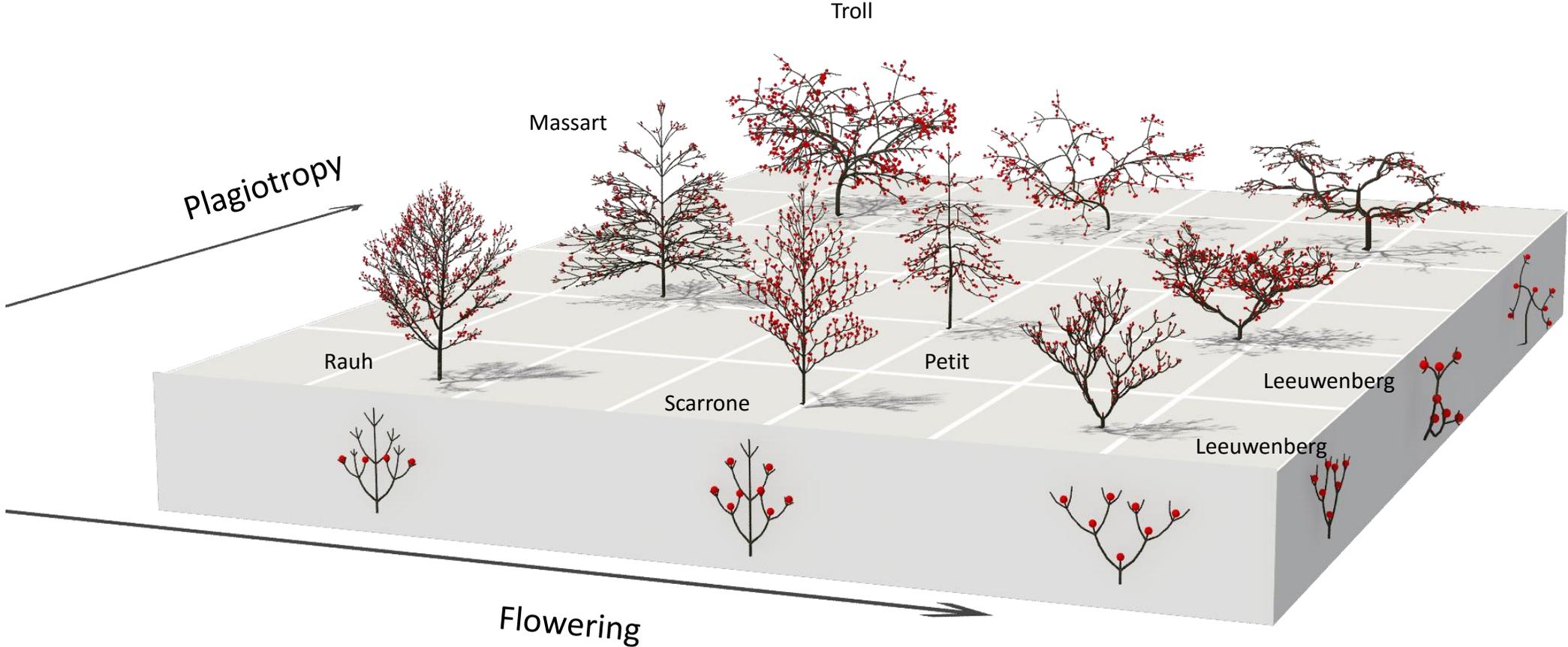
Flowering



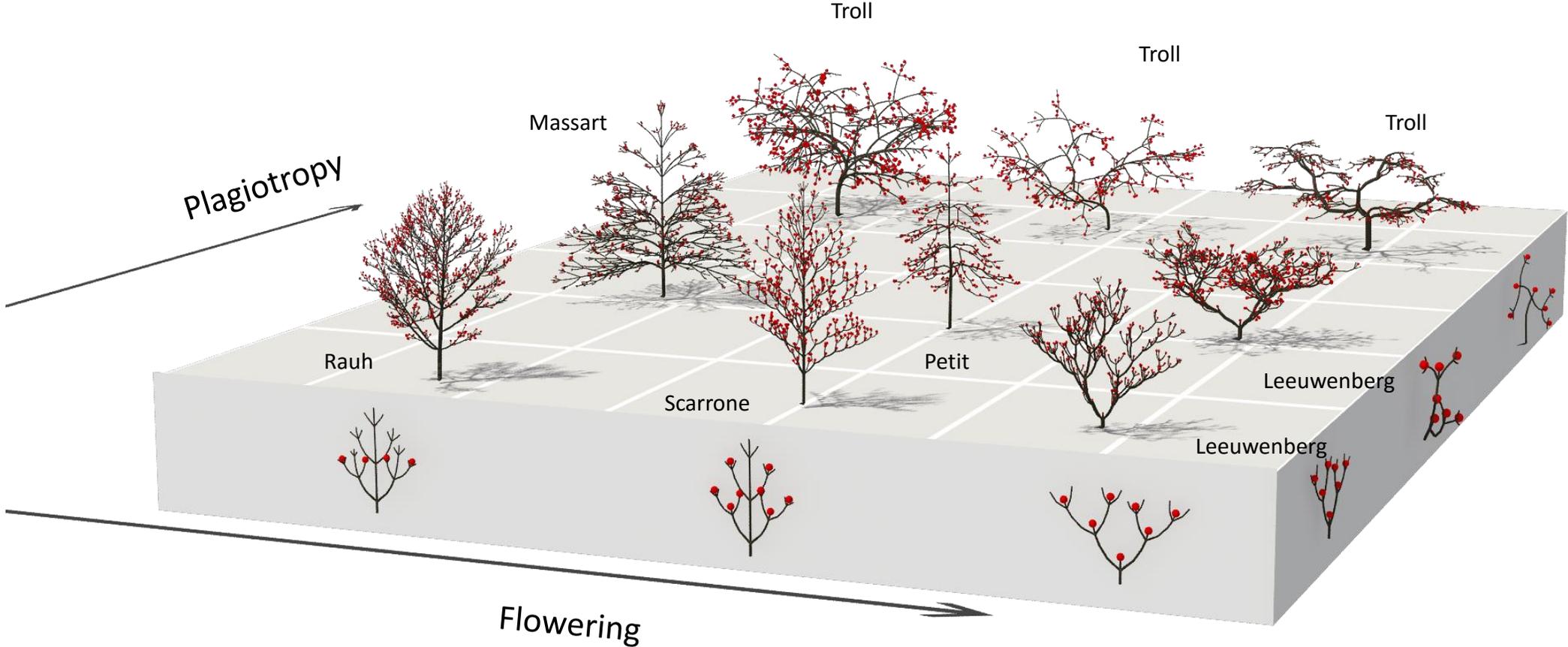
Plagiotropy x Flowering



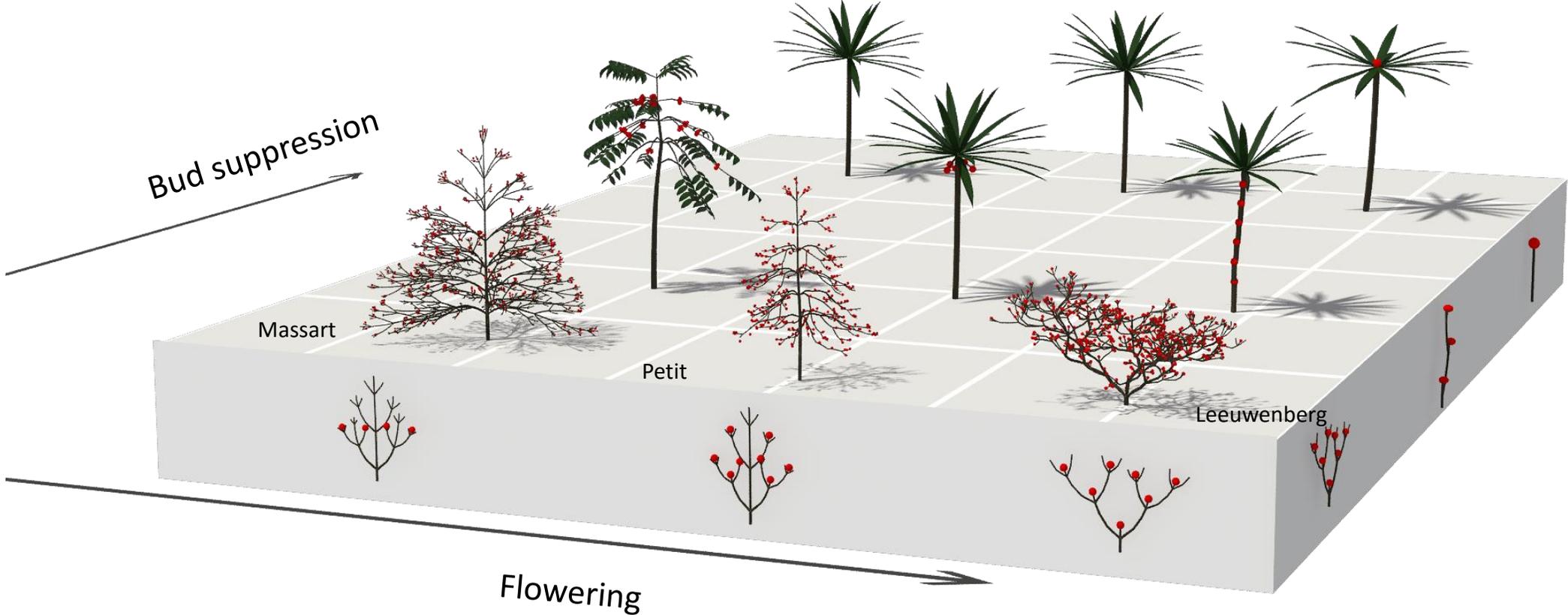
Plagiotropy x Flowering



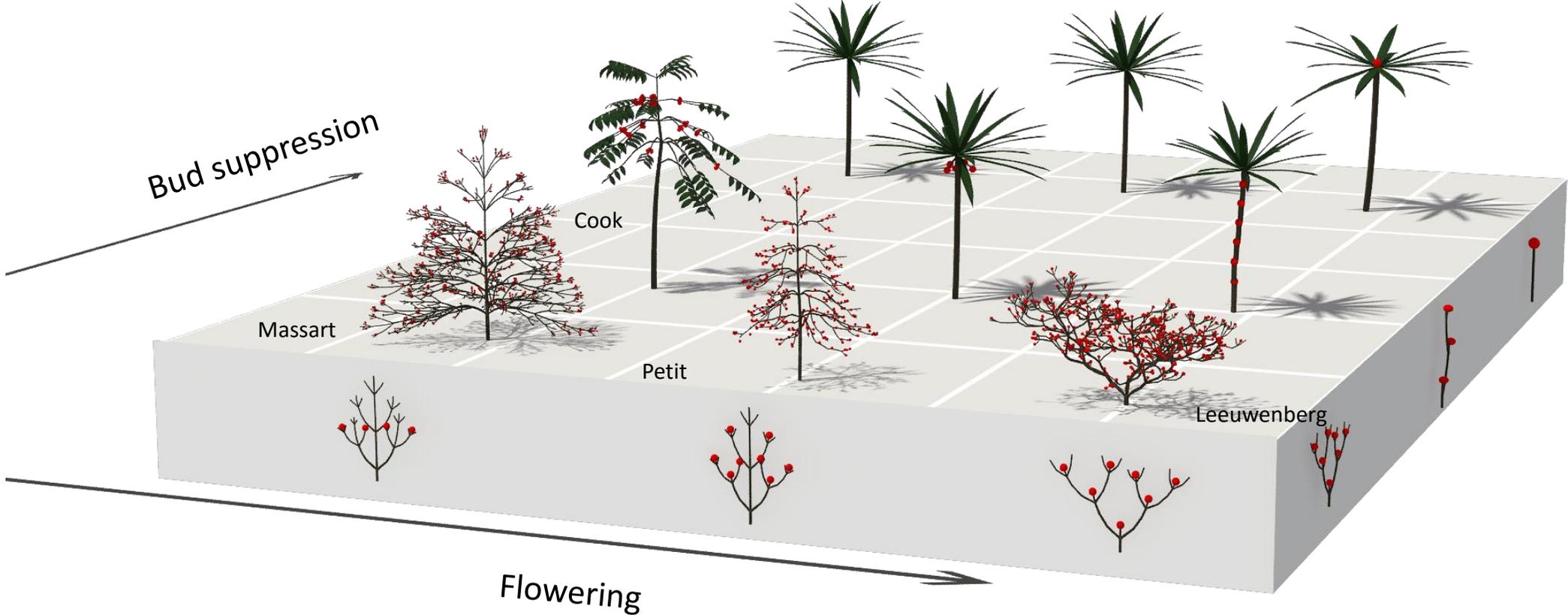
Plagiotropy x Flowering



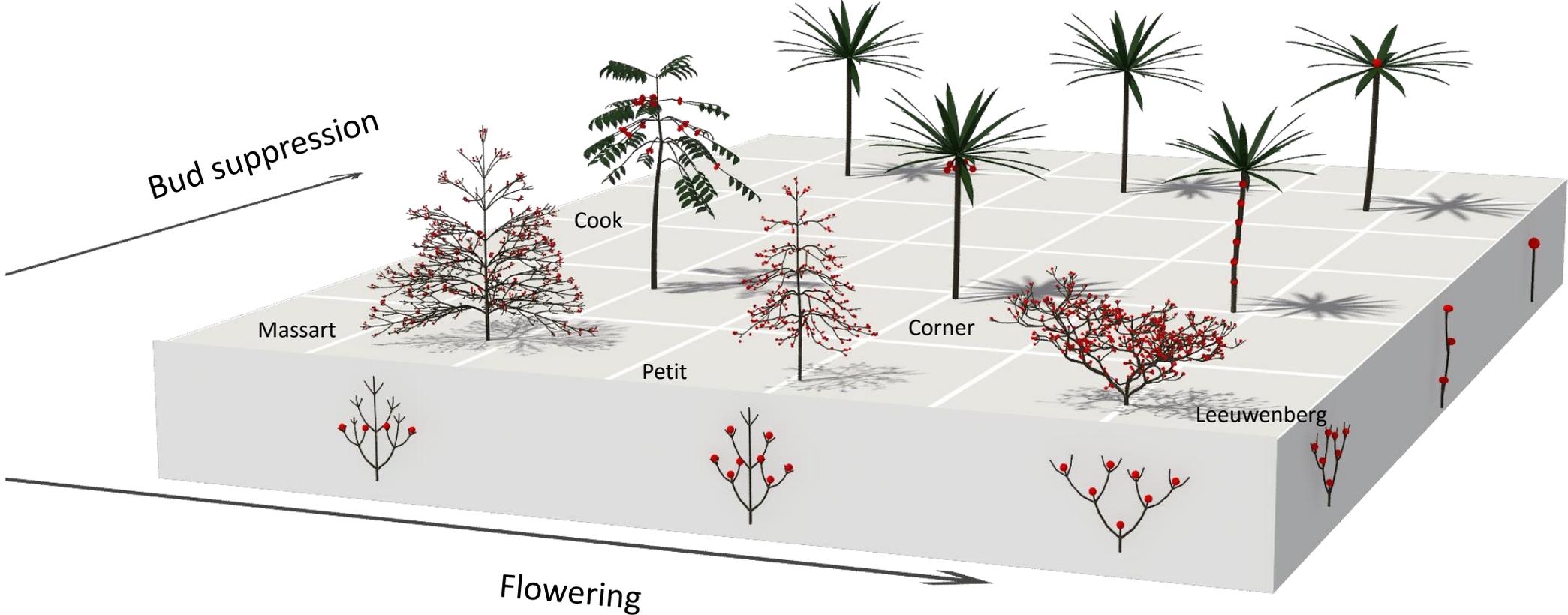
Bud Suppression x Flowering



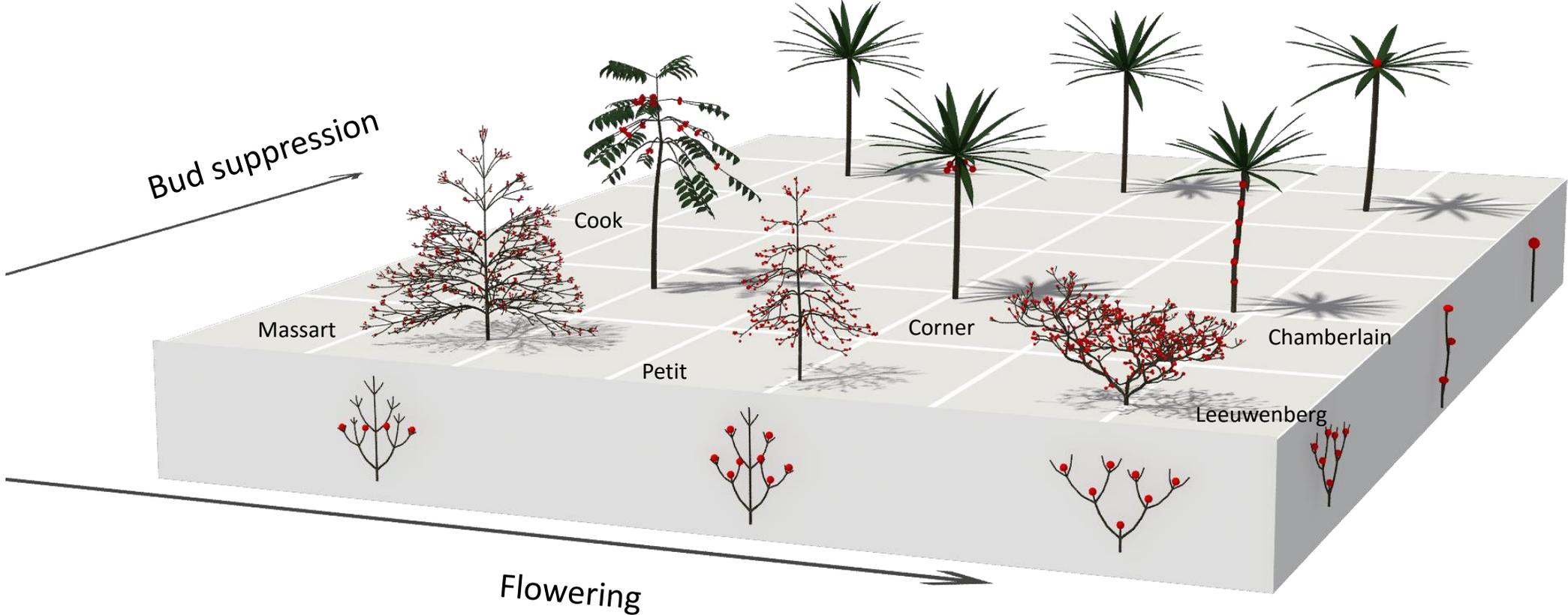
Bud Suppression x Flowering



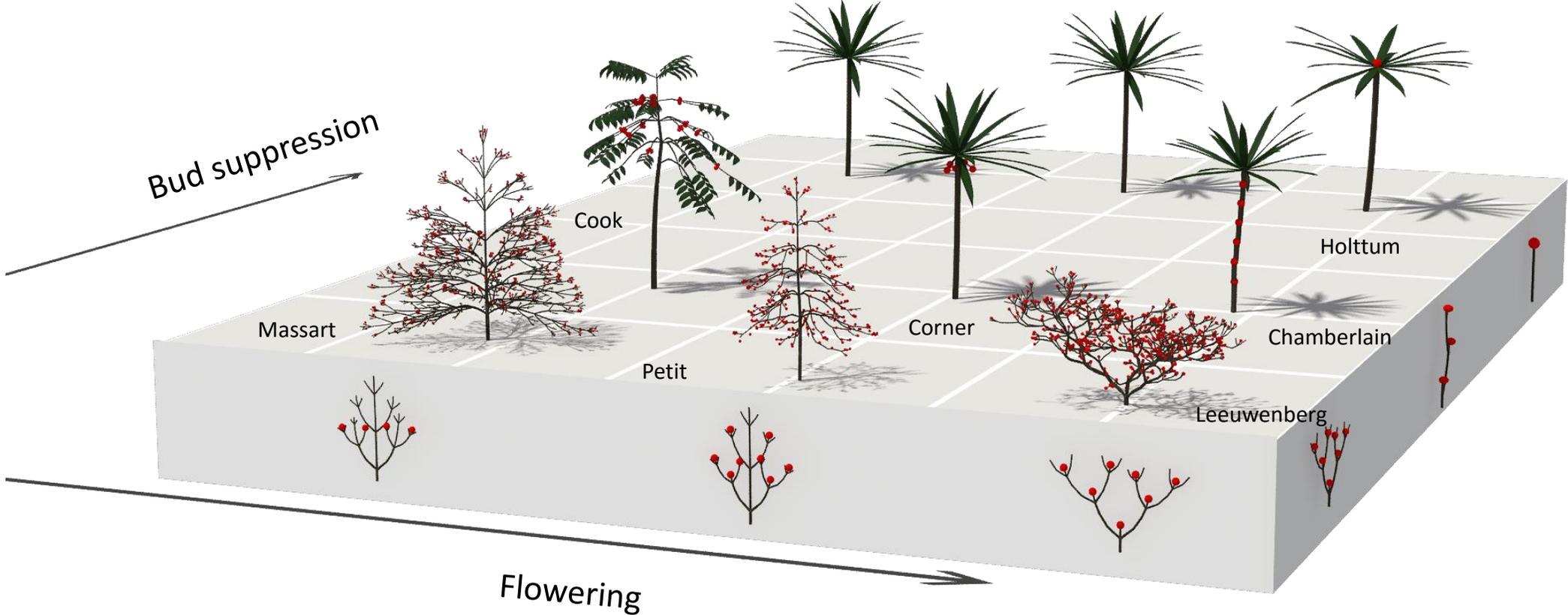
Bud Suppression x Flowering



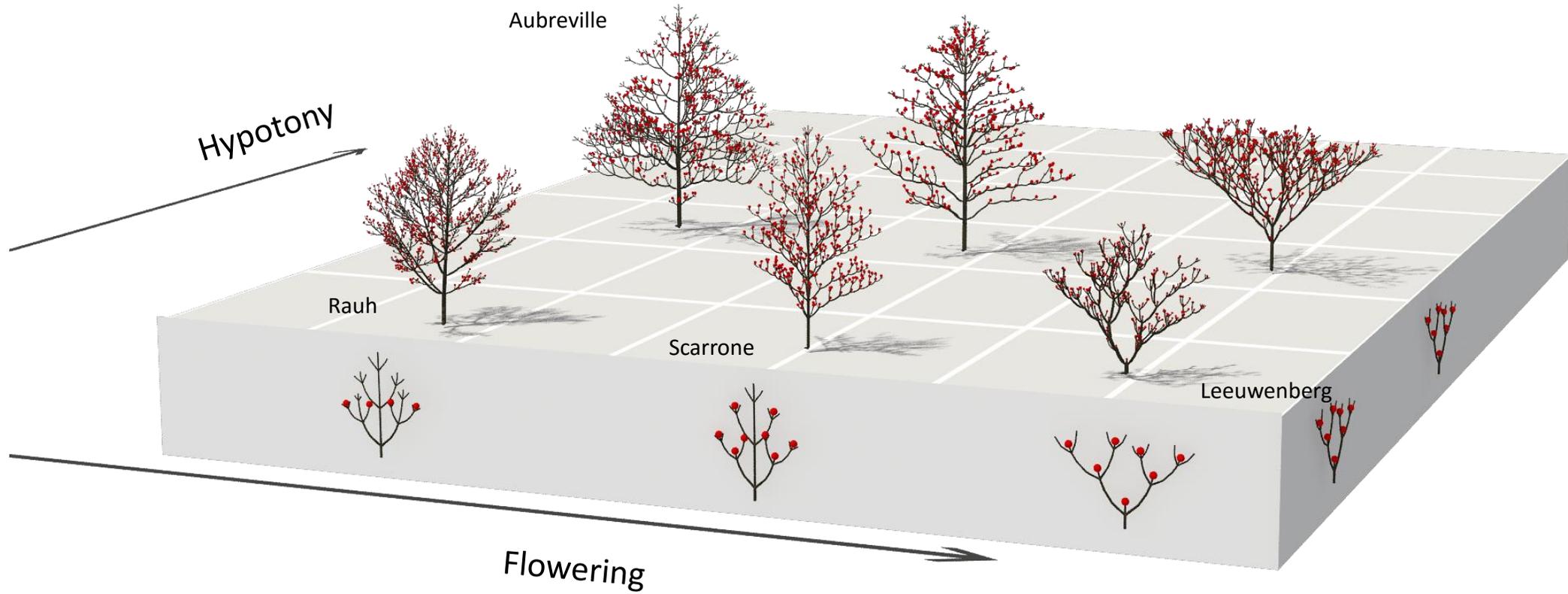
Bud Suppression x Flowering



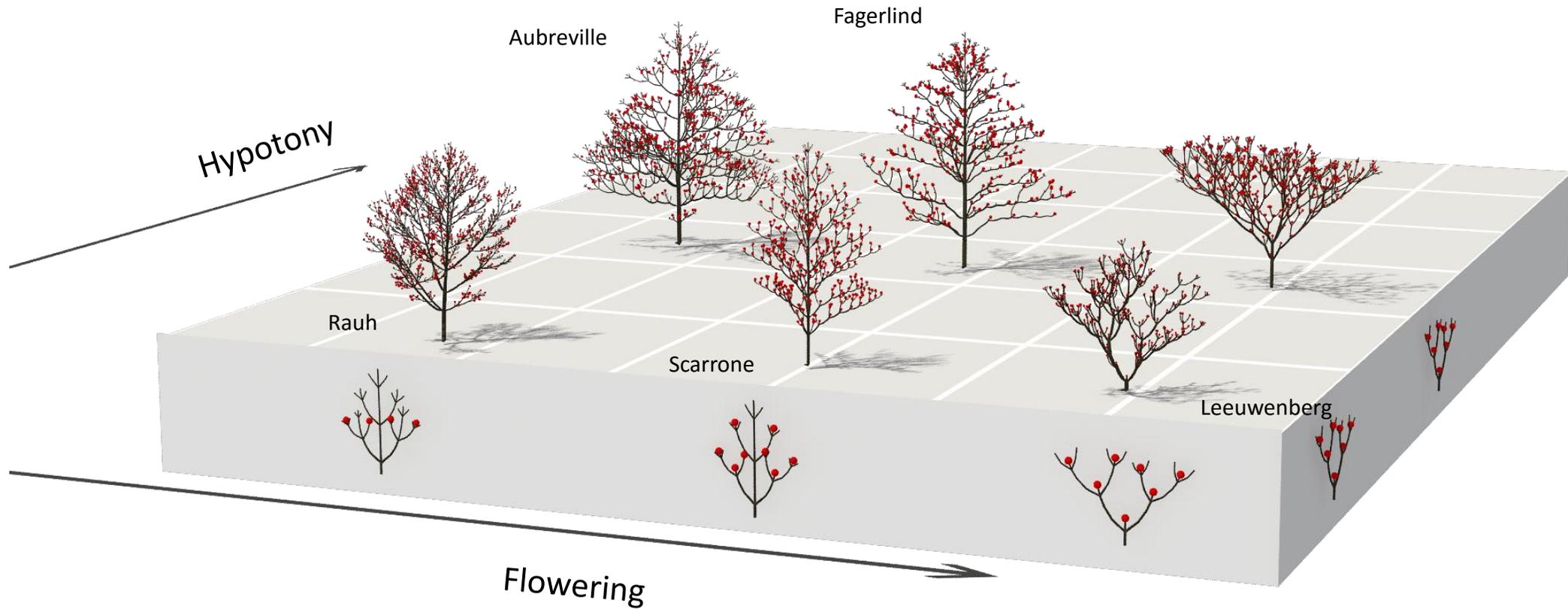
Bud Suppression x Flowering



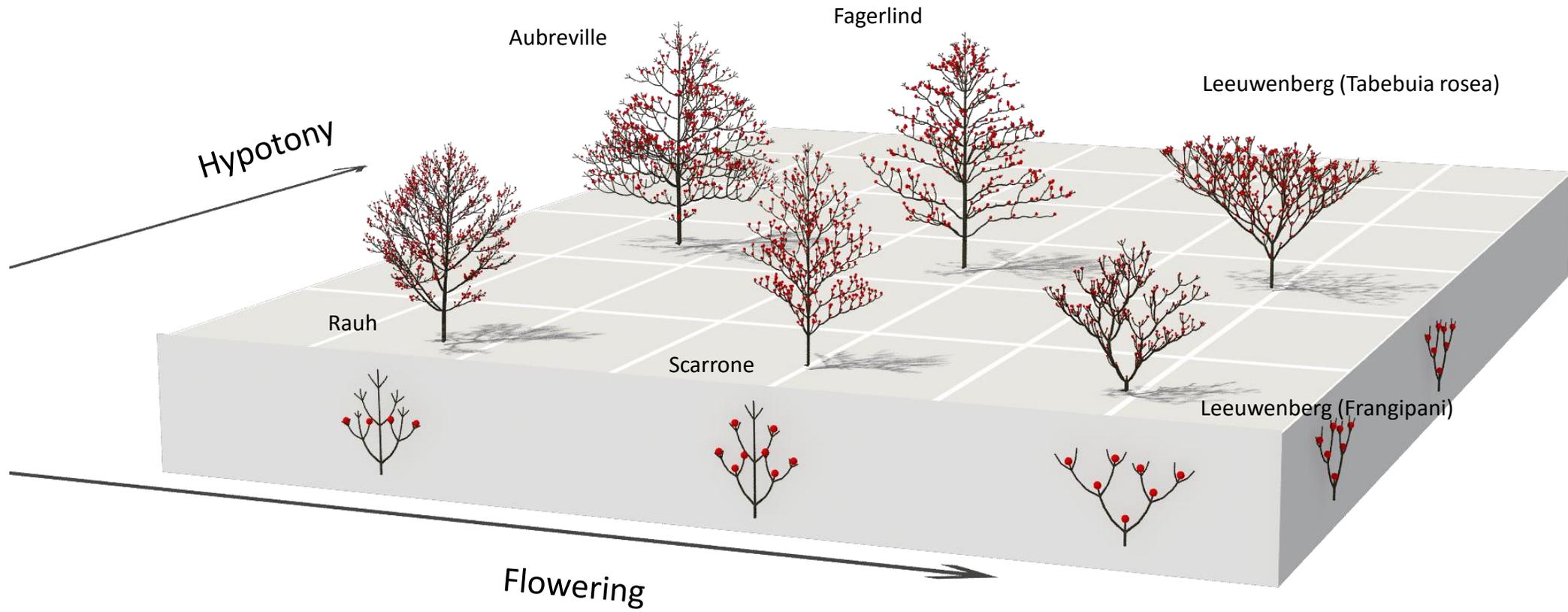
Hypotony x Flowering



Hypotony x Flowering



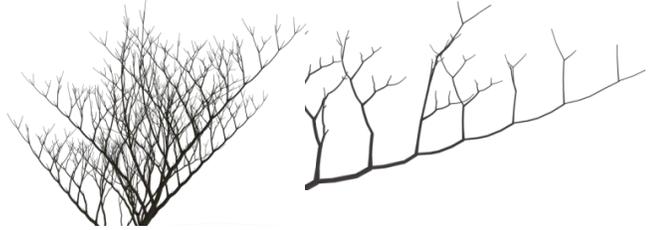
Hypotony x Flowering



Observations

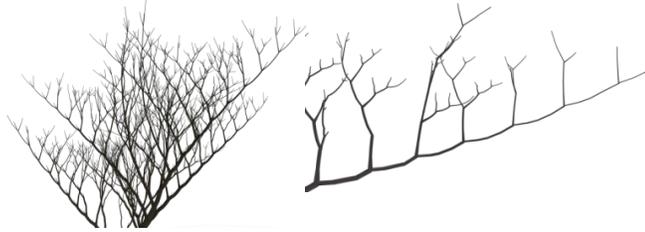
- 20 Architectural Models are captured with the growth model
- Self-organization is fundamental for understanding plant architecture
- Highlights the key plant processes to describe a variety of different tree forms, presented in a mechanistic model of development

Comparison to real trees



Tabebuia rosea (Model of Leeuwenberg)

Comparison to real trees

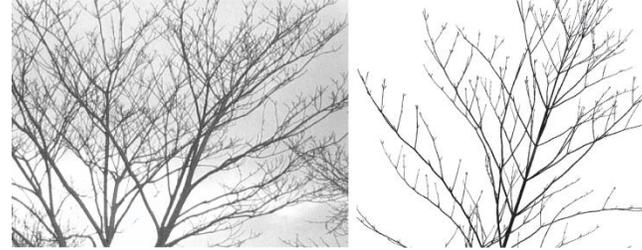


Tabebuia rosea (Model of Leeuwenberg)



Sequoia sempervirens (Model of Massart)

Comparison to real trees



Phellodendron chinense (Model of Scarrone)

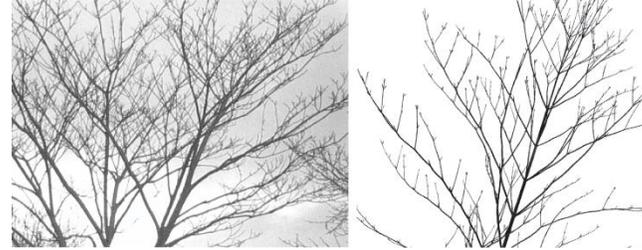


Tabebuia rosea (Model of Leeuwenberg)



Sequoia sempervirens (Model of Massart)

Comparison to real trees



Phellodendron chinense (Model of Scarrone)



Tabebuia rosea (Model of Leeuwenberg)



Delonix regia (Model of Troll)



Sequoia sempervirens (Model of Massart)





L-Systemy: modelowanie struktur drzewiastych

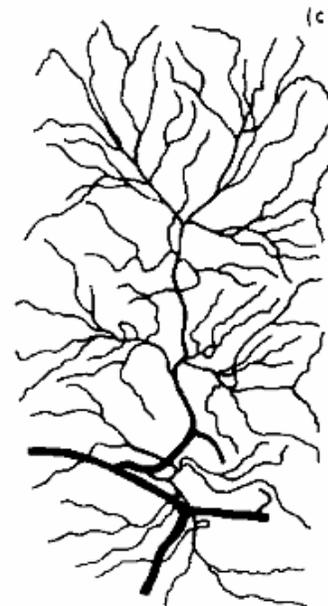
- D0L-Systems
- Parametric L-Systems
- Stochastic L-Systems
- Context-sensitive L-Systems
- Modeling in 1(.5) dimensions



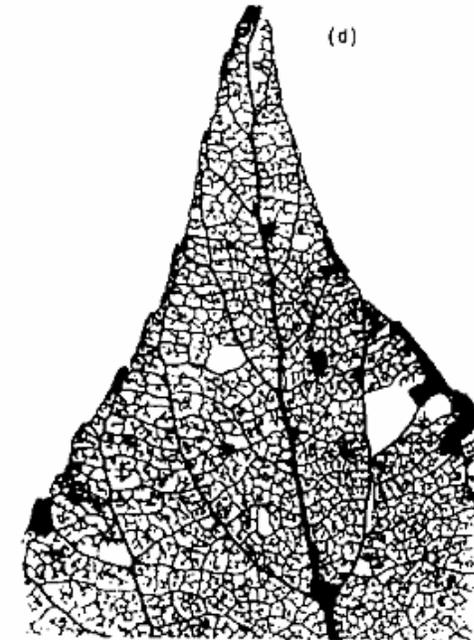
komórki nerwowe



naczynie krwionośne

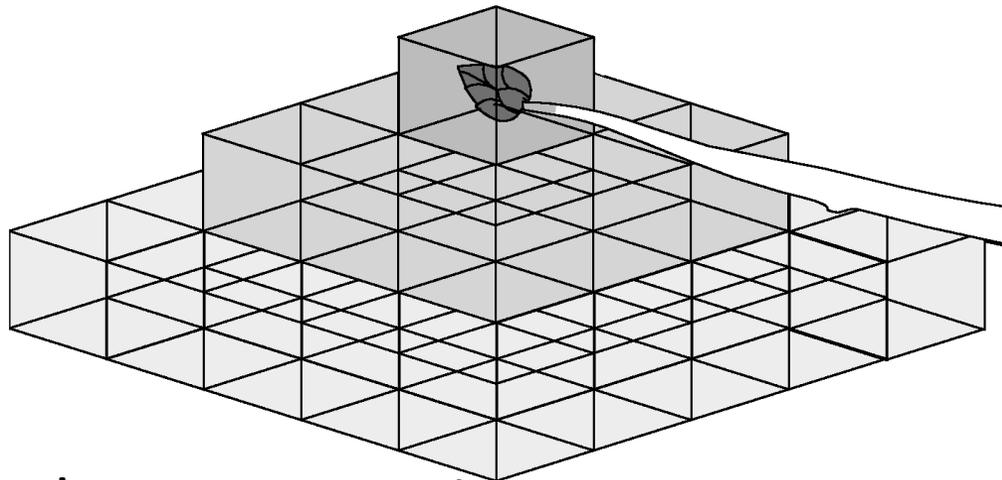


tchawica



tkanka przewodząca

Implementacja Voxel Space + L-System

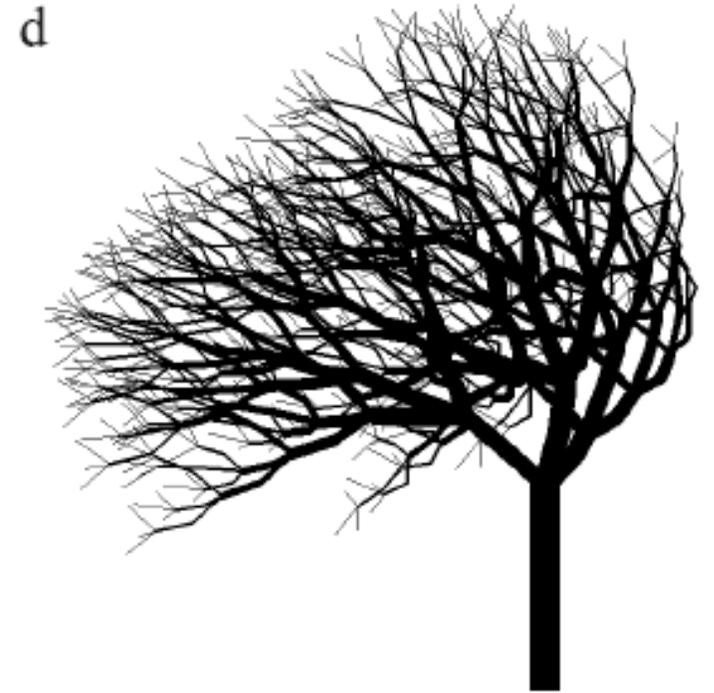
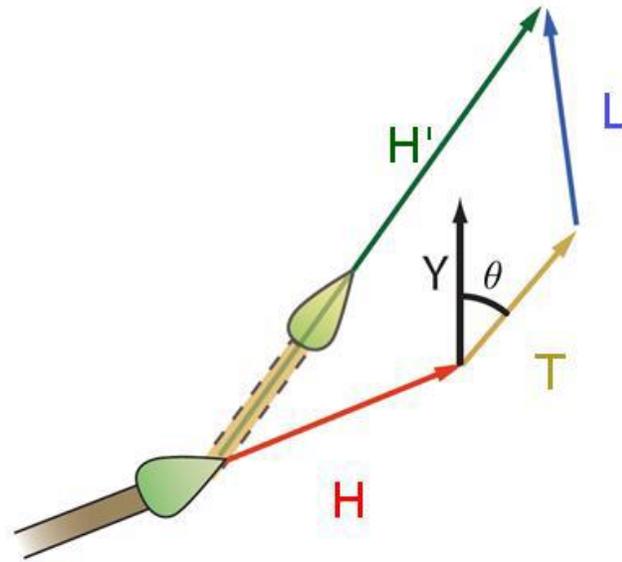


Shadow propagation



Tropizmy

- Przedstawić co to są tropizmy i jakie botanika opisuje
- Zastanowić jak wyrazić matematycznie tropizmy
- Stworzyć w L-Py interaktywna aplikacje ilustrującą tropizmy



Implementacja wzrostu rytmicznego



Continuous
development



Rhythmic
development



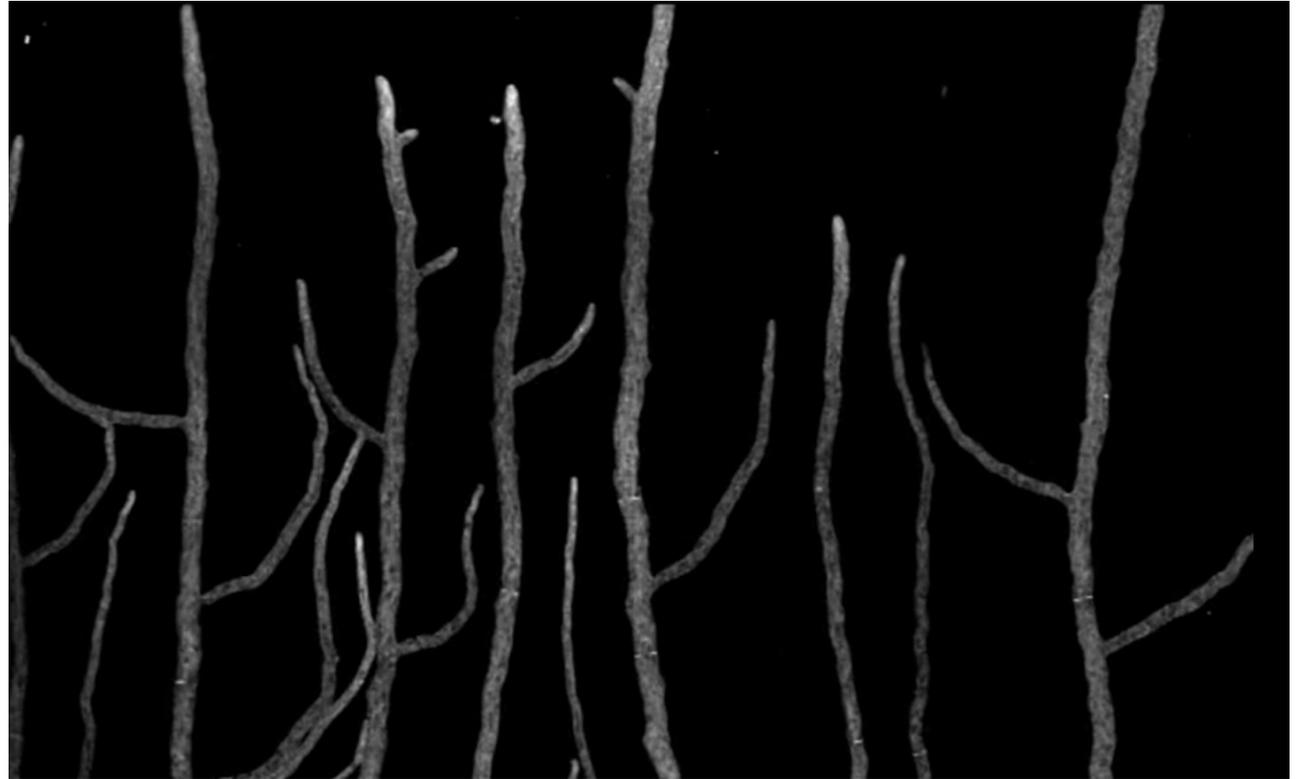
Widowiskowa symulacja wzrostu kwiatu, oparta o przepływ informacji (context-sensitive)

- Stworzyć hipotezę na model hormonalny który kontroluje rozwój jakiegoś kwiatu
- Zaimplementować ja jako L-System

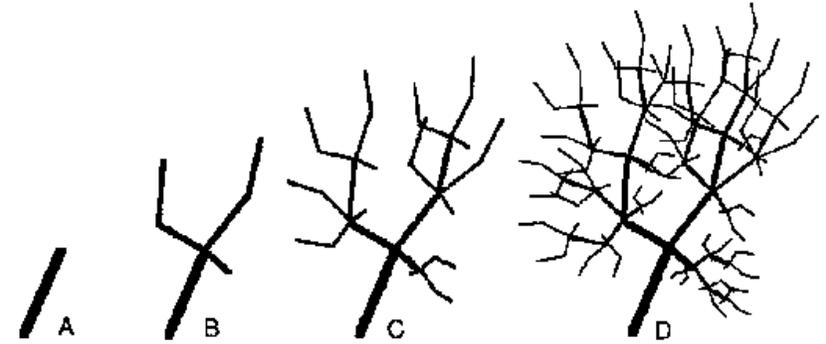


Model Mycelium – gałęzie omijają się 2D

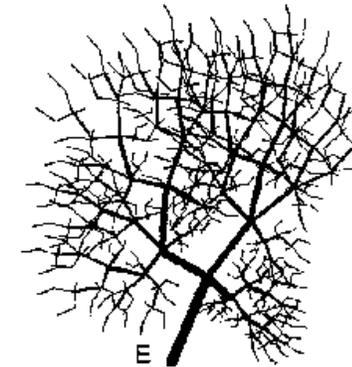
- Jak matematycznie obsługiwać kolizje pomiędzy gałęziami



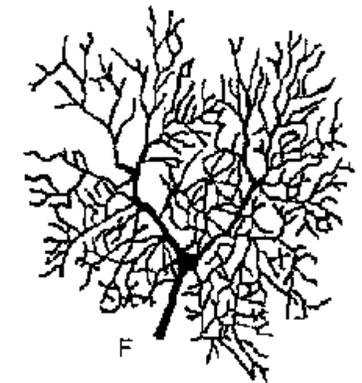
Model rozwoju neuronów



-http://fractogene.com/89_fractal/89_fractal.html
-McCormick, Bruce H.; Mulchandani, K., L-system modeling of neurons, Proceedings of the SPIE, Volume 2359



FRACTAL PURKINJE MODEL



GUINEA PIG PURKINJE CELL

Projekty

1. Samo-organizacja gałęzi w przestrzeni + model drzewa (voxel space)
2. Model z interaktywnymi tropizmami
3. Implementacja wzrostu rytmicznego (symulacja wzrostu korzeni 3D)
4. Widowiskowa symulacja wzrostu kwiatu
5. Modelowanie Mycelium w 2D gdzie gałęzie omijają się
6. Model rozwoju neuronów
7. Własna propozycja...